Forecasting Earthquakes

Lecture 19
Earthquake Recurrence
1) Long-term prediction - Elastic Rebound Theory

- slow accumulation of stress & strain that deforms rock on either side of fault
- weakest rocks or those at point of highest stress fracture and fault ruptures

- deformation can be monitored
- characteristic rupture return periods

Fault rupture and rebound

Stress & strain accumulation
Elastic rebound theory of tectonic earthquakes

Proposed by H F Reid (1910) to explain mechanism of 1906 SF earthquake

1. The fracture of the rock, which causes a tectonic earthquake, is the result of elastic strains, greater than the strength of the rock can withstand, produced by relative displacement of neighbouring portions of the Earth’s crust

2. These relative displacements are not produced suddenly at the time of the fracture, but attain their maximum amounts gradually during a more or less long period of time.

3. The only mass movements that occur at the time of the earthquake are the sudden elastic rebounds of the sides of the fracture towards positions of no elastic strain; and these movements gradually diminishing, extend to distances of only a few miles from the fracture.

4. The earthquake vibrations originate in the surface of fracture; the surface from which they start has at first a very small area, which may quickly become very large, but at a rate not greater than the velocity of compressional elastic waves in the rock.

5. The energy liberated at the time of an earthquake was, immediately before the rupture, in the form of energy of elastic strain of the rock.
Shimazaki - Model A (Fixed recurrence)

Slip on fault and time interval between large earthquakes are fixed.
Time between large earthquakes is proportional to the amount of slip preceding the earthquake.
Shimazaki - Model C  (Slip-predictable)

Time interval between large earthquakes is proportional to the slip amount of earthquake about to occur
Cumulative moment and seismic slip in a zone of the Calaveras fault (1962-77)

The arrow at the top right indicates the anticipated time of intersection of the slip-rate line and the projected seismic slip assuming an average rate of small ($M \leq 2.5$) earthquake activity in the interim (from Bufe, Harsh & Burford, 1977).

Favours model B - time predictable
Predicted seismic activity

In model B the length of period of high activity of earthquakes prior to a large earthquake is constant, regardless of interval between large earthquakes.

In model C the length of the active period varies and the length of the inactive period is constant.
Seismic activity Nankai trough

(a) Shows three large earthquakes that occurred after a short interval (100 yrs) following previous large earthquake

(b) Shows four large earthquakes that occurred after a long interval following previous large earthquake

Both show 50 yrs of precursory activity

Favours model B - time predictable
Review: Poisson model - discrete

If have a Poisson process, $N$ is the number of events in time, $t$, $\lambda$ is the rate, then the probability function for $N$ is:

$$\Pr[N = x] = p_N(x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t} \quad x = 0, 1, 2, ...$$

If we take unit time ($t=1$) and take $x$ to be an interval of time, $\lambda$ is the mean number of earthquakes per year, $\lambda^{-1}$ is the mean interval between earthquakes or recurrence time.

$$\Pr[N = x] = p_N(x) = \frac{\lambda^x}{x!} e^{-\lambda} \quad x = 0, 1, 2, ...$$
Review: Poisson model - continuous

If $T$ is the interval of time between two consecutive earthquakes then $T$ has *exponential probability density function*:

$$\Pr[T < x \leq T + \Delta T] = \lambda e^{-\lambda x} \Delta T$$

Note that the discrete and continuous Poisson equations can be used interchangeably, and one can be derived from the other, but the continuous version defines a frequency distribution in continuous $x$:

$$f_T(x) = \lambda e^{-\lambda x}, \quad x > 0$$

The continuous version is used most because it is easy to simulate in a computer.

The version for large earthquakes by Epstein and Lomnitz (1966) is used widely in engineering for assessing earthquake hazard.
Poisson model: magnitude distribution

a) Magnitude distribution
Remember the Gutenberg-Richter *magnitude* distribution:

\[ f_M(x) = \beta \, e^{-\beta x} \]

Here \( x \) is defined as \( M - M_t \) (where \( M_t \) is the threshold magnitude)
\( \beta^{-1} \) estimates mean magnitude
note: \( \beta = b \ln 10 \approx 2.3 \, b \)

Note we say that when the b-value rises, the mean magnitude is falling and vice-versa
The time intervals $T$ between earthquakes are assumed to be independently distributed with constant hazard function:

$$h(x) = \lambda$$

where $\lambda$ is the mean number of events per unit time.

The hazard function $h(x)$ is defined as:

$$h(x).dx = \Pr[\text{earthquake occurs in } x, x+dx \text{ given it does not occur before time } x]$$

$1/\lambda$ is the mean interval or interoccurrence time of the process. This is a standard or “pure” Poisson distribution. The interval distribution is exponential:

$$f_T(x) = \lambda e^{-\lambda x}, \quad x > 0$$

These two distributions, magnitude and interval, define the model: they are formally the same.
Non-Poissonian statistics

- Hazard function for Poisson model:
  \[ h(x) = \lambda \]

- Can generalize to:
  \[ h(x) = n \lambda x^{n-1} \]

where \( n \) is Weibull index for a Weibull process.

Poisson process is a special case with \( n = 1 \)
Shimazaki model - constant strain rate $k$

- In workshop asked to show:
  \[ F_T(x) = 1 - \exp[-(k/2)x^2] \]
- This is a Weibull distribution with $n = 2$

- The hazard function is:
  \[ h(t) = kx \]
- This is a linear function - the probability of rupture is linear function of strain accumulation
Chinese Seismograph Vase

Nearly 2000 years ago, the ancient Chinese made a special vase that had several sculpted dragons mounted all around the sides of the vase. Each dragon held in its mouth a metal ball. When the ground shook, some of the balls would fall from the mouths of the dragons into the waiting mouths of the sculpted frogs to show how the ground had moved.

Jericho

Archaeologist points to the base of that mudbrick wall. All agree that the wall fell down, but differ on the date. The most informed date the destruction of the wall to the time of Joshua (1400 B.C.)
Extreme value distributions

Poisson model:

- Magnitude distribution
- Interval distribution

This is the simplest two-parameter model of seismicity and it agrees amazingly well with the data.

The larger the size of the earthquake to be predicted the better the agreement: which is expected because

- Large earthquakes on a fault are likely to be widely spaced in time
- Large earthquakes occurring one after another in time are likely to be far spatially far apart
- In either case the dependence between earthquakes is weak

The extreme-value approach makes uses of this
Extreme value distributions

- Let $y$ be the largest event in any year
- Suppose only $y$ is listed in catalogues
  - More reliable since larger earthquakes are better recorded
  - Poisson fit is bound to improve since only large events are used
  - Loss of information is more than compensated for by being able to go further back in time

The cumulative distribution function of $y$ is:

$$F_M(y) = \exp(-\alpha e^{-\beta y})$$

$\alpha$ is the mean annual number of earthquakes and $\beta$ is (mean magnitude)$^{-1}$.
Mean return times for different magnitudes is:

$$T(y) = \alpha^{-1} \exp(\beta y)$$
Extreme value distributions: example

\( \alpha \) and \( \beta \) are estimated from earthquake catalogues by regression. For California, \( \log \alpha = 11.43 \) meaning there are \( e^{11.43} \) earthquakes (M>0) per year. \( \beta \approx 2.0 \)

Can calculate for magnitude \( y = 8 \) should occur on average every 100 years. (Note this is the result of extrapolation.)

The probability that a maximum annual earthquake of magnitude \( y \) will occurs in any \( D \)-year period is the earthquake hazard:

\[
R_D(y) = 1 - \exp(-\alpha D e^{-\beta y})
\]

e.g., estimate the hazard for a maximum annual earthquake of M=7.1 in any 10-year period in California:

\[
R_D(7.1) = 1 - \exp(-e^{11.43} \times 10 \times e^{-2 \times 7.1}) = 0.5
\]

or if we calculate the mean recurrence period for M=7.1 earthquakes:

\[
T(7.1) = \alpha^{-1} \exp(\beta y) = \exp(2.0 \times 7.1) / \exp(11.43) = 15.8 \text{ yrs}
\]