Statistics, Lambda and closed Universes

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Statistics and detecting Lambda I

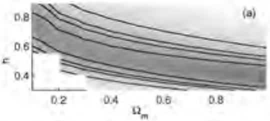
- First met Ofer while giving a graduate course in the CMB
- We then worked with each other as an early example of Cavendish/IoA cooperation
- First joint paper was

arXiv:astro-ph/9802109 [pdf, ps, other]

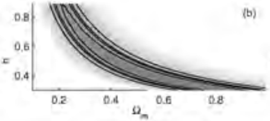
Joint estimation of cosmological parameters from CMB and IRAS data

Matthew Webster, S.L. Bridle, M.P. Hobson, A.N. Lasenby, Ofer Lahav, Graca Rocha

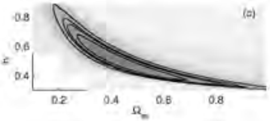
- The idea of this was to break the degeneracies inherent in the CMB alone, or LSS alone, by combining the two



CMB alone



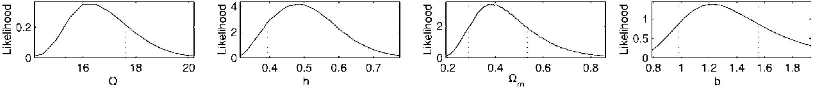
IRAS 1.2 Jy redshift survey



Combined

Statistics and detecting Lambda II

- The Bayesian statistical framework adopted was pretty much similar to what would be done today - following are 1d marginalised results



> ## 6. DISCUSSION
>
> The results of this joint optimization are in reasonable agreement with other current estimates. The relatively low value of $\Omega_m \approx 0.4$ is close to that found by others (White et al. 1993; Bahcall, Fan, & Cen 1997) and is in line with recent supernovae results (Perlmutter et al. 1998). However, given the assumption of a flat universe, it requires a very high cosmological constant $(\Omega_\Lambda = 0.6)$.

Supernovae

- The *Supernovae Cosmology Project* results published July 1997 (Perlmutter *et al.*, ApJ, **483**, 565) had found

> Comparing light-curve width-corrected magnitudes as a function of redshift of our distant ($z = 0.35$–0.46) supernovae to those of nearby Type Ia supernovae yields a global measurement of the mass density, $\Omega_M = 0.88^{+0.69}_{-0.60}$ for a $\Lambda = 0$ cosmology. For a spatially flat universe (i.e., $\Omega_M + \Omega_\Lambda = 1$), we find $\Omega_M = 0.94^{+0.34}_{-0.28}$ or, equivalently, a measurement of the cosmological constant, $\Omega_\Lambda = 0.06^{+0.28}_{-0.34}$ (<0.51 at the 95% confidence level).

- And constraints on Ω_Λ from radio-selected gravitational lenses (Falco, Kockhanek & Munoz, ApJ, **494**, 47), published February 1998, gave $\Omega_\Lambda < 0.62$ at 2σ
- So our joint CMB and LSS result for Λ came in a context where people weren't necessarily expecting a large Ω_Λ!
- May object that our analysis was limited to flat models
- Most were at that time, but in fact we already had pretty good evidence for high $\Omega_{\rm tot}$ from the CMB at the time

CMB results ∼ 1996

Constraints on cosmological parameters from recent measurements of cosmic microwave background anisotropy

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Accepted 1997 July 31. Received 1997 July 11; in original form 1996 September 19

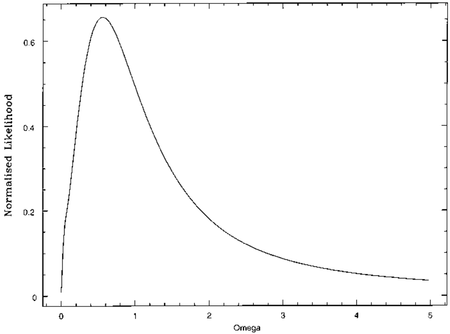
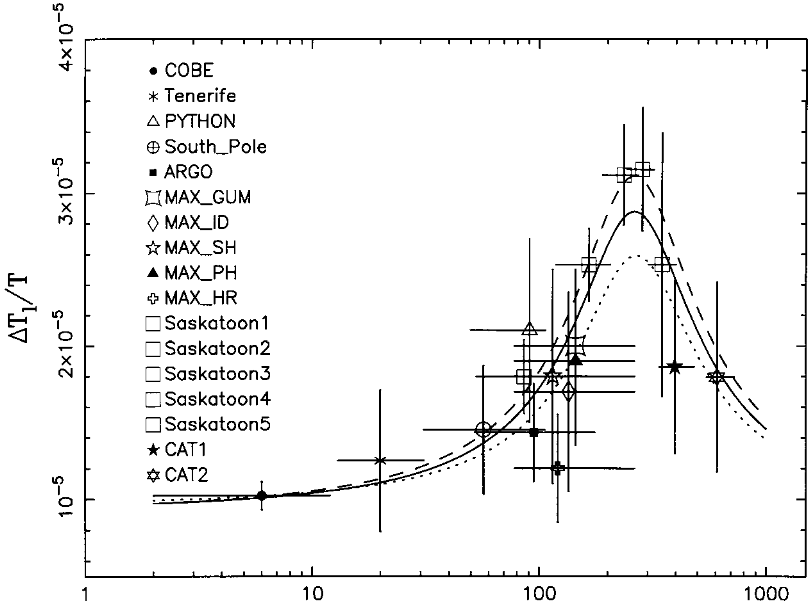


Figure 4. The 1D marginal likelihood curve for Ω.

CMB data ∼ 1996



Hyperparameters

Bayesian 'hyper-parameters' approach to joint estimation: the Hubble constant from CMB measurements

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Accepted 2000 April 28. Received 2000 April 14; in original form 1999 December 8

We show below that if the prior probabilities for $\ln(\alpha)$ and $\ln(\beta)$ are uniform then one should consider the quantity

$$-2\ln P(\mathbf{w}|D_A, D_B) = N_A \ln(\chi^2_A) + N_B \ln(\chi^2_B) \qquad (4)$$

instead of equation (1). Note that in this case our method is equivalent to assuming that we are ignorant of the relative scale of the errors in each experiment.

- This results from generalising the usual equation for combining two data sets A and B

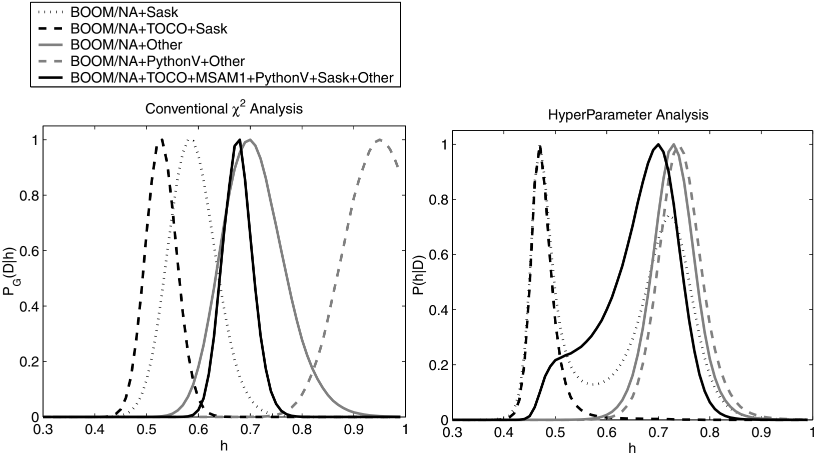
$$\chi^2_{\text{joint}} = \chi^2_A + \chi^2_B$$

- We include hyperparameters α and β in the form

$$\chi^2_{\text{joint}} = \alpha\chi^2_A + \beta\chi^2_B$$

which are then marginalised over with a uniform log prior

Hyperparameters — the (year 2000) results for H_0



Recent Bayesian work I

Bayesian sparse reconstruction: a brute-force approach to astronomical imaging and machine learning

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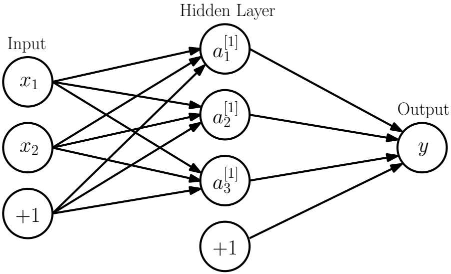
Accepted 2018 December 3. Received 2018 November 25; in original form 2018 September 10

- Idea here is to explore a fully Bayesian approach to fitting functions and machine learning
- Moreover one in which we can ensure sparsity via choices of prior
- Model a signal as a sum of N basis functions, of a type T belonging to discrete classes and with a set of continuous controlling parameters
- Sparsity can be enforced e.g. by a prior on N
- Full exploration of the posterior allows one to carry out the equivalent of ‘dictionary learning’

Recent Bayesian work II

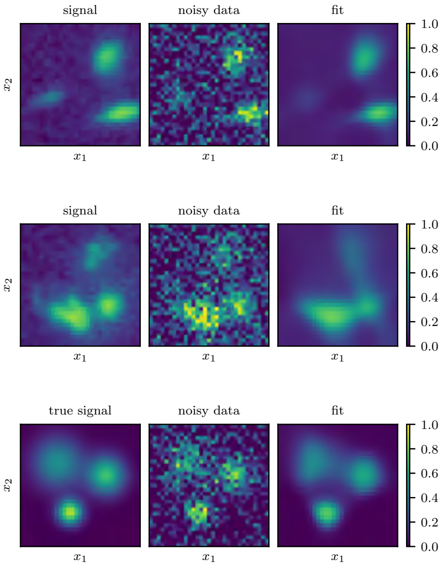
- Implied 'trans-dimensional' sampling is dealt with by a product space approach, and 'Evidence' not needed as such — we get directly the posterior ratios of different models as subsets of the total space
- This is computationally pretty heavy, so have only applied to toy problems so far
- But qute interesting nevertheless — e.g. one application in paper is to neural network architecture

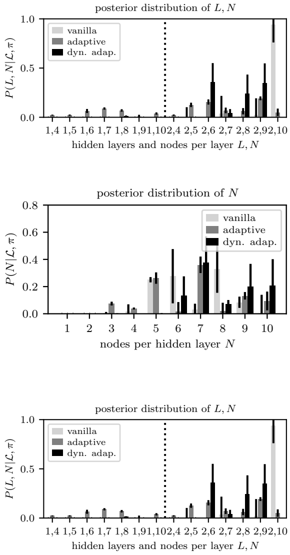


We can let the parameters of this architecture, such as the number of nodes in a hidden layer, be part of the parameters of our problem

Can then 'marginalise' over these — quite novel

2-D examples: generalised Gaussians & HST images





Recent Bayesian work I

- Note the computational side of this was made possible by using the new slice-sampling replacement for MultiNest, called *Polychord* and a *Dynamic Nested Sampling* program called DyPolyChord
- See Handley, Hobson & Lasenby, MNRAS, **450**, L61 (2015), and MNRAS, **453**, 4384 (2015) for PolyChord
- And Higson *et al.* Astrophysics Source Code Library, record ascl:1902.010 for the DyPolyChord program (significant speed-ups with both)

Closed universes

- For some while have been interested in a closed model of the universe satisfying a particular boundary condition
- This is that the total elapse of conformal time, as t goes from a big bang start to $t = \infty$, should be $\eta = \pi/2$
- Can think of this condition as being 'half' of what a photon would do in pure de Sitter space, but where midpoint of the embedding has been replaced by the big bang

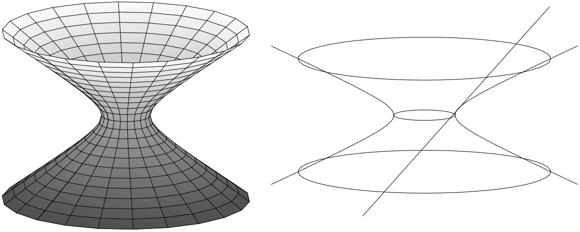
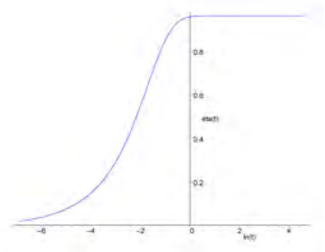


Figure 2: *Two-dimensional de Sitter Space.* The timelike direction is vertical, and spatial sections are closed. The right-hand diagram shows a null geodesic, which is a straight line in the embedding space.

Closed universes I

- This very simple requirement (which can be motivated in a variety of alternative ways), when coupled with inflation, has some surprising effects
- This is because inflation ‘eats up’ some conformal time, but then makes it difficult for the universe to achieve more conformal time except by lasting a very long (cosmic) time



- $\eta = \int dt/R(t)$ increases rapidly then saturates at about 0.967
- Therefore need about 0.604 in η to be made up in rest of development of universe

Closed universes II

- But the $R(t)$ by this stage (end of plot) is already 2.44×10^{22}!
- So universe is going to have to last a very long time further to make up the deficit in η
- Assume evolution from $t = t_0 = 14.56$ onwards satisfies a $R \propto (t - t_0)^{2/3}$ behaviour (crude approx but ok order of magnitude)
- Equation need to solve to find the age T of the rest of the universe is then:

$$\frac{(100 - 14.56)^{2/3}}{2.44 \times 10^{22}} 3T^{1/3} = 0.604$$

- Therefore need the universe to exist for a time equivalent to 1.63×10^{61} Planck lengths to build up the rest of η (this is about 2.8×10^{10} years).
- This gives a good illustration of how large numbers, such as the age of the universe in Planck times, can come out of inflation plus our boundary condition

Closed universes III

- This has implications for the cosmological constant, and in fact can show that

$$\boxed{\Lambda \approx e^{-6N} l_{\mathrm{pl}}^{-2}}$$

- Here N is the number of e-folds
- $N \approx 46$ (typical for closed universe inflation and nothing to do with our boundary condition) implies $\Lambda \sim 10^{-122} l_{\mathrm{pl}}^{-2}$, exactly in the right ball park!
- So all of this was first gone through in the 2005 paper Lasenby & Doran, Phys.Rev.D **71**, 063502
- First full Bayesian comparison with the observations (in which it did quite well!) in Bridges et al. (2007) (MNRAS, **381**, 68)
- Then further tests in Vazquez *et al.*, MNRAS **442**, 1948 (2012) and JCAP, **08, 001 (2013)**
- Again did well, but more recently model is in trouble, since based on ϕ^2 potential and this now effectively ruled out by current B-mode constraints

Closed universes IV

- You are probably thinking isn't curvature itself now ruled out?

- Planck 2018 Parameters paper gives results at right:
- But CAMSPEC for same data gives $\Omega_K = -0.037^{+0.019}_{-0.014}$
- And if add CMB lensing and then BAO get successively $\Omega_K = -0.0106 \pm 0.0065$ and $\Omega_K = 0.0007 \pm 0.0019$ though clearly there is some tension underneath this increase in precision

The combination of the *Planck* temperature and polarization power spectra give

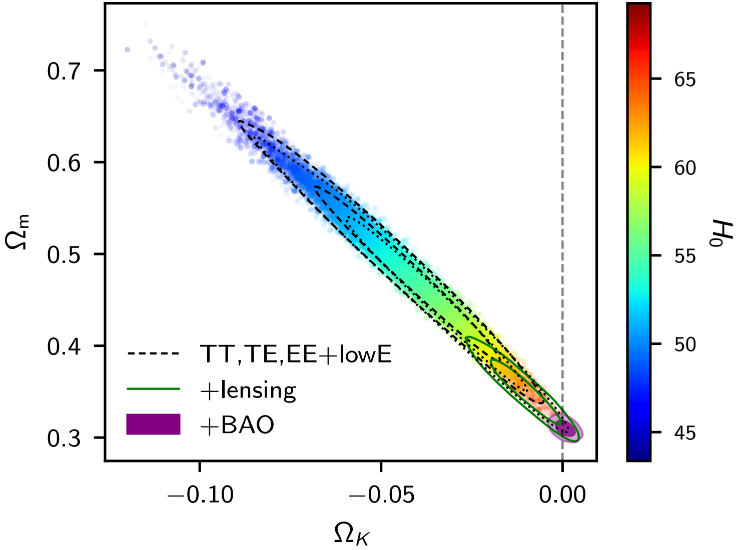
$$\Omega_K = -0.056^{+0.028}_{-0.018} \quad (68\,\%,\ \textit{Planck}\ \text{TT+lowE}), \qquad (46a)$$

$$\Omega_K = -0.044^{+0.018}_{-0.015} \quad (68\,\%,\ \textit{Planck}\ \text{TT,TE,EE+lowE}), \qquad (46b)$$

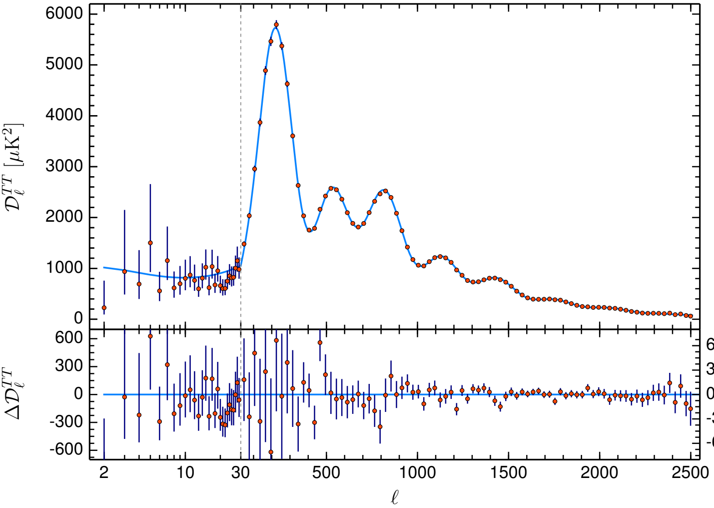
an apparent detection of curvature at well over $2\,\sigma$. The 99 % probability region for the TT,TE,EE+lowE result is $-0.095 < \Omega_K < -0.007$, with only about 1/10000 samples at $\Omega_K \geq 0$.

- In examples below will use what we can call a 'CMB value' of $\Omega_K = -0.01$

Planck 2018 curvature constraints

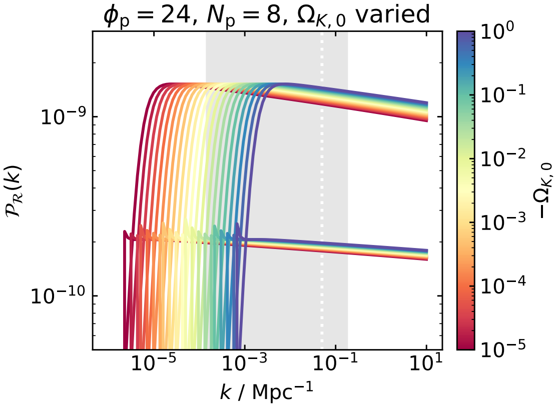


The 2018 Planck power spectrum



- Generally somewhat low in the low-ℓ region and a possible feature around $\ell = 20 - 30$ — signs of curvature?

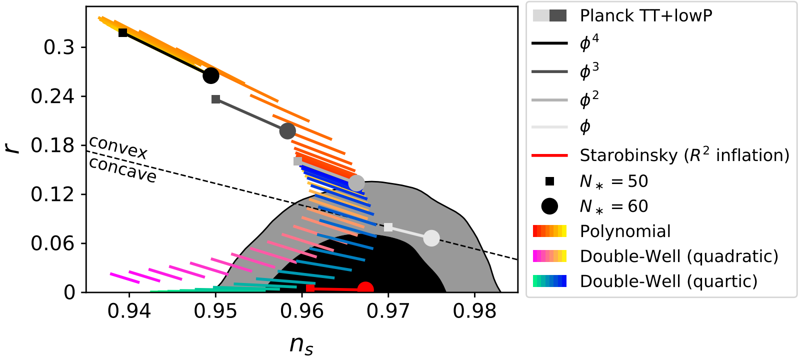
Cutoff in power spectrum versus Ω_k



(Lukas Hergt *et al.*, in preparation)

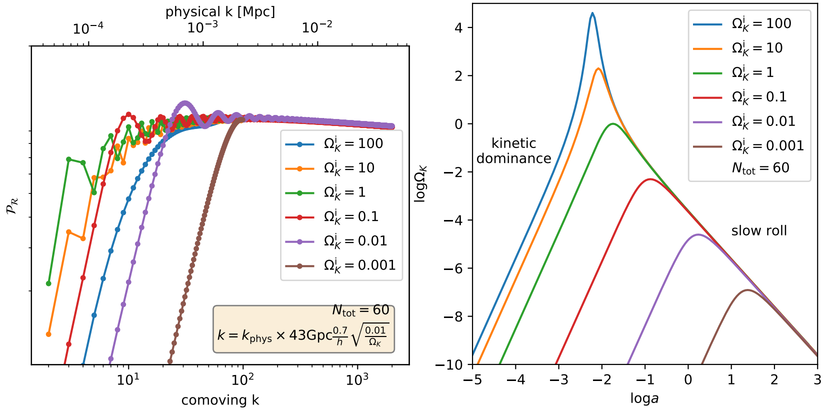
- This computed using slow roll approximation for spectrum and with ϕ^2 inflaton potential (same as for previous studies)
- Need to extend to integrating full Mukhanov-Sasaki mode equations and to more general potentials

r versus n_s for ϕ^2 and ϕ^4 combinations



(Lukas Hergt *et al.*, in preparation)

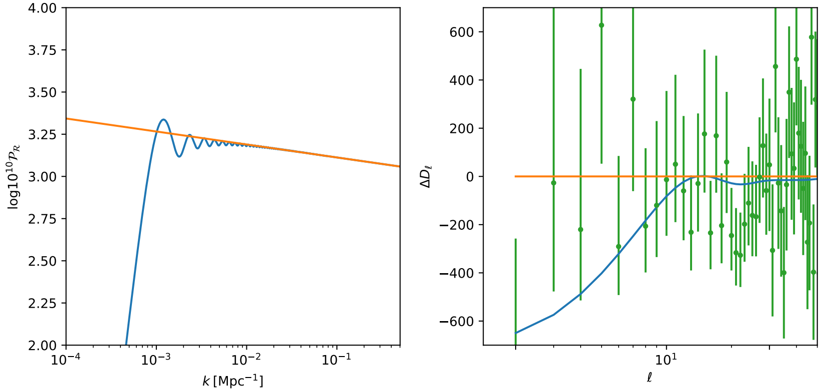
Primordial power spectrum shape versus curvature



(Will Handley *et al.*, in preparation)

- This uses full Mukhanov-Sasaki mode evolution in closed case (however still for ϕ^2 potential)
- Bunch-Davies initial conditions are set at peak curvature

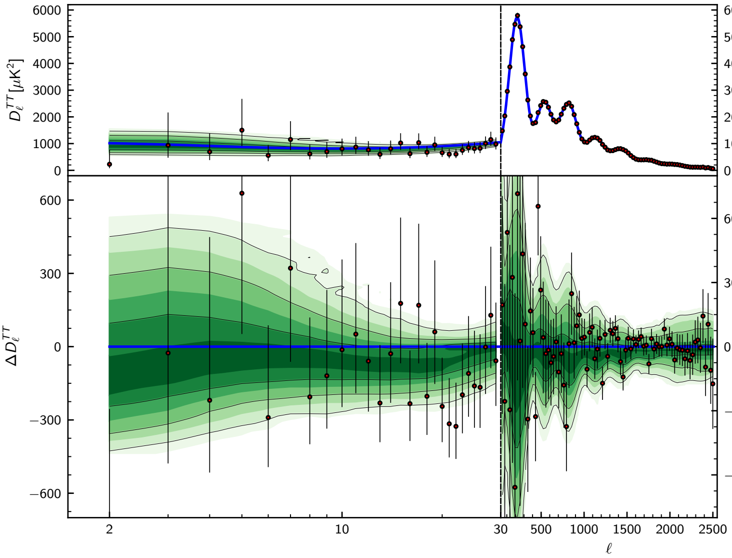
Effect on CMB power spectrum



(Will Handley *et al.*, in preparation)

- Worth noting that a free-form reconstruction of the primordial power spectrum from the data, and then transferring *this* through to the C_ℓs, indicates a lower amplitude of feature favoured in any case

Effect on CMB power spectrum



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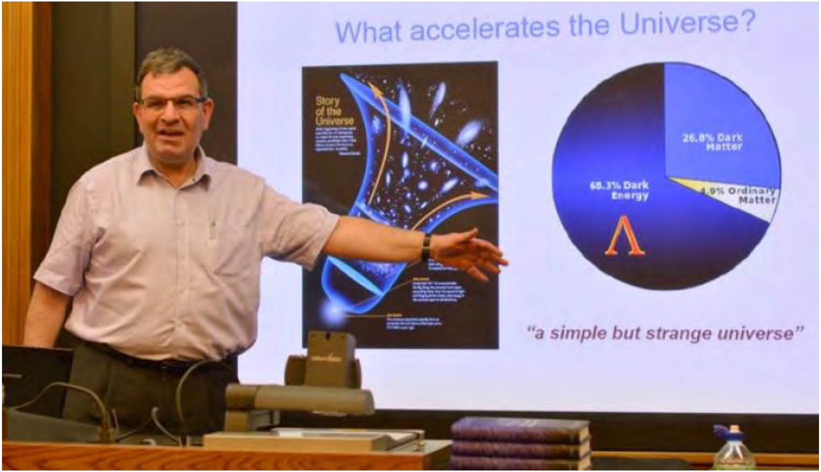
What accelerates the Universe?
Story of the Universe
26.8% Dark Matter
68.3% Dark Energy
4.9% Ordinary Matter
Λ
"a simple but strange universe"