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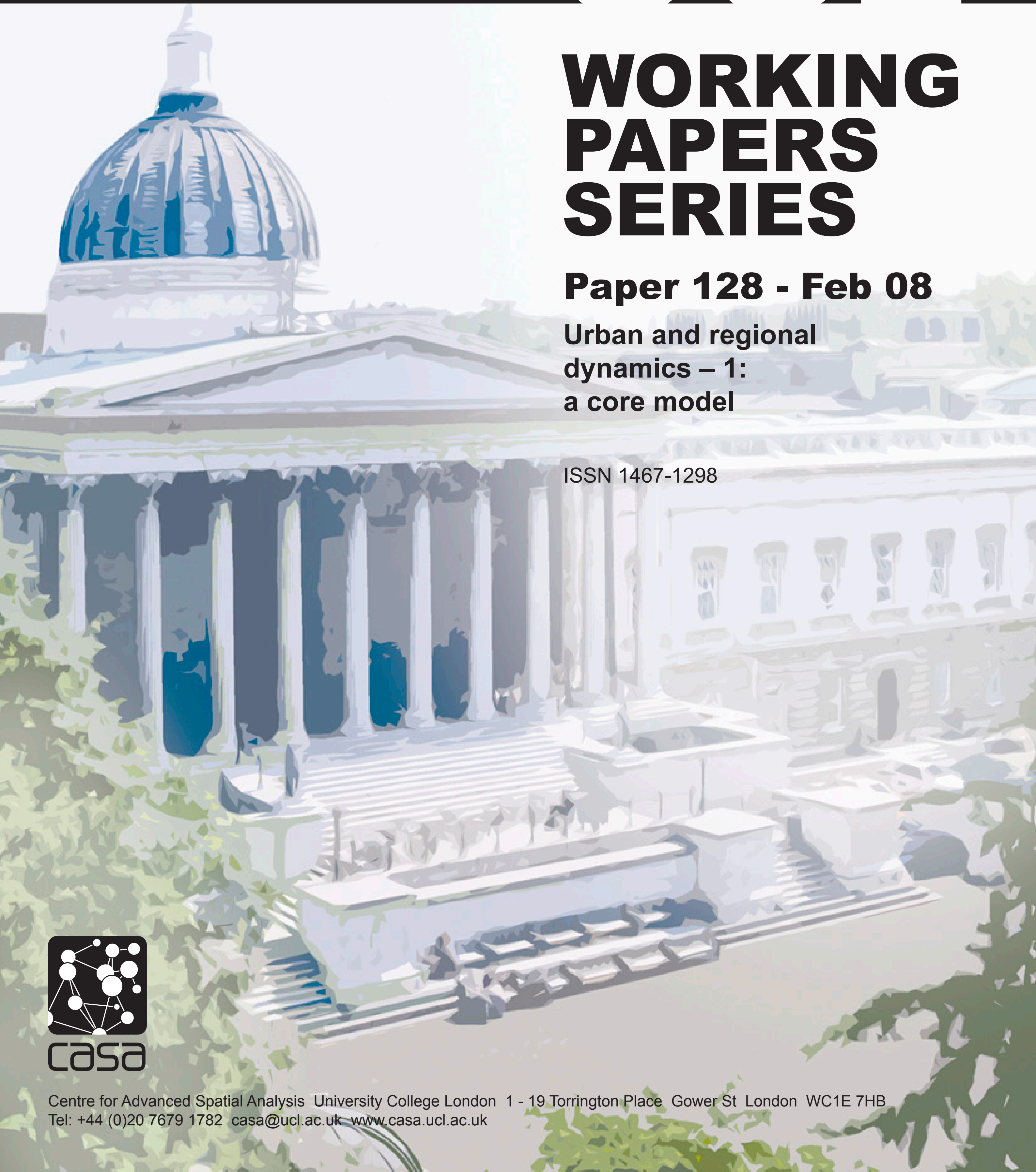

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Urban and regional dynamics – 1: a core model

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Urban and regional dynamics – 1: a core model

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Abstract

It is generally recognised that there are important interdependencies that govern the development of urban and regional systems and that these can in principle be captured through the building of appropriate models. In this paper, the task is addressed of specifying the simplest demonstration model that will capture the most important dependencies. In particular, this involves linking the well-known methods of spatial interaction and location modelling with input-output analysis. A model is developed and presented and its potential uses are explored.

1. Introduction.

Urban modelling has reached a point at which there is a huge amount of experience of sectoral models – for example, in retail and transport analysis – and there are many applications of comprehensive models, usually rooted in Lowry's model. In the case of the sectoral models, and the retail model in particular, there are the beginnings of understanding of the mechanisms of dynamics. This is rooted in spatial interaction modelling with limited connections to demographic and economic input-output modelling. What has not been achieved is the full integration of these developments in sectoral modelling within a comprehensive dynamic framework. This is because of the sheer scale of the problem. In an earlier paper (Wilson, 2007-A), a general notation was introduced to facilitate the assembly of a comprehensive dynamic model and it was estimated that a set of arrays to achieve an appropriate base might have of the order of 10^{13} variables.

It remains important to make further progress with this challenge (a) to establish the core science and (b) because many contemporary urban policy questions are rooted in the interdependencies of the comprehensive picture. The objective of this paper, therefore, is to present a demonstration model which is much simpler than the 10^{13}-variable model but which can be used to demonstrate the benefits of comprehensiveness – and in particular to be able to account for key interdependencies . It can be thought of as a toy model, but one that will illustrate what can be achieved in the future.

We proceed in three initial stages: first to specify a system description (section 2); secondly, to articulate the underlying accounting equations (section 3); thirdly, to specify the model (section 4). In section 5, we then explore the workings of the model. In section 6, we discuss the range of analysis that can be accomplished with the model and in section 7, we offer concluding comments and discuss next steps.

This is the first of three linked papers which are designed to provide the theoretical underpinnings of a future research programme. The second, Wilson (2008-A), shows that there are four different spatial scales at which model building is relevant – forming a hierarchy - and that the kind of core model presented in this paper could, appropriately adapted, be used at each level. The third paper, Wilson (2008-B), focuses on the structural variables that determine future dynamics and seeks to identify the 'DNA' and the 'genes' of the system as a basis for generating a typology of areas and a range of potential policy applications.

2. System description

2.1. Principal variables

We begin by assuming that we have an urban system of interest divided into zones of a suitable size – perhaps a grid – with zone centroids being used to characterise locations. We then make fundamental distinctions between *infrastructure* at and between locations, which is relatively slowly changing, *activities* at a location, and *interactions* between locations. For brevity and convenience, we will use the term 'structure' for 'infrastructure'. We then need to delineate the smallest number of sectors and subcategories that will make the model realistic. We take the following as the principal sectors:

- Population
- Housing
- Public services[1] such as schools and hospitals
- Retail services
- Capacity in the economy – buildings, offices, equipment etc
- Government[2]

Labour is generated by converting the population at a location into an active workforce. We then make a distinction between the capacity of a sector at a location and the corresponding level of activity. (The population/workforce distinction is one example: there may be under- or over-employment.)

The principal spatial interactions are:

- The journey to work
- Population to housing
- Population to public services
- Population to retail centres
- Business to public services
- Business to retail services
- Business to business
- Government (spending) to population, public services, retail services and the economy

To achieve a degree of realism even in a demonstration model, we need some subdivisions – for example, population by income, retail facilities by type and so on. We now depart from the notation of the previous paper which was abstract anchored around an array $\{Y^{mng}_{ij}\}$ as the flow of a good (or service), g, from sector m in i to sector n in j. Vectors $\{X^{mg}_{i}\}$ and $(Z^{ng}_{j}\}$ are used to represent row and column totals. In this simpler model, we label sectors by m and n as before but we drop the g superscript and use separate letters to represent the main origin and destination totals and the flows. There is some loss of generality – and a struggle to find enough distinctive letters! - but an increase in transparency for a demonstration model. This leads to the following definitions of key variables:

- P_i^m, the number of type m people in zone i
- H_i^k, the number of type k houses in zone i
- V_j^n, the capacity of type-n consumer services in j; F_j^{mn}, the take-up (on a suitable measure – school places e.g.) by m-type people

[1] 'Public services' is an approximate description, used for convenience for sectors such as education and health in which the bulk of the provision is government funded

[2] We will assume an aspatial 'pool' into which taxes flow providing the funds for government spending

- W_j^n, the capacity of type-n retail facilities in j; D_j^{mn}, the take-up of these facilities by m-type people
- Q_j^n, the capacity of the nth economic sector in zone j; X_j^n, the product of sector n in j
- G_j^n, government spend in j[3].

In the case of the population and housing at a location, we are taking the variables to be measures of both capacity and activity – an approximation in the case of housing because it means that we are neglecting vacancies for example. In the other cases, we make a distinction. We distinguish the public services sector (such as education and health) and the retail sector from the general set of economic sectors to facilitate model construction and policy interpretation later. This is again in contrast to the paper cited earlier. That is, we are distinguishing the V and W sectors from the other X-sectors (and the population sector, P, in so far as it is being treated as a producer of labour).

The main interaction variables are[4]:

- Y_{ij}^{mVn}, Y_{ij}^{mWn}, Y_{ij}^{mQn}, the flows to work in sectors V, W and Q from population group m in zone i to sector n in zone j for V, W and Q respectively;
- N_{ij}^{mnk}, the allocation of type m people who work in sector n in j to type k houses in i;
- U_{ij}^{mn}, the flow of type m people in zone i to consumer services of type n in j;
- S_{ij}^{mn}, the flow to retail facilities of type n;
- J_{ij}^{mn}, the flow of goods from sector m in i to the consumer services sectors n in j;
- K_{ij}^{mn}, the flow of goods from sector m in i to the retail services sectors n in j;
- M_{ij}^{mn}, the flow of goods from sector m in i to sector n in j.

We will also need to define variables that represent aggregates:

- E_j^{mn}, total employment of type m-people in sector n at location j, which is, of course, $\sum_i(Y_{ij}^{mVn} + Y_{ij}^{mWn} + Y_{ij}^{mGn})$
- I_i^m, total income of type m people, resident in i;
- T_j^n, ..., tax take from various sectors, n, in j;
- G_j^n, government spending in n in j.

The transport system is implicit in this formulation. The structural variables are the modal networks and the vehicles but we are most interested in the generalised cost of travel between zones, c_{ij}, which, in due course, we will have to disaggregate.

[3] At some stage it would be necessary to distinguish between revenue and capital

[4] We note the possibility of indices such as m and n themselves being lists

2.2. Input-output tables.

These variables can be arranged within an input-output table, as in Figure 1.

	P	H	V	W	X	G	Exports	Output totals
P			Y	Y	Y	T^P		
H	N					T^H		
V	U					T^V		F
W	S					T^W		D
Q			J	K	M	T^Q	Exp	X
G	G^P	G^H	G^V	G^W	G^Q			
Imports					Imp			
Input totals								

Figure 1. Input output table for the core model

A column has been added to record exports and a row to record imports. This is particularly important at the level of urban analysis because much of what is needed for example by retailers to serve their consumers will be imported into the city. For later modelling use to enable us to retain a conventional notation (and for the purposes of interpretation), it is convenient to turn this table through 90^o. This can be seen as a row view of users choosing suppliers and is consistent with the interaction definitions given above.

	P	H	V	W	Q	G	Imports	Input totals
P		N	U	S		G^P		
H						G^H		
V	Y				J	G^V		
W	Y				K	G^W		
X	Y				M	G^X	Imp	
Exports					Exp			
G	T^P	T^H	T^V	T^W	T^Q			
Output totals			F	D	X			

Figure 2. Users choosing suppliers

3. Accounting equations

3.1. Introduction

In Figure 1, the usual convention was adopted that outputs are recorded along rows and inputs – the 'consumers' of outputs from elsewhere – down columns. Conventionally, the final column would be 'final demand' but we wish to retain the possibility of there being elements that behave like final demand in other parts of the table. For example, much of the 'P' column will be final demand. We can then write the relevant accounting equations that underpin the model – and these are useful in their own right. It is well known that assumptions have to be made about what is driving the model (Macgill, 1977-A, B , Wilson et al, 1981). If it is final demand together with all the inputs necessary to produce it, then the equality constraints will be on inputs. In the case of outputs, the accounting equations will typically be inequalities which will indicate that there is at least sufficient capacity to produce the appropriate deliveries (which will be inputs elsewhere). There are of course, other possible assumptions, for example that in some cases resource limitations drive the model – in which case not all final demand might be met. For present illustrative purposes, we will make the assumption that 'final demand plus exports' drives the model. The first set of accounting equations can then be associated with production (outputs) and the second with absorption (inputs). However, the framework presented can be used to incorporate a wider range of rich hypotheses.

Figure 2 is produced from Figure 1, as we have seen, by rotating it through a 90^{o}. This enables us to maintain consistency with the more usual model notation. A particular example is associated with retail flows: we model a flow of expenditure from residents of zone i to shops in zone j, S_{ij} – the population choosing where to shop. This is taken to be the active direction of causality.

3.2. The accounts.

We then present first the output constraints (columns) and then the input constraints (rows) based on the structure of Figure 2 and the assumptions and definitions of variables above. These are the accounting equations that will underpin the model. A brief commentary is offered after each equation.

Output constraints

$$\sum_{jn} Y_{ij}^{mVn} + Y_{ij}^{mWn} + Y_{ij}^{mGn} \leq y_i^m P_i^m \quad (1)$$

The left hand side represents the total labour force of type m produced in zone i needed as workforce inputs in the rest of the system and this must be less than the potentially available workforce, shown on the right hand side – the relevant populations multiplied by activity rates, y_i^m.

$$\sum_{jmn} N_{ij}^{mnk} = H_i^k \quad (2)$$

The housing of type k in zone i is set equal in this case – an approximation neglecting vacancies and homelessness – to the populations to be housed, taking into account their workplaces.

$$\sum_{im} U_{ij}^{mn} \leq V_j^n \quad (3)$$

This is the output of consumer services of type n in j and the constraint asserts that the demand for consumer services of type n in j must be met.

$$\sum_{im} S_{ij}^{mn} \leq W_j^n \quad (4)$$

Similarly, this represents the output of the retail sector and the demand for retail services must be met. These two sets of conditions will be relaxed slightly when the models extend into dynamics.

$$\sum_{jn} (J_{ij}^{mn} + K_{ij}^{mn} + M_{ij}^{mn}) + Exp_i^m \leq Q_i^m \quad (5)$$

The left hand side represents the total production of goods (or services) in sector n in zone j and the right hand side is a measure of the capacity to produce these goods and this must not be exceeded[5].

In each case, the equation is saying that what is produced must be within the capacity of the producer.

Input constraints

$$\sum_{im} Y_{ij}^{mVn} = v_j^n V_j^n \quad (6)$$

The left hand side represents the workforce input flows into consumer services and the right hand side (with an appropriate input coefficient) what has to be delivered to sustain the capacity, V. v_j^n is an appropriate rate.

$$\sum_{im} Y_{ij}^{mWn} = w_j^n W_j^n \quad (7)$$

These are equivalent equations for retail services.

$$\sum_{im} Y_{ij}^{mXn} = x_j^n X_j^n \quad (8)$$

These are the equivalent equations for the rest of the economy

$$\sum_{ik} N_{ij}^{mk} = E_j^m \quad (9)$$

This is the population complement of equation (2).

$$\sum_{jn} U_{ij}^{mn} = u^m P_i^m \quad (10)$$

This represents the delivery (inputs) of consumer services to the m-population of zone i.

$$\sum_{jn} S_{ij}^{mn} = e^m P_i^m \quad (11)$$

These are the equivalent equations for the retail sector.

$$\sum_{im} (J_{ij}^{mn} + G_i^{Vn}) + Imp_j^n = v_j^n V_j^n \quad (12)$$

These are the inputs to the consumer services sector [other than labour which was dealt with in equation (6) above]

$$\sum_{im} (K_{ij}^{mn} + G_j^{Wn}) + Imp_j^n = w_j^n W_j^n \quad (13)$$

These are the equivalent for the retail sector.

$$\sum_{im} (M_{ij}^{mn} + G_j^{Xn}) + Imp_j^n = x_j^n X_j^n \quad (14)$$

These are the equivalents for the rest of the economy. v, w and x are coefficients representing the unit inputs needed to produce unit outputs of V, W and X respectively.

Interaction constraints.

For each of the flows, we have to make an assumption about the total amount of resource available to fund the interaction – that is, to meet transport costs. Using an obvious notation, we have

$$\sum Y_{ij}^{mVn} c_{ij}^{YmVn} = C^{YmVn} \quad (15)$$

Where a Y has been added to the cost superscripts to identify the costs associated with the Y-flow. Similarly, we would define c_{ij}^{Umn} and C^{Umn} for costs of travel to retail centres and there would be constraint equations for all the interaction arrays defined above, similar to (15). We will leave these to be understood.

[5] This is essentially the usual multi-region input-output equation. For the sake of this demonstration model, we can assume that all the intermediate products are in the 'economy' and the two service sectors, along with exports, are treated as final demand. We would then have to make an assumption along the lines of

$$M_{ij}^{mn} = a^{mn} m_{ij}^{mn} X_j^n$$

Where the a^{mn} are the technical coefficients and m_{ij}^{mn} is the proportion of the (i,m)-product that goes to (j, n).

Land use constraints.

Finally, we need to account for land use. Let L_i be the total amount of land in zone i and let λ, with appropriate subscripts and superscripts represent unit land needed for different kinds of activity. (This has to have a zonal subscript to represent areas where high-rise building means that some activities require less land for a given amount of floorspace.) Then:

$$\sum_k \lambda_j^{Hk} H_j^k + \sum_n \lambda_j^{Vn} V^n + \sum_n \lambda_j^{Wn} W_j^n + \sum_n \lambda_j^{Xn} X_j^n \leq \sum L_j \quad (16)$$

3.3. Introduction of prices.

We have not so far specified units explicitly. Mostly we have implicitly worked in terms of person or volume flows. It is particularly important to present the accounting equations in terms of money flows and to do this we need to specify a set of prices. If we were now to assume that the flows were measures of volume, then we could introduce prices – which would be price indexes for groups of goods or services. Using an obvious notation, these could be p_i^{Hn}, p_j^{Vn}, p_j^{Wn}, p_j^{Xn}, and so on, assuming that in at least some cases, housing for instance, they would be zone dependent. It would then be possible to trace income flows and to estimate variables such as I_i^{me}, total income of type m people, resident in i, working in e-type jobs[6].

4. A demonstration model

4.1. Introduction

The model should be anchored in the accounting equations. This will show how the steady state functions – and, if we use the kind of assumptions that Lowry used, how equilibrium builds up. We will illustrate the consequence of adopting Lowry-like assumptions in this framework, but, in effect, he assumed all the dynamics to be 'fast' and we will want to refine that. In either case, it is necessary to articulate the directions of causality and then to see the effects pulled through the accounts.

We begin with the steady state model. Assume that we have fixed values for the main structural arrays: P, H, V, W and Q and we can estimate the employment needs of each element at each location. Then the workforce can be allocated to residential locations through a submodel rooted in equations (2) and (9). The journey-to-work accounts are recorded in equation (1) – from the perspective of the population – and in equations (6)-(8) as inputs into the economy[7]. The resident populations consume services [equations (3), (6)

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[6] We would have to transform all the variables along the following lines (cf. Wilson, 1974):

$$\hat{X}_j^m = X_j^m p_j^{Xm}$$

so that

$$X_i^m = \hat{X}_j^m / p_j^{Xm}$$

It would then be possible to substitute for X_j^m (and the equivalent variables throughout the accounting equations) and to interpret the tables in Figures 1 and 2 as money flows. This will be particularly important for the 'money flows' version of equation (14). These are the column sums of the input output tables and at this point can be used to estimate prices. However, in our demonstration model, we will assume an initial set of prices as given and then make adjustments as part of the slow dynamics.

[7] This means that the two sets of equations have to be reconciled at some point. We need a constraint such as the following:

$$\sum_k N_{ij}^{mnk} = \sum (Y_{jj}^{mVn} + Y_{ij}^{mWn} + Y_{ij}^{mQn})$$

and (10)] and retail services [(4), (7) and (11)]. The economy generates the goods that are needed – the outputs being recorded in equation (5) – and these are delivered as inputs as appropriate via equations (12)-(14).

The Lowry assumptions can be easily incorporated by designating an initial set of basic employment, generating the resident populations associated with that, and then those populations will generate further employment through consumer and retail services and these workers have to be assigned residential locations. And so on. All the dynamics are then 'fast' in the sense that the underpinning structures (H, V, W and Q) are assumed to spring up at each iteration as 'demanded' by new populations. We will continue the argument to build the model on the basis of the steady state assumptions and then generalise by adding 'slow' dynamics for the structural variables.

4.2. The fast dynamics: the interaction arrays.

The next step is to estimate all the interaction variables – Y, N, U, S, J, K and M - in terms of the structural variables and the transport system. This can be done by setting up an entropy function in the interaction variables as in Wilson (2007-A). The G-spend is assumed to be exogenous. We need aggregate employment, E_j^{mn}, total employment of a type (taken as a superscript m, assumed connected to person type) in a sector (n) at location j – as defined in section 2. Where needed and using an obvious notation, we divide this into components E_j^{Vmn}, E_j^{Wmn} and E_j^{Xmn}. The results are:

$$Y_{ij}^{mVn} = A_i^{Ym}B_j^{Yn}P_i^{m}E_j^{Vmn}\exp(-\beta^{YV}c_{ij}) \quad (17)$$
$$Y_{ij}^{mWn} = A_i^{Ym}B_j^{Yn}P_i^{m}E_j^{Wmn}\exp(-\beta^{YW}c_{ij}) \quad (18)$$
$$Y_{ij}^{mXn} = A_i^{Ym}B_j^{Yn}P_i^{m}E_j^{Xmn}\exp(-\beta^{YX}c_{ij}) \quad (19)$$
$$N_{ij}^{mnk} = A_i^{Nk}B_j^{Nn}H_i^{k}E_j^{mn}\exp(-\beta^{H}c_{ij}) \quad (20)$$
$$U_{ij}^{mn} = A_i^{Um}P_i^{m}V_j^{m}{**}\alpha^{U}\exp(-\beta^{U}c_{ij}) \quad (21)$$
$$S_{ij}^{mn} = A_i^{Um}P_i^{m}W_j^{m}{**}\alpha^{W}\exp(-\beta^{W}c_{ij}) \quad (22)$$
$$J_{ij}^{mn} = Ai_J^{m}Bj_J^{n}X_i^{m}X_j^{n}\exp(-\beta^{J}c_{ij}) \quad (23)$$
$$K_{ij}^{m} = A_i^{Km}B_j^{Kn}X_i^{m}X_j^{n}\exp(-\beta^{K}c_{ij}) \quad (24)$$
$$M_{ij}^{mn} = A_i^{Mm}B_j^{Mn}X_i^{m}X_j^{n}\exp(-\beta^{M}c_{ij}) \quad (25)$$

These are mostly conventional spatial interaction models with the balancing factors, the As and the Bs, being calculated in the usual way. However, it should be noted that the expressions for the A- and B-terms in the J, K, M equations are more complex. The A-terms include the export flows and the B-terms both the import flows and input-output coefficients. This is a distinctive feature: the links between spatial interaction and input-output relations are not normally made explicit. The formulation becomes more complicated when not all the constraints 'bite' – that is, 'inequality' is satisfied and 'equality' is not needed but this can be handled.

This would be a refinement of the spatial interaction models to be presented below.

4.3. The drivers of the model.

These interaction models then allow us to expand our description of the drivers of the model. Equations (20)-(22) show the extent to which the model is driven by the population. In turn, we have choice of workplace and housing (simultaneously), and choice of consumer and retail services. The demands for goods and services are then channelled through the J, K and M flows. The fact that the model is rooted in accounting equations means that everything is forced to add up. This means also that refinements have to be added to deal with real world complexities. We have already mentioned vacancies in housing, or homlessness. It is also the case, the mechanisms that allocate people to services, such as schools, for example, may be very different in practice. However, this can all be handled, as has been indicated in the past, but we retain minimal complexity for the purposes of exposition. If and when we need to modify these assumptions in the context of future discussions, then this will be done. We should, however, recall one trick which can be useful. We have not been explicit about the core indices such as k, m or n. We have noted in earlier work that these can themselves be lists. We can usefully consider that they can be expanded or contracted as convenient. In the case of the population, for example, at times we need 'm' to incorporate age, level of education, income, occupation and so on.

4.4. The slow dynamics: structural evolution.

Bearing this in mind, we retain the simplicity but complete the model by adding the slow dynamics. The first step is to decide what is exogenous and what is endogenous. There are various possibilities for what to include within the 'exogenous' group.

- We do not have the ability to model some particular variables and so we have to manage with an exogenous specification.
- A different kind of model is needed for some other variables that can be treated as distinct. The population model can be treated in this way. Some elements can be estimated using conventional multi-regional demographic models, but there are others – notably the large migration flows common in many places at the present time – for which modelling is difficult or impossible. These flows are the import and export cells associated with the population rows and columns of Figure 2 and at the present time would almost certainly have to be specified exogenously.
- This argument can be extended to other import and export flows. This also indicates that in so far as modelling is possible for these flows, it would have to be carried out on a multi-region basis.
- The most interesting case of exogenous specification is that of variables representing public service facilities that can be planned. A plan can then be conceived as an exogenous input and its efficacy can be tested through the model. The accounting equations are particularly important here: they would indicate very quickly, for example, if there was insufficient capacity to meet, say school or hospital demand. In some of these cases, it would be possible to specify and run a 'free market' model of service provision to get an idea of what the optimum plan might look like under certain assumptions.

On the basis of this analysis, we offer hypotheses for retail dynamics and for the main economic variables and we assume that other variables will be specified exogenously, either through other more or less independent models as discussed or through plans to be tested – for example, for schools and hospitals.

In the retail case, the principles are now well known. The total attracted to zone j to a centre of the size, W_j, can be calculated from the interaction model equation - in this case (22):

$$D_j^{*n} = \sum S_{ij}^{*n} \tag{26}$$

If this is a measure of revenue and k_j^{Wn} is a measure of the unit cost of running the centre, W_j^n, then it has been shown (Harris and Wilson, 1978) that a suitable dynamic is:

$$dW_j^n/dt = \varepsilon^W[D_j^{*n} - k_j^{Wn}W_j^n]W_j^n \tag{27}$$

These equations are strongly nonlinear. The equilibrium solutions are path dependent – that is, highly dependent on initial conditions and indeed, as the characterisation implies, on the whole of the preceding path in state space. The form of $\{W_j\}$ structures that emerge as solutions to these equations are important indicators of the type of system

In the case of the economic arrays, we can use a similar argument:

$$dX_i^{mg}/dt = \varepsilon^X[\sum_{jn} J_{ij}^{mng} + K_{ij}^{mng} + M_{ij}^{mng} + Exp_i^{mg} - k_j^X X_j^{mg}]X_j^{mg} \tag{28}$$

This takes revenue from equation (5) and makes a simple assumption about production costs as in the retail case.

We should also bear in mind that we could now incorporate price adjustments on the same basis:

$$dp_j^{Wn}/dt = \varepsilon^{PW}[D_j^{*n} - k_j^W W_j^n]W_j^n \tag{29}$$

and

$$dp_j^{Xmg}/dt = \varepsilon^{PX}[\sum_{jn} J_{ij}^{mng} + K_{ij}^{mng} + M_{ij}^{mng} + Exp_i^{mg} - k_j^X X_j^{mg}]X_j^{mg} \tag{30}$$

4.5. Further refinements

In practice, of course, developers and those who control the structural variables, do not take stock and make all their decisions at the year end. That is a mathematical convenience in a demonstration model. It would be possible to have finer time divisions and to construct probabilities of structural change at each interval.

It would be particularly interesting to explore the version of the model stated in money units and with prices explicit. In the steady state model, the amount to be spent on retailing, and the choice of destination, would be a function of this demand./budgeting process. The model represents the outcome of intersecting supply and demand 'curves'. If we accept this as the 'physiology', then it raises the interesting possibility of calculating the C-variables in the interaction constraints such as equation (15) as a function of the expenditure determined from the demand-supply modelling. This would then make the β parameters in the interaction model equations endogenous. This in turn would have implications for the way we analyse phase changes.

5. The workings of the model

Consider Figure 3. The boxes represent the main infrastructure variables and the

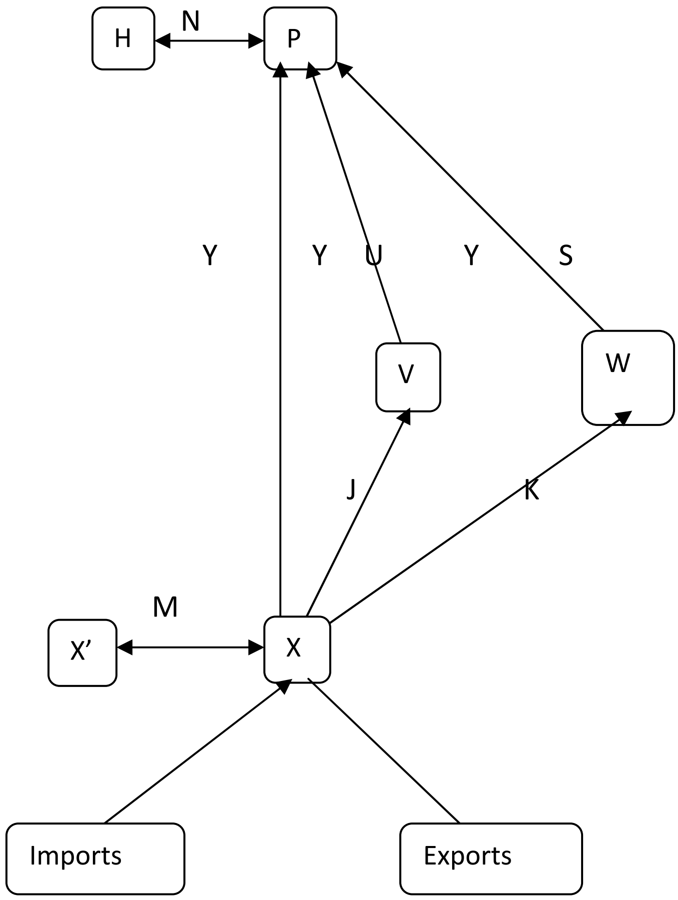

Figure 3. Structure and flows

the arrows the main flows. An additional X-box is shown so that we can represent inter-business flows. The structures and flows that are shown are symbolic and illustrative of course: the real system would have the stocks in each of a large number of zones and there would be corresponding interaction matrices. However, this is useful in giving an account of the workings of the model.

Consider what happens in the period t to t+1, say in one year. The population supplies labour through the flows Y to the three main economic sectors.[8] We make the simplifying assumption that the population has been allocated to housing by location in relation to employment through the $\{N_{ij}^{km}\}$ model.[9] What can then be thought of as 'final demand' is the set of demands for consumer and retail goods and services that are delivered through these sectors (V and W) to the population through the $\{U_{ij}\}$ and $\{S_{ij}\}$ interactions. These have to be supplied through the X-sectors and the $\{J_{ij}\}$ and $\{K_{ij}\}$ flows.

[8] This illustrates a key point of the model: each of the flows, presented as one-way, are actually two-way. Y represents a flow of people to work and (with suitable coefficients – in this case wage rates), a flow of income back to the place of residence (and this in turn can be used to purchase, for example, retail goods). There is a corresponding argument for each flow: the set of prices introduced earlier allow a flow of goods and services in one direction to be converted into money flows in the other.

[9] There are many more complex hypotheses that could be incorporated here of course.

Business sectors in turn demand inputs from other sectors, and these are the $\{M_{ij}\}$ flows. The flows in the interaction models have been calculated to ensure that all the accounting constraints are met.[10,11]

At the end of each period, as we have seen, we assume that there will be a stocktaking of the structural variables represented by the slow dynamics' equations and adjustments will be made. We need to bear in mind, of course, that the nonlinear nature of these models means that there can be phase changes – shifts to a different kind of structure that happen suddenly.

At this point we need to bear in mind the discussion of the previous section on the specification of exogenous variables. We have implicitly assumed here that there will be a distinct demographic model. This will handle conventional demographic change but there are also likely to be substantial migration flows that can only be handled exogenously. Where there are large net in-migration flows, for example, the impacts on housing, services and the labour market can be calculated. These kinds of changes can themselves bring about phase shifts.

Imports and exports, too, play critical roles in urban economies (as in national) in two ways. First, they provide crucial balancing in the steady state interactions. A tour round a supermarket shows how many goods are imported to meet retail demand; exports are a crucial element of 'final demand' in the model, because they generate much of the income that is then spent on goods and services within the city. Secondly, they should be considered to be important parts of the infrastructure, albeit outside the city boundaries and the model should ultimately be refined to reflect this. We should also note that there is a 'balance of payments' concept for urban economies just as there is for national ones. If there is an adverse balance of payments, then the balancing will come through capital flows and external stakeholders will begin to 'own' the city.[12]

Much investment will also have to be handled exogenously, particularly in major facilities for consumer services such as schools and hospitals and for transport infrastructure. However, these are structural variables, and the model can still be used to predict the interaction matrices and can be used in this way to evaluate alternative investment plans.

6. Analysis with the model

To show what can in principle be achieved with this model base, we can consider the levers available to Government for future urban planning. The principal ones are

- Land use zoning and planning controls
- Investment in public facilities, for example:

[10] There is much detail to be added here, particularly in relation to the accounting equations that are inequalities. Zones have to be divided for each of these equations intro those in which the constraints are binding (equalities) and those that are not. This is technically messy, but can be done. It will not be shown explicitly here in order to retain a relatively simple core argument.

[11] It is the business accounting equations that contain input-output coefficients, as we have seen, and this means that there is an element of matrix inversion in the iterative solution to the accounting equations. It would be interesting to explore the possibility, if the model is being run iteratively, period by period, to see if this process could be approximated by the first term of as series expansion.

[12] There is an interesting modelling challenge to represent this phenomenon.

- Housing
- Incentives for economic development – for example in regions
- Schools, colleges and universities
- Hospitals and health facilities
- The criminal justice system
- Cultural facilities
- Transport infrastructure and systems

These G-arrays, in our terminology, are contributors to 'final demand' when seen through the perspective of an input-output model. A challenge for Government will be to invest in such a way as to encourage complementary private investment. These interventions will impact on the {P, H, V, W, X, L. p, c} arrays. The model system can then be used to predict the possible futures evolving from alternative plans.

We can consider some examples of the kinds of policies that could be tested in this way. Consider a proposed investment in public transport. This will be reflected in a changed $\{c_{ij}\}$ array and people will make different mode and destination choices as reflected in the spatial interaction submodels. It will be particularly important to represent each c_{ij} as *generalised* costs, as sums of different kinds of time that are valued in different ways. It would be useful, for example, then to test a conjecture: that suburban densities have to exceed some threshold figure for public transport options to be viable.

A second conjecture is based on the following hypothesis: thriving towns and cities have households with (at least relatively) good incomes. They can then at least maintain the housing stock, for example, and can probably press for good consumer services (or pay for them privately if they are not supplied). Good retail services will be supplied through the market. The good incomes will derive from good qualifications and well-paid employment. The roots, therefore, are in the education and training systems and in the ability of the place to attract employment. There is an education-employment-income-housing loop

A third question, in some ways related to the first two, is this: consider an area – a zone, say of an inner city, which is obviously run down – under-investment in maintaining the stock for example – with a population with inadequate income and skills. What can be done for that area? There will be no unique answer, of course. What is needed is a portfolio of possibilities that can be applied in different kinds of situations. A pointed example of this question arises in the context of the super casino project for a run-down area of Manchester in the UK. This would offer 2,700 jobs and was seen as the key building block for a massive regeneration programme. The Prime Minister has argued that there must be a better way of supporting regeneration. But how?

This argument by example can then be generalised. Government investment in model terms is a form of final demand for an initial period, both in its own terms and through the employment it generates which is essentially based on 'export activity' for the area. A policy or a plan is the specification of the exogenous variables of the model. The challenge, therefore, can be presented as: identify the optimum components of 'final demand' that would generate a sustainable improvement for the area[13].

[13] Connect back to footnote 7

This brings three other points into focus.

- First, our analysis for a city can be applied to a small area of the city[14] (as we have seen through Figure 5 above). In input-output terms, the city needs a positive balance of payments; so does any small area. This should be one of the key performance indicators.
- However, secondly, it is also clear that the impacts of different kinds of investment are differentially spread over future time, and this also needs to be incorporated in the model-based analysis.
- Thirdly, we do need a good set of benefit measures, and these should include the indirect benefits of reductions in the costs of benefits, health and criminal justice in areas that can be radically improved.

7. Next steps: concluding comments

On the basis of this theoretical analysis, we can identify a number of steps to support model-based planning and policy analysis in the future. We need to be able to

- Run the model for reasonably realistic situations at different levels in a hierarchy – and see Wilson (2008-A)
    - for subareas
    - cities
    - for regions
    - and for systems of cities
- To provide the means of identifying what might be exogenous 'final demand' variables and to trace the flows of causality
- To offer a range of performance indicators. For example, to be able to calculate and interpret 'balance of payments' indicators for areas of any size
- To be able to run the model dynamically, for example to demonstrate the accessible area of phase space – possible futures – under various assumptions of initial conditions and investment plans. See Wilson (2008-B) for the related 'DNA' argument. Some structures rule out possible development paths. For example, public transport investment may be worth very little in areas that do not exceed a threshold density. We need to be able to identify both downward and upward spirals.
- To identify possible policy and planning levers which would usually be represented by exogenous variable specifications (or through zoning constraints)
- To identify bifurcation points that would add leverage to initial investments
- Since investment would be limited, to articulate the 'choices' that have to be made
- To chart the range of 'problems' to be solved. Planning for growth? Planning for effective competition with other urban areas?
- The methods can be used to model the evolution of urban and regional networks – as described in Wilson (2007-B)

[14] There is also the perennial question: what constitutes the city: administrative boundaries, journey-to-work areas or whatever?

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