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# ROTIONS: A program for the calculation of rotational excitation cross sections in electron–molecular ion collisions

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## Abstract

ROTIONS is a program for the calculation of cross sections for rotational excitation of linear molecular ions by electron impact. The cross sections can be obtained from input  $T$ -matrices and from the Coulomb–Born approximation for dipole ( $\Delta j = 1$ ) and quadrupole ( $\Delta j = 2$ ) transitions. The Coulomb–Born results can be used to augment the low partial-wave  $T$ -matrices to an effectively infinite number of partial waves. © 1998 Elsevier Science B.V.

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Keywords: Rotational excitation cross sections; Coulomb–Born approximation; Electron impact

## PROGRAM SUMMARY

*Title of program:* ROTIONS

*Catalogue identifier:* ADJA

*Program Summary URL:*

<http://www.cpc.cs.qub.ac.uk/cpc/summaries/ADJA>

*Program obtainable from:* CPC Program Library, Queen's University of Belfast, N. Ireland

*Licensing provisions:* none

*Computer for which the program is designed and others on which it is operable:*

*Computers:* Digital 8400 (known as Columbus), CRAY J90, IBM RS6000 workstations; *Installations:* Rutherford Laboratory, local

*Operating systems under which the program has been tested:* Unix

(OSF/1, UNICOS 8, AIX 4)

*Programming language used:* Fortran 90

*Memory required to execute with typical data:* 3 Mwords

*No. of bytes in distributed program, including test data, etc.:* 794164

*Distribution format:* uuencoded compressed tar file

*CPC Program Library subprograms used:* COULFG [1] (catalogue number: ABNK)

*Keywords:* rotational excitations cross sections, Coulomb–Born approximation, electron impact

*Nature of physical problem*

ROTIONS calculates rotational cross sections from body-frame  $T$ -matrices and the Coulomb–Born (CB) approximation for

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electron–molecular ion collisions.

#### Method of solution

The body-frame  $T$ -matrices are frame transformed into the lab-frame, and the corresponding cross sections obtained. At the same energies, the CB approximation is used to obtain both total and partial rotational cross sections. The low- $l$  partial-wave contribution to the total CB cross section is substituted by the  $T$ -matrix cross section [2].

#### Restrictions on the complexity of the problem

ROTIONS is restricted to the study of linear targets. The CB approximation is only applicable when the target is charged.

#### Typical running time

The only part of the program that takes significant time is the

calculation of CB cross sections for quadrupole transitions. It can be up to 0.5 s per energy on Columbus.

#### Unusual features of the program

ROTIONS uses the NAG routine D01AKF [3] (not included) to perform a numerical integration.

#### References

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- [2] I. Rabadán, B.K. Sarpal, J. Tennyson, *J. Phys. B* 31 (1998) 2077.
- [3] NAG Fortran Library, The Numerical Algorithms Group Ltd. (Oxford, UK, 1996).

## LONG WRITE-UP

### 1. Introduction

Cross sections for the rotational excitation of molecular ions by collisions with electrons have been traditionally calculated using the dipole Coulomb–Born (CB) approximation [1–3,5]. In the CB approximation, only the long-range interactions are taken into account and the dipole interaction leads to only  $\Delta j = 1$  transitions.

However, our recent work [6], I hereafter, shows that the short-range interactions can be as important as the long-range ones for dipolar interactions, and the main interaction for higher multipoles. In I, we studied the cases of  $\text{HeH}^+$ , an ion with a large permanent dipole, and  $\text{NO}^+$ , an ion with a large permanent quadrupole. For  $\text{HeH}^+$ , the inclusion of short-range interactions increased the rotational excitation cross section by almost a factor of 2. It was also shown that the, previously disregarded, cross sections for quadrupole transitions are only a factor of 2 smaller than dipole ones, which can lead to significant population of the  $j = 2$  rotational level [7] and, under favourable conditions, observation of  $j = 2 - 1$   $\text{HeH}^+$  emissions in interstellar plasmas. For  $\text{NO}^+$ , quadrupole CB rotational cross sections were found to be dominated by low- $l$  partial waves. However, low- $l$  electron partial waves sample the internal structure of the molecular ion and, thus, are subject to short-range interactions. In I, the inclusion of short-range effects produced substantially different rotational cross sections than those given by the CB approximation.

Short-range interactions in electron–molecular ion collisions can be introduced using close-coupling (CC) methods. In the case of electron-neutral molecule collisions, CC methods have been implemented to study rotational, vibrational and electronic transitions [8]. However, implementations in the case of electron–molecular ion scattering only cover, so far, vibrational and electronic excitation [9]. With ROTIONS, we extend the range of applications to the study of rotational transitions.

ROTIONS uses, as its starting point, the body-frame (BF)  $T$ -matrices produced by reliable CC methods to obtain, through a frame-transformation to the laboratory frame, rotational cross sections. As the number of partial waves used to represent the projectile electron in CC methods is necessarily limited, the cross sections thus obtained lack the contribution of high partial waves. In ROTIONS, the contribution of these high-partial waves to the rotational cross section is evaluated using the CB approximation. This approximation has been implemented only for transitions with  $\Delta j \leq 2$ . Transitions with  $\Delta j > 2$  can be significant [4,6], but the high- $l$  CB contribution to the rotational cross sections is negligible for these transitions, and they can be safely evaluated using BF  $T$ -matrices alone. On the other hand, ROTIONS can be used to calculate CB rotational excitation cross sections with  $\Delta j \leq 2$  alone.

In I, we calculated rotational cross sections from  $T$ -matrices obtained for three different treatments of nuclear motion: (i) the fixed-nuclei approximation, in which the  $T$ -matrices are obtained at the target equilibrium geometry; (ii) the adiabatic approximation, with the  $T$ -matrices obtained by vibrational averaging fixed-nuclei  $T$ -matrices calculated at different geometries; and (iii) the non-adiabatic approximation, in which the  $T$ -matrices are obtained from wavefunctions that include nuclear motion terms. Results from fixed-nuclei and adiabatic  $T$ -matrices were found in I to be very similar, while those from non-adiabatic  $T$ -matrices contained Feshbach resonances associated with nuclear excited states of the molecular Rydberg states. These resonances appear as oscillations around a background cross section similar to the fixed-nuclei and adiabatic ones. Their effect tends to be washed out when averaging in energy around the resonance, so this detailed treatment is probably unnecessary to obtain reliable rotational excitation rates.

Rotational excitation cross sections calculated from BF  $T$ -matrices and the CB approximation do not have the correct behaviour at threshold due to the assumed degeneracy of rotational states. Threshold effects are important at low energy collisions, especially for light systems such as  $\text{HeH}^+$  [6], and a kinematic correction [10,11] is implemented in ROTIONS (see Section 2.4).

ROTIIONS has been implemented within the UK molecular  $R$ -matrix package [9,12]. This means that the  $T$ -matrices are read according to the output format of the  $R$ -matrix package. However, as long as the  $T$ -matrix files are produced in this format (detailed in Section 5), ROTIONS is ‘CC-method’ independent.

When several spin states of the total system have to be considered in the rotational excitation of a target state from BF  $T$ -matrices, each spin-state requires a separate calculation with ROTIONS. Similarly, if the system is homonuclear, molecular symmetries  $g$  and  $u$  need to be treated separately. To obtain the total rotational cross section including contributions from different spin states and symmetries  $g/u$ , when applicable, the user has to sum the separate results allowing for spin-statistical weights; the present spin weighting is appropriate for a  $^1\Sigma^+$  target state. On the other hand, rotational cross sections in the CB approximations are independent of the total spin-state and, in the case of homonuclear molecules, the rotational transitions  $j = 0 \rightarrow 2$  include both even and odd  $l$ -waves (i.e.,  $g$  and  $u$  symmetries).

The next section contains the outline of the theory implemented in ROTIONS, Section 3 presents the structure of the code and the task of each subroutine, Section 4 describes the input data required and Section 5 the format of the  $T$ -matrix file. Section 6 describes the test data, test run and output.

## 2. Theory

The method ROTIONS implements is described in detail in I, which made extensive use of earlier work on this problem [2,11,13–15]. Here, we summarise the method.

### 2.1. $T$ -matrix rotational cross sections

ROTIIONS starts with energy dependent BF  $T$ -matrices. These are frame-transformed to the laboratory-frame (LAB),

$$T_{j'v',jl}^J \simeq \sum_{A=-1}^1 A_{j'v'}^{JA} T_{v',l}^A A_{jl}^{JA}, \quad (1)$$

using the rotational frame transformation matrix  $A$  [11]. The cross sections are, then, obtained using the equation

$$\sigma^{\text{TM}}(j \rightarrow j') = \frac{\pi}{k_i^2 (2j+1)} \sum_{J=0}^{\infty} \sum_{l=|J-j|}^{J+j} \sum_{l'=|J-j'|}^{J+j'} (2J+1) |T_{j'v',jl}^J|^2, \quad (2)$$

where  $k_i$  is the projectile kinetic momentum,  $T_{j'l',jl}^J$  are the LAB  $T$ -matrices obtained from the BF  $T$ -matrices  $T_{j'l',jl}^A$ ,  $l$  labels the projectile angular momentum,  $j$  the target rotational state and  $J$  is the total angular momentum of the target plus projectile system. The sum over  $J$  is, in practice, limited by the number of partial waves included in the  $T$ -matrices,  $LMAX \geq l, l'$ .

## 2.2. Coulomb–Born rotational cross sections

CB cross sections are (optionally) evaluated at the energies where the BF  $T$ -matrices are given or at a user-specified energy grid if no  $T$ -matrices are given. If  $\Delta j = 1$ , the dipole CB cross section is obtained; if  $\Delta j = 2$ , the corresponding quadrupole CB cross section. No implementation has been made for transitions with  $\Delta j > 2$ .

The total dipole CB cross section is obtained by the closure formula

$$\sigma_1^{\text{CB}}(j \rightarrow j') = \frac{3Q_1^2}{4\pi} \frac{2j'+1}{k_i^2} \begin{pmatrix} j & j' & 1 \\ 0 & 0 & 0 \end{pmatrix} f_{E1}(\eta_i, \xi), \quad (3)$$

where  $Q_1$  is the permanent dipole moment of the target,  $\eta = -1/k$ ,  $\xi = \eta_f - \eta_i$  and  $f_{E1}$  is a dipole Coulomb excitation function expressed in terms of hypergeometric functions.

The partial dipole CB ( $\sigma_1^{\text{PCB}}$ ), total quadrupole CB ( $\sigma_2^{\text{CB}}$ ) and partial quadrupole CB ( $\sigma_2^{\text{PCB}}$ ) cross sections are obtained using the equation

$$\sigma_\lambda^{\text{CB,PCB}}(j \rightarrow j') = 16\pi \frac{k_f}{k_i} Q_\lambda^2 \frac{2j'+1}{2\lambda+1} \begin{pmatrix} j & j' & \lambda \\ 0 & 0 & 0 \end{pmatrix} \sum_{l_i l_f} (2l_i+1)(2l_f+1) \begin{pmatrix} l_i & l_f & \lambda \\ 0 & 0 & 0 \end{pmatrix}^2 \left| M_{l_i, l_f}^{-\lambda-1} \right|^2, \quad (4)$$

where  $\lambda = 1$  for the dipole and  $\lambda = 2$  for quadrupole transitions, and  $M_{l_i, l_f}$  are radial matrix elements defined as

$$M_{l_i, l_f}^{-\lambda-1} = \frac{1}{k_i k_f} \int_0^\infty F_{l_f}(k_f r) r^{-\lambda-1} F_{l_i}(k_i r) dr. \quad (5)$$

Here,  $F_l(kr)$  is the regular radial Coulomb function of angular momentum  $l$ .

The sum in Eq. (4) is performed up to  $l_i = l_f = LMAX$ , in the case of the partial CB cross sections, and up to  $l_i = l_f = LMAXQ$  for the total quadrupole CB cross section;  $LMAXQ$  has to be high enough to obtain convergence ( $LMAXQ=40$  is a safe value).

The integrals in Eq. (5) are evaluated numerically only for  $M_{02}^{-3}$  and  $M_{20}^{-3}$ . The remaining elements are obtained using recurrence relations. In the dipole case, all radial matrix elements are obtained with recurrence relations from the first two monopoles, which have an analytical solution in terms of a hypergeometric function, see I.

## 2.3. Final rotational cross section

The final cross section, which includes the low- $l$  partial wave information from the  $T$ -matrices, and the high- $l$  cross section from the CB calculations, is obtained simply by

$$\sigma(j \rightarrow j') = \sigma^{\text{TM}}(j \rightarrow j') + \sigma_\lambda^{\text{CB}}(j \rightarrow j') - \sigma_\lambda^{\text{PCB}}(j \rightarrow j'). \quad (6)$$

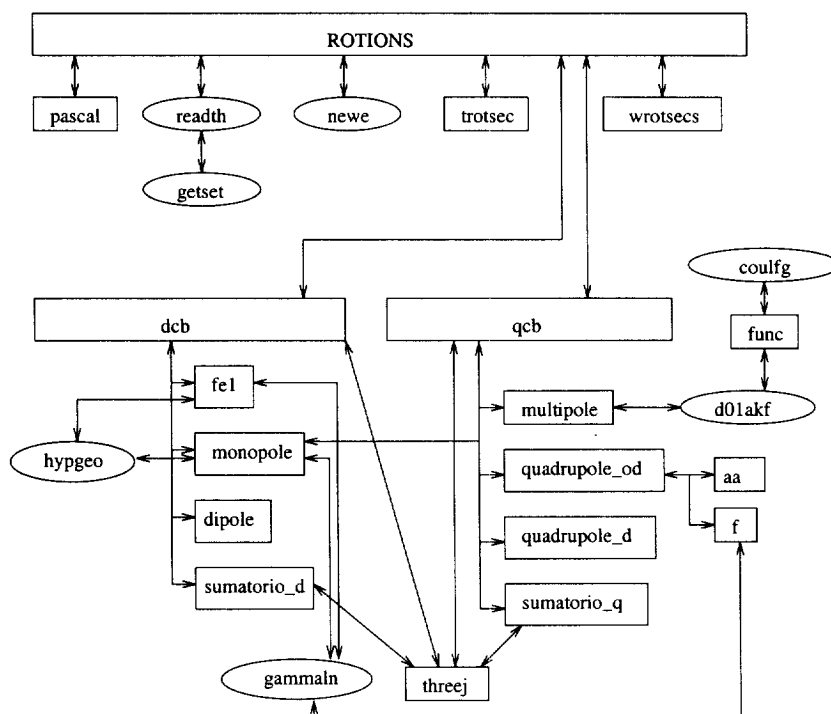


Fig. 1. Flow diagram of the program ROTIONS. Routines in ovals have been taken from existing libraries.

#### 2.4. Threshold correction

The theory implemented in ROTIONS assumes that the rotational states of the target are degenerate, which leads to rotational excitation cross sections that do not go to zero at threshold. This is (optionally) corrected by multiplying the rotational cross sections by a kinematic ratio [10,11]

$$\sigma^{\text{lc}}(j \rightarrow j') = \frac{k_f}{k_i} \sigma(j \rightarrow j'), \quad (7)$$

where  $k_i$  and  $k_f$  are the initial and final projectile momentum, respectively. We note that this correction can lead to cross sections that do not obey detailed balance [7] and de-excitation cross sections should be evaluated, in this case, from the excitation ones, using the detailed balance relation.

### 3. Description of the program

Fig. 1 shows the linkage of the subroutines in ROTIONS. What follows is a description of each subroutine. Further comments are given within the code.

**ROTIIONS** is the central driving routine. It reads the input parameters and drives the calculation accordingly.

**PASCAL** initializes a  $100 \times 100$  matrix with binomial coefficients to use in the calculation of  $3j$ -symbols.

**READTH**, **GETSET**, **NEWE** are routines taken from the UK molecular  $R$ -matrix package [12] to read the  $T$ -matrix files, and set the collision energy range.

**TROTSEC** calculates the  $T$ -matrix rotational excitation cross sections ( $\sigma^{\text{TM}}$ ) from BF  $T$ -matrices by performing the frame transformation of Eq. (2).

**DCB** calculates the dipole CB rotational cross sections,  $\sigma_1^{\text{CB,PCB}}$ .

**FE1** evaluates the total CB rotational cross sections of Eq. (3).

**MONOPOLE** obtains the monopole radial matrix elements: Eq. (5) for  $\lambda = 0$ .

**DIPOLE** uses monopole radial matrix elements to calculate dipole ones ( $\lambda = 1$ ).

**SUMATORIO.D** performs the sum of Eq. (4).

**QCB** evaluates the quadrupole CB cross sections,  $\sigma_2^{\text{CB,PCB}}$ .

**MULTIPOLE** obtains two quadrupole radial matrix elements by numerical integration of Eq. (5). It can also be used to obtain any multipole of the same equation.

**QUADRUPOLE\_OD** computes the off-diagonal quadrupole radial matrix elements. It uses functions **AA** and **F** that specify the recurrence relations between monopole radial matrix elements and quadrupole ones.

**QUADRUPOLE.D** computes the diagonal quadrupole radial matrix elements.

**SUMATORIO.Q** obtains the partial and total CB rotational cross sections by performing the sum (4) up to  $l_i, l_f = \text{LMAX}$  and  $l_i, l_f = \text{LMAXQ}$ , respectively ( $\text{LMAXQ} \geq \text{LMAX}$ ).

**FUNC** is a function that contains the integrand of Eq. (5).

**HYPGEO** is a routine to evaluate complex hypergeometric functions taken from Numerical Recipes [16].

**D01AKF** is a NAG routine [17], not provided, which evaluates the integral of Eq. (5). It integrates the function supplied in function **FUNC** over the range  $AQ$  to  $BQ$  and returns the result in  $XM$ . **D01AKF** uses an 1-D quadrature, adaptative, finite interval, method suitable for oscillating functions.

**COULFG** is a CPC library routine (catalogue number **ABNK**) [18], not supplied. It evaluates the regular radial Coulomb function for a given energy and at a given distance.

**GAMMLN** evaluates  $\ln \Gamma(z)$ , where  $z$  a complex number. It is a modified version of a Numerical Recipes [16] routine.

**THREEJ** Obtains the 3j-symbols required.

**WROTSECS** is the routine to print out the rotational cross sections obtained. It makes the units conversions required and sorts the different cross sections into columns, suitable for display with a graphics package.

#### 4. Input data

A single namelist **/RXSECIN/** is required. The default values are given in brackets. This is read from the standard input.

<b>NAME</b>	Character*80 variable to contain a title for the output.
<b>ANR_OP</b>	[1] = 1, threshold corrected cross sections, see Eq. (7), are computed; = 0, no threshold correction.
<b>BE</b>	Rotational constant of the ion. Units: $\text{cm}^{-1}$ .
<b>CB_OP</b>	[0] = 0, only $T$ -matrices are used to obtain rotational cross sections. = 1, $T$ -matrix and CB rotational cross sections are obtained. = 2, only the CB rotational cross sections are obtained. CB_OP = 1, 2 will only work if the target is charged and if $\Delta j = 1$ or 2.
<b>EMIN</b>	[0.0] Minimum scattering energy (in units as specified by <b>ieunit</b> ).
<b>EMAX</b>	[ $1 \times 10^8$ ] Maximum scattering energy (in units as specified by <b>ieunit</b> ).
<b>GRID</b>	[1] Energy grid mode to use when CB_OP=2: 1=linear, 2=logarithmic.

ITFORM	['F'] T-matrix data format: 'F' = Formatted, 'U' = Unformatted.
IEUNIT	[1] Units of the input and output scattering energies. 1 = eV, 2 = Hartree, 3 = Ryd.
ION	[1] Charge of the target.
IPRNT	[6*0] Debug print switches: IPRNT(1) = 1 print all input data. IPRNT(4) = 1,2,3 are increasing debug levels for the rotational cross section calculations.
IWRITE	[6] Logical unit for standard output.
IXSN	[1] Units of the cross sections on output. 1 = $a_0^2$ , 2 = $\text{Å}^2$ , 3 = $\pi a_0^2$ .
IXSOUT	[8] Logical unit to output the cross sections.
J	[0] Initial rotational quantum number of the target.
JP	[1] Final rotational quantum number of the target.
MJJ	[6] Maximum total angular momentum to be considered in the rotational cross sections calculations from BF T-matrices.
LUTMV	[NSYM*12] Vector of dimension NSYM containing the logical unit numbers for the T-matrix data sets.
MAXI	[1] Label of highest initial state for which cross sections are required.
MAXF	[0] Label of highest final state for which cross sections are required.
NTSET	[1] Set number of T-matrix input.
NSYM	[0] Number of molecular symmetries to be considered.
NUMENER	[0] Number of energies to be read from the T-matrix files. If = 0, all energies will be read. If CB_OP=2, this is the number of energies to set up an energy grid.
Q1	[0.0] Permanent dipole moment of the target (in Debye).
LMAX	[5] Maximum value of $l$ (partial waves) to be used in the partial CB rotational cross sections.
LMAXQ	[40] Number of partial waves to use for the total CB quadrupole rotational cross sections.
Q2	[0.0] Permanent quadrupole of the target in atomic units.
AQ	[0.1] Lower limit for the numerical integral of the quadruple radial matrix elements M_02 and M_20.
BQ	[ $2 \times 10^4$ ] Upper limit for the numerical integral of the quadrupole radial matrix elements M_02 and M_20.

## 5. *T*-matrix file format

The files containing the *T*-matrices can be formatted or unformatted. For the formatted option, the header of the files is read with the following code (or equivalent):

```

READ(LUTMT,FMT='(5I5)') KEY,NSET,NREC
READ(LUTMT,FMT='(A80)') TITLE
READ(LUTMT,FMT='(5I5,F15.8)') MGVN,STOT,GUTOT,NCHAN,NAPPR,MAXNE,NVIB,
&                                NDIS,NTARG,MAXCHI,MAXCHF,R
DO I=1,MAXCHF
  READ(LUTMT,*) ICHL(I),LCHL(I),MCHL(I),ECHL(I)
END DO
DO IE=1,MAXNE
  READ(LUTMT,*) I,NEREP(I),(EINC(J,I),J=1,2)
END DO

```

while the *T*-matrices are read with

```

DO IE=1,NE
  READ(LUTMT,FMT='(3I5,F10.6)') MVI,MVJ,MVD,EN(IE)
  READ(LUTMT,FMT='(4E20.13)') ((TR(I,J,IE),I=1,MVI),J=1,MVJ+MVD)
  READ(LUTMT,FMT='(4E20.13)') ((TI(I,J,IE),I=1,MVI),J=1,MVJ+MVD)
END DO

```

For the unformatted option, the code used is the same but without specifying the formatting labels.

The meaning of the variables is as follows:

KEY	12.
NSET	<i>T</i> -matrix data set number within the file.
NREC	Number of records in the file. If the file is formatted, then $NREC = 1 + NETOT * (2 * ((MAXCHI * MAXCHF + 3) / 4) + 1) + 1 + MAXCHF + MAXNE$ ; if the file is unformatted, $NREC = 1 + 3 * NETOT + 1 + MAXCHF + MAXNE$ .
TITLE	A character*80 title.
MGVN	Molecular symmetry of the electron plus target system. 0 = $\Sigma$ , 1 = $\Pi$ , 2 = $\Delta$ , etc.
STOT	Spin multiplicity of the total system.
GUTOT	g/u symmetry of the total system. 1 = g, -1 = u, 0 = not applicable.
NCHAN	Number of channels.
NAPPR	Theory approximation: 1 = fixed-nuclei <i>T</i> -matrix, 2 = adiabatic <i>T</i> -matrix, 3 = non-adiabatic <i>T</i> -matrix.
MAXNE	Number of energy subranges.
NVIB	Number of vibrational levels.
NDIS	Number of dissociative channels.
NTARG	Number of target electronic states.
MAXCHI	Maximum number of initial channels.
MAXCHF	Maximum number of final channels.
R	Internuclear distance ( $a_0$ ).
ICHL(I)	Number of the electronic state of the Ith channel.
LCHL(I)	<i>l</i> -value of the Ith channel.
MCHL(I)	<i>m</i> -value of the Ith channel.
ECHL(I)	Threshold energy of the Ith channel.
NEREP(IE)	Number of energies in the IEth energy subrange.



EINC(1,IE)	Initial value of the energy in the IEth subrange.
EINC(2,IE)	Increment in the energy value in the IEth subrange.
MVI	Number of initial channels.
MVJ	Number of final channels.
MVD	Number of dissociative channels.
EN	Vector with the energies where the $T$ -matrix has been calculated (in Ryd).
TR	Real part of the $T$ -matrix.
TI	Imaginary part of the $T$ -matrix.

## 6. Test data

The present test run corresponds to the calculation of the rotational excitation cross section of  $\text{HeH}^+$  ( $j = 0 \rightarrow 1$ ) by electron impact. We use the  $T$ -matrices for electron– $\text{HeH}^+$  collisions at the  $\text{HeH}^+$  equilibrium geometry calculated by Sarpal et al. [19]. Four molecular symmetries are considered:  $^2\Sigma^+$ ,  $^2\Pi$ ,  $^2\Delta$  and  $^2\Phi$ , which should be available in files fort.50, fort.51, fort.52 and fort.53, respectively. These  $T$ -matrices were obtained using three target states and electron partial waves up to  $l = 5$ .

The standard input for the present test run is given below. ROTIONS will read this input and produce a standard output plus the file fort.8 containing the rotational excitation cross sections. The standard output contains information about the  $T$ -matrices that are input and, depending of the options specified, total rotational excitation cross sections from  $T$ -matrices and/or from the CB approximation. The columns of data in the file fort.8 correspond to the several cross sections in Eq. (6). The results are threshold corrected according to Eq. (7).

## Acknowledgements

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**TEST RUN INPUT**

```
&rxsecin
name=' HeH+ equilibrio rotational XS ',
numener=10, nsym=4,
lutmtv=50,51,52,53, itform='F',
ieunit=1, ixsn=2,
anr_op=1, cb_op=1,
lmax=5, q1=1.66, q2=1.397864,
j=0, jp=1, mjj=6,
be=33.47278,
aq=1.d-3,bq=2.d3,
maxi=3, maxf=6,
&END
```

**TEST RUN OUTPUT**

The (edited) standard output of ROTIONS for the test run is

Program ROTIONS (last modified 20-Mar-1998 )

HeH+ equilibrio rotational XS

The charge of the target is 1

The cross sections will include the ANR correction.

Coulomb-Born cross sections will be obtained.

-----T-matrix 1

T-matrices will be read from set 1 on unit 50  
(HEH 2-PHI T-MATRIX )

Symmetry data MGVN = 0 STOT = 2 GUTOT = 0

Fixed nuclei data for R = 1.455

Number of target states = 3

Number of scattering channels = 18

Channel data corresponding to stored T-matrix elements

Element	Target	L	M	Energy
1	1	0	0	0.00000
2	1	1	0	0.00000
3	1	2	0	0.00000
4	1	3	0	0.00000
5	1	4	0	0.00000
6	1	5	0	0.00000
7	2	0	0	1.59352
8	2	1	0	1.59352
9	2	2	0	1.59352
10	2	3	0	1.59352
11	2	4	0	1.59352
12	2	5	0	1.59352
13	3	0	0	1.93844
14	3	1	0	1.93844
15	3	2	0	1.93844
16	3	3	0	1.93844
17	3	4	0	1.93844
18	3	5	0	1.93844

Input Energy grid (Ryd)

Number of points 400

Initial values 0.00200

Increments 0.00200

The number of partial waves is 6 (l= 0 to 5)

Cross-sections can be computed for 400 energies in the T-matrix file  
from 0.27211E-01 to 10.884 eV

Only the first 10 T-matrices will be used

```

---> T-matrix 1; starting pointers with next=      1

T-matrices in symmetry 1 have been read for      10 energies

....
lines deleted
....

```

\*\*\*\* Rotational excitation cross sections from T-matrices

	energy (eV)	X Section(A <sup>2</sup> )	Col. strength
1	0.02721	282.628	0.643
2	0.05442	158.606	0.721
3	0.08163	110.816	0.756
4	0.10885	85.787	0.780
5	0.13606	70.390	0.800
6	0.16327	59.950	0.818
7	0.19048	52.392	0.834
8	0.21769	46.658	0.849
9	0.24490	42.151	0.862
10	0.27211	38.511	0.876

\*\*\*\* Coulomb-Born rotational cross sections.

	ei (eV)	ef (eV)	TCB (A <sup>2</sup> )
1	0.027211	0.018911	816.409
2	0.054423	0.046123	487.382
3	0.081634	0.073334	357.758
4	0.108846	0.100545	288.423
5	0.136057	0.127757	244.846
6	0.163268	0.154968	214.655
7	0.190480	0.182180	192.336
8	0.217691	0.209391	175.056
9	0.244903	0.236602	161.211
10	0.272114	0.263814	149.819

\*\*\* Task has been successfully completed \*\*\*

The file fort.8, with the rotational excitation cross sections for HeH<sup>+</sup>( $j = 0 \rightarrow 1$ ), reads

```

# HeH+ equilibrio rotational XS
#
# key= 3 R-matrix & TCB & ACB XS
# Number of energies= 10
#
# Rotational transition: j= 0--> 1
#
# Energy in eV ; Cross sections in ANGS**2
#
# Threshold energy 0.83001958E-02 eV
#
# Number of partial waves in PCB (lmax) = 5
#
# Energy          TM+TCB-PCB      TM CS          TCB CS          PCB CS
0.27211400E-01  1018.5171      282.62834     816.40898      80.520252

```

0.54422800E-01	596.13701	158.60627	487.38157	49.850828
0.81634200E-01	430.69199	110.81645	357.75774	37.882199
0.10884560	342.66886	85.786554	288.42278	31.540476
0.13605700	287.65414	70.389888	244.84618	27.581928
0.16326840	249.75565	59.949535	214.65547	24.849351
0.19047980	221.89748	52.391744	192.33595	22.830218
0.21769120	200.45071	46.657863	175.05625	21.263401
0.24490260	183.36010	42.151415	161.21076	20.002074
0.27211400	169.37262	38.510827	149.81919	18.957388