GEOPHYSICS (08/430/0012)

SEISMIC WAVES AND EARTH OSCILLATIONS

<u>*OUTLINE*</u>

Elasticity

Stress, strain, Hooke's law, elastic moduli, Poisson's ratio

Seismic Waves

Types of seismic wave
- body waves: P–waves and S–waves
- surface waves: Rayleigh and Love waves
- guided waves and interface waves
- seismic wave recordings

Seismic wave propagation
- ray theory: Snell's law, mode conversion
- rays and wavefronts
- rays in layered media and media with varying velocities
- seismic travel time curves, velocity estimation
- simple harmonic waves: $V = f\lambda$
- dispersion of surface waves, diffraction and scattering of seismic waves.

Earth Oscillations

Modes of oscillation, application to the Earth's deep structure and density

Reading: Fowler §4.1 and §4.4; Lowrie §3.1-3.3 and §3.6 (reading around the maths).

ELASTICITY

Stress and strain

- stress = force/area;   unit: 1 Nm^{-2} = 1 Pa (pascal);

  1 atmosphere = 1.013^5 Pa;   1 p.s.i.(pound/sq.in.) = 6895 Pa

- strain: (a,b) longitudinal strain = (change in length)/(original length),

  (c) volume or bulk strain = (change in volume)/(original volume),

  (d) shear strain = shear angle (in radians)

These types of strain are illustrated in the figure overleaf.

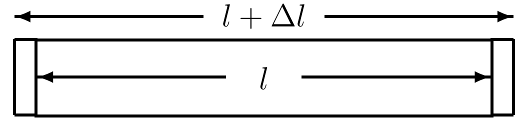
Hooke's Law

Stress is proportional to strain for elastic materials.
Away from the seismic source, the strains in seismic waves are very small and well within the elastic portion of the stress–strain curve of all rocks.
From Hooke's law one can define four elastic moduli corresponding to the four types of strain (a) to (d).

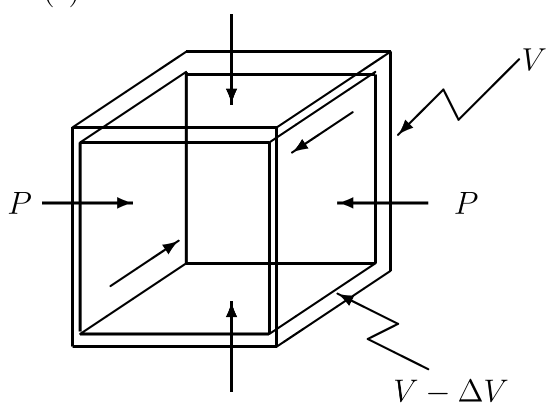
ELASTICITY: TYPES OF STRAIN

(a) longitudinal strain

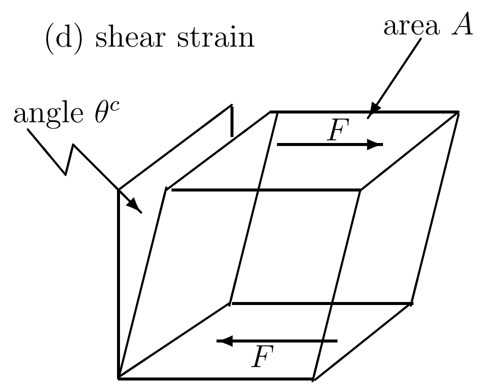
(b) longitudinal strain



(c) bulk strain



(d) shear strain



ELASTIC CONSTANTS (1)

Elastic moduli

These are defined by: elastic modulus = stress/strain *See figures (a) to (d) above*

(a) Young's modulus Y applies to longitudinal stressing of rods and wires:

$$Y = \frac{F/A}{\Delta L/L}$$

(b) The axial modulus Ψ applies to longitudinal stressing when there is no lateral strain (wave propagation through a large volume of material):

$$\Psi = \frac{F/A}{\Delta l/l}$$

(c) The bulk modulus K applies to bodies under uniform external pressure P:

$$K = \frac{P}{\Delta V/V}$$

(d) The shear modulus μ applies to bodies under shear stress:

$$\mu = \frac{F/A}{\theta}$$

ELASTIC CONSTANTS (2)

POISSON'S RATIO σ:

$$\sigma = \frac{\text{lateral (contractional) strain}}{\text{longitudinal(extensional)strain}} = \frac{\Delta b/b}{\Delta l/l}$$

RELATIONS BETWEEN ELASTIC CONSTANTS
Only two elastic constants are needed to describe the elastic properties of isotropic materials (materials having the same properties in all directions). That is, only two of the elastic constants are independent:

$$Y = 3K(1-2\sigma); \quad \Psi = K + \frac{4}{3}\mu;$$

$$Y = 2\mu(1+\sigma): \quad \sigma = \frac{3K-2\mu}{6K+2\mu}$$

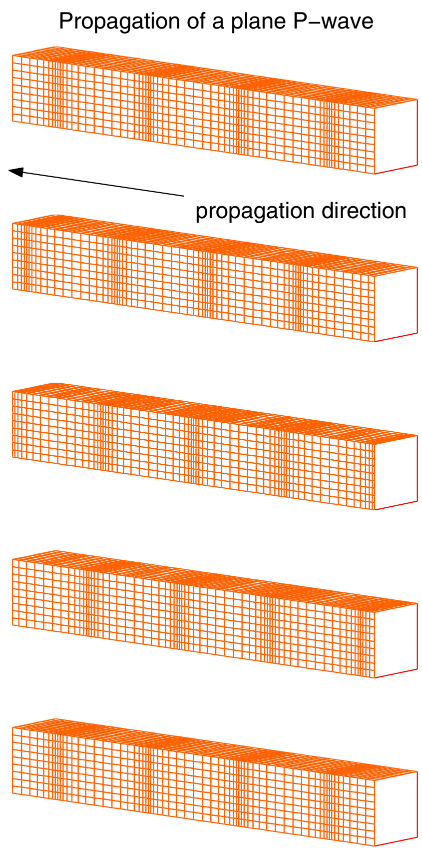
TYPES OF SEISMIC WAVE (1): BODY WAVES

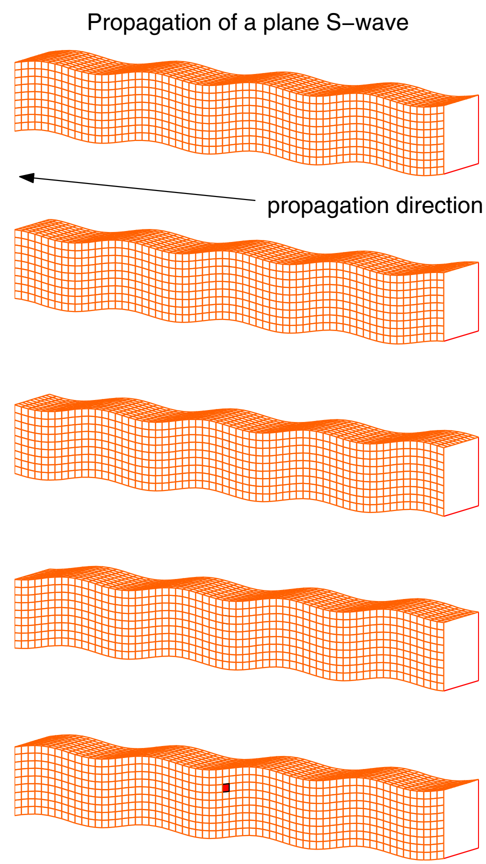
	P–waves		S–waves
strain	compression/rarefaction /dilatation (push/pull)		shear rotational (twisting)
particle motion	longitudinal		transverse
velocity	$\sqrt{(K+\frac{4}{3}\mu)/\rho}$	(ρ = density)	$\sqrt{\mu/\rho}$
in fluids	P–waves = acoustic waves		no S–waves ($\mu = 0$)

P– and S–waves are analogous to longitudinal and transverse waves on an elastic spring.

SCHEMATIC DIAGRAM OF P AND S WAVES

The passage of a P–wave compresses and extends a small cube of material parallel to the direction of travel of the wave, a square cross–section becoming rectangular.
The passage of an S–wave shears or twists a small cube of material perpendicular to the direction of travel of the wave, a square cross–section becoming rhomboidal.





ELASTIC MODULI FROM VELOCITIES

Calculating the bulk modulus requires both v_P and v_S.

P-wave velocity v_P
S-wave velocity v_S
density ρ

→

$$\mu = \rho v_S^2$$
$$K = \rho v_P^2 - \frac{4}{3}\mu$$

→

bulk modulus K
shear modulus μ

Example: lab.measurements at 40 MPa from Han et al. (1986)

sample	density (g/cc)	clay content	porosity	v_P (km/s)	v_S (km/s)	K (GPa)	μ (GPa)
4	2.39	0.00	0.154	4.81	3.10	24.67	22.97
28	2.30	0.03	0.216	3.95	2.39	18.37	13.14
57	2.35	0.27	0.150	3.99	2.13	23.20	10.66

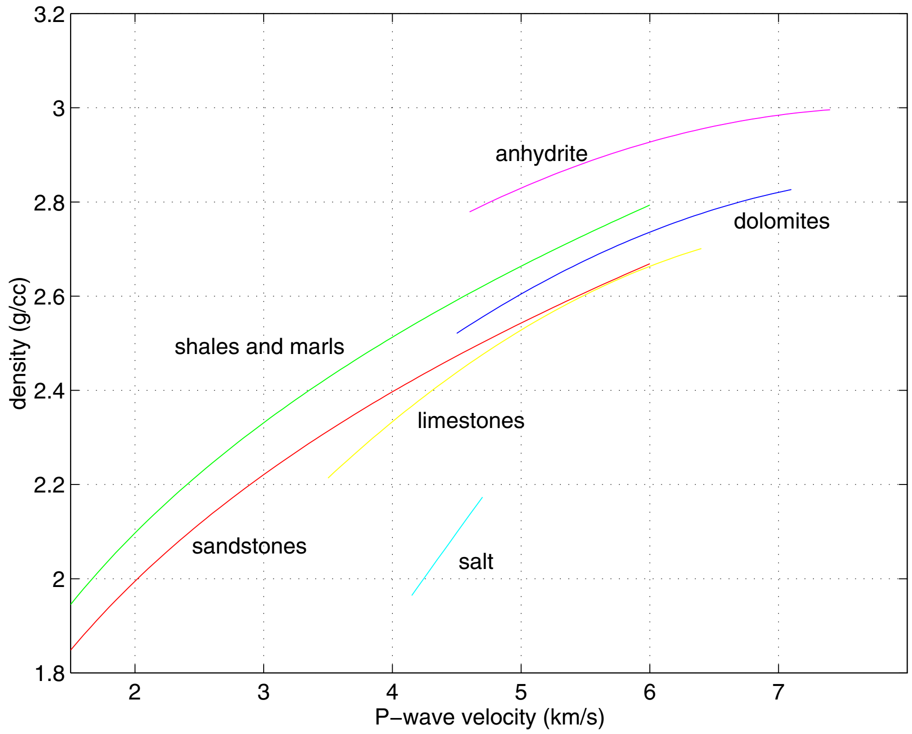
Cf. quartz: $K \simeq 37$ *GPa,* $\mu \simeq 44$ *GPa.*

The 'effective' moduli K and μ of porous rocks can be very different from the 'intrinsic' or grain-matrix moduli of the non-porous rock. Velocities in porous rocks are also much more influenced by stress and temperature. The bulk modulus is affected by fluids in the pores and cracks; the shear modulus is not (at seismic frequencies).

VELOCITY-DENSITY TRENDS

In most rocks the moduli K and μ *correlate* with the density ρ such that elastic wave velocity tends to increase with density. However when brine replaces gas in a rock, the density increases without any increase in shear modulus and the shear velocity drops. Thus, when other factors are equal, velocity varies inversely with density. It is incorrect to say that "velocity increases *because* density increases". This confuses causation with correlation.

The figure below shows velocity-density trends for sedimentary rocks.



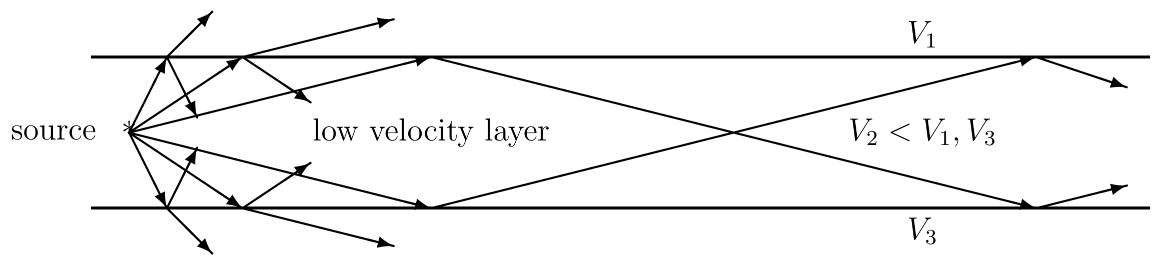
TYPES OF SEISMIC WAVE (2): SURFACE AND GUIDED WAVES

Surface waves

These are guided by the Earth's surface. Their amplitude decays with depth and inversely with wavelength. Their velocity depends on wavelength and layer thicknesses as well as on elastic constants.
Two types of surface waves are commonly seen on earthquake seismograms: (a) Rayleigh waves, and (b) Love waves.

Guided waves

Other types of guided wave occur. For example, channel waves are guided by a low–velocity layer. Low–angle rays are trapped in the layer.

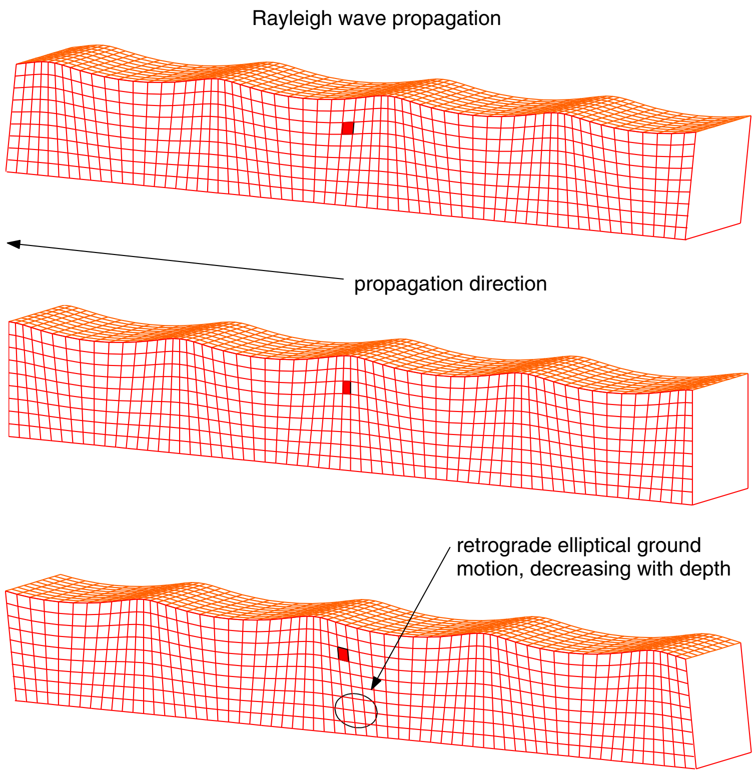


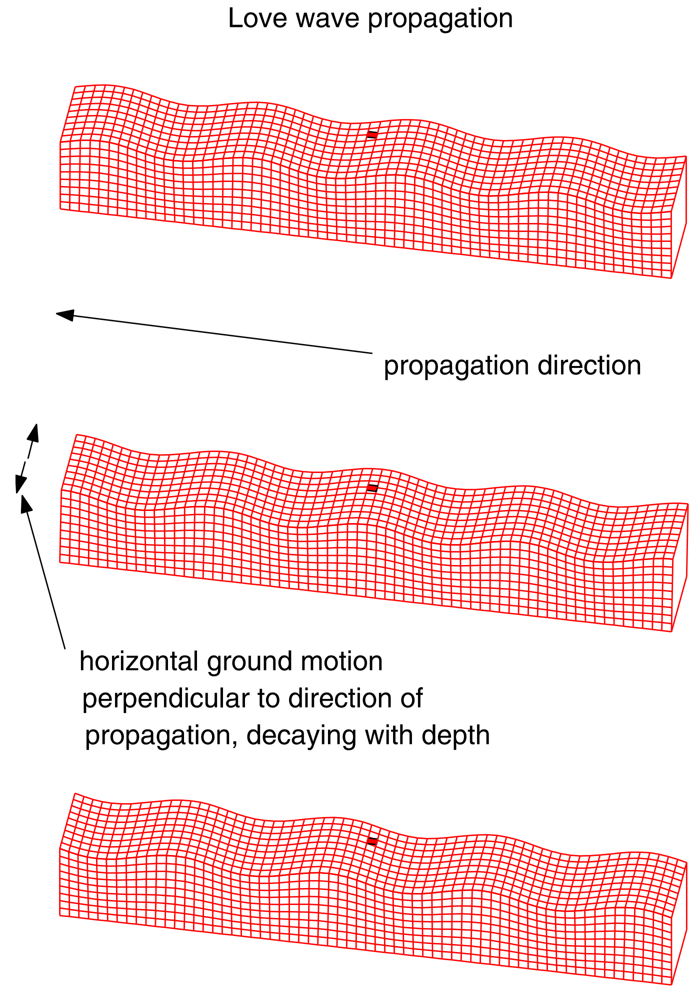
Interface waves

A commonly observed interface wave is the so-called "refracted wave" which travels along the interface between a higher-velocity substratum underneath a lower-velocity overburden and which emerges beyond the critical distance, the distance corresponding to the angle of critical reflection. Really this wave is a *head wave* similar to the bow wave from a speedboat. The wave travelling in the higher-velocity substratum generates its own "bow wave" as it travels along the interface with the overburden.

SCHEMATIC DIAGRAM OF RAYLEIGH AND LOVE WAVES

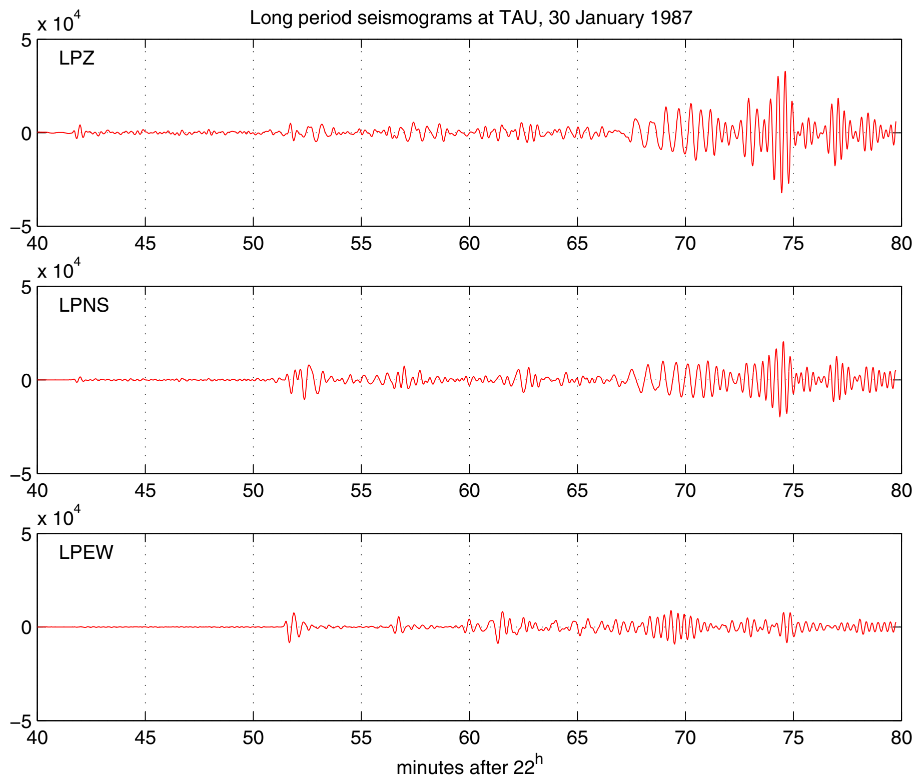
Rayleigh waves have retrograde elliptical particle motion.
Love waves have horizontal transverse particle motion.



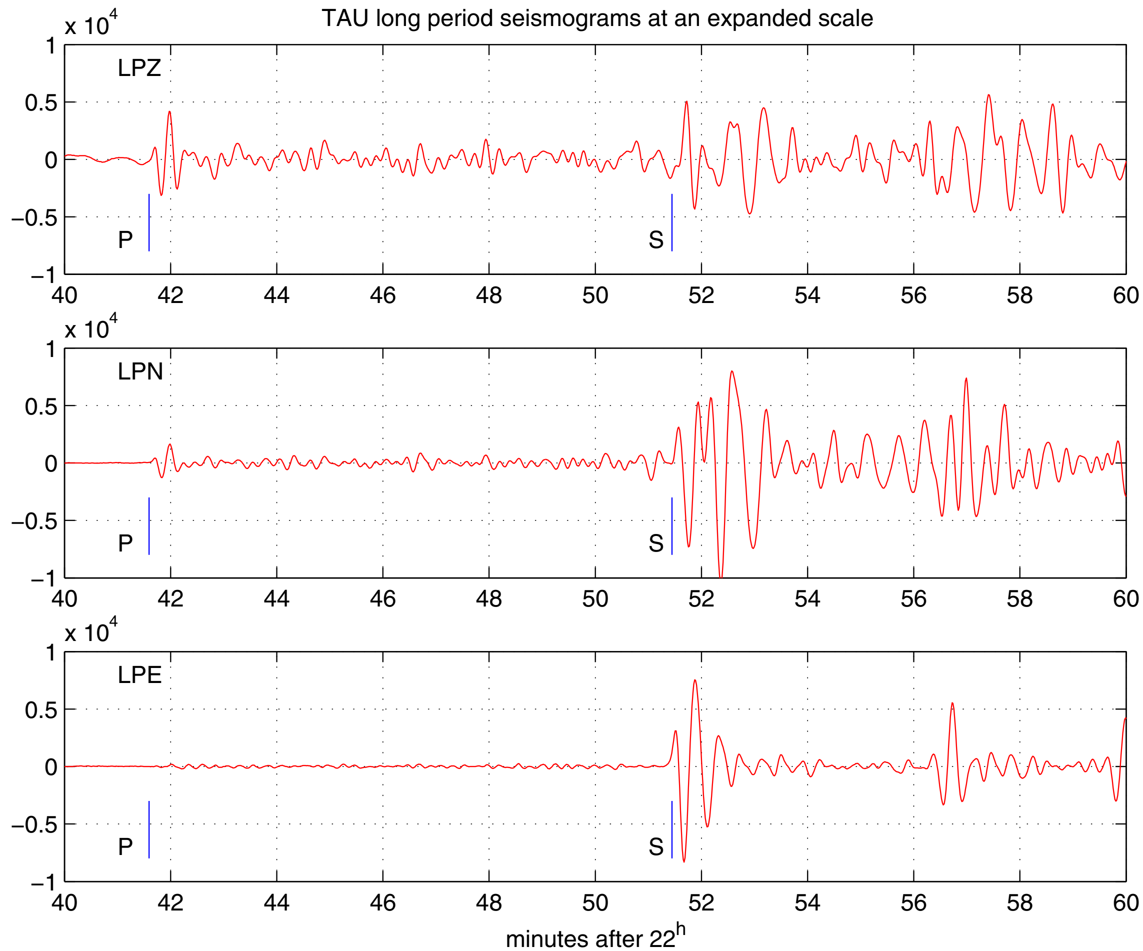


LONG-PERIOD SEISMOGRAMS SHOWING P, S AND SURFACE WAVE RECORDINGS

The figure below shows the three-component long-period seismograms recorded at the Tasmanian seismograph station TAU from an earthquake in Antarctica. LPZ is the long-period vertical seismogram, LPNS the the long-period north-south seismogram, and LPEW the long-period east-west seismogram. The first half of these seismograms is plotted at an expanded scale on the next page. The earthquake was almost due south of TAU at a geocentric distance of 77.19°. The P-wave arrives just before 22^h42^m, the S-wave arrives just before 22^h52^m, the Love wave at $\sim 23^h01^m$ and the Rayleigh wave at $\sim 23^h07^m$. The P-wave and Rayleigh wave are recorded on the LPZ and LPNS seismometers; the LPZ and LPNS also record SV-waves while the LPEW records SH and Love waves.



THE P AND S WAVE RECORDINGS AT TAU



TRAVEL TIMES OF THE P AND S WAVE RECORDINGS AT TAU

The origin time of the earthquake was $22^h\ 29^m\ 39.8^s$. Given that the geocentric distance of TAU from the earthquake's epicentre is 77.19°, use the travel times extracted from the IASPEI tables (below) to check that the expected P-wave and S-wave onsets are at the times indicated on the figure above.

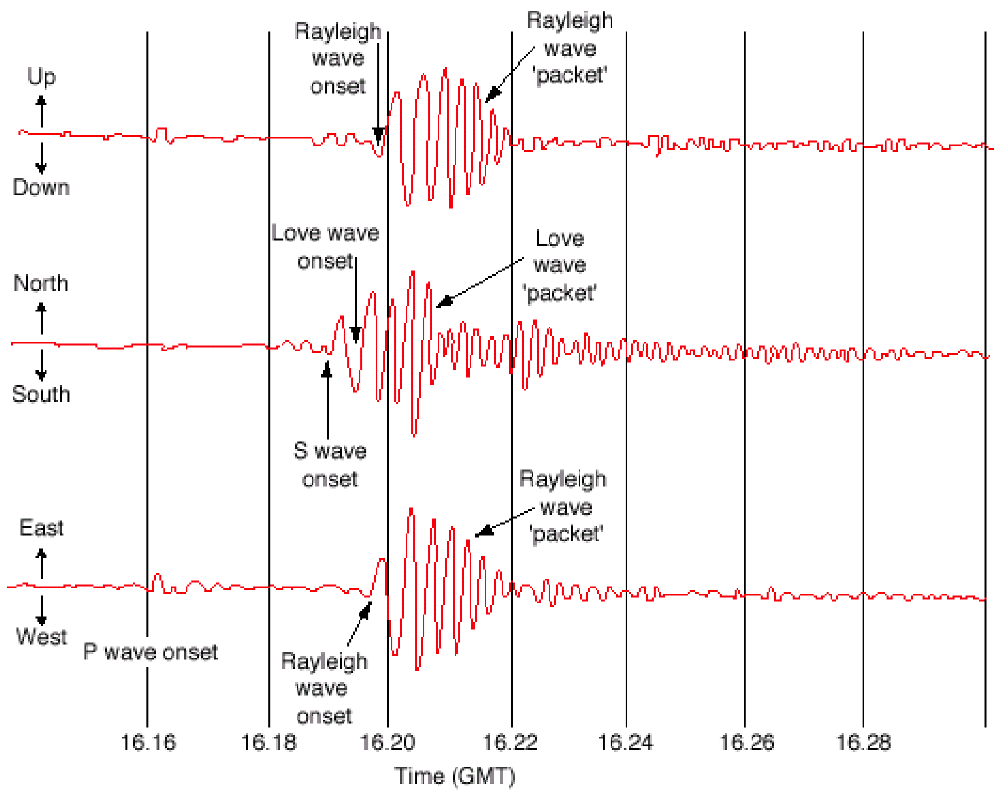
P AND S WAVE TRAVEL TIMES FROM A SURFACE SOURCE
from the IASPEI 1991 Seismological Tables

$\Delta°$	$t(P)$		t(S)	
	m	s	m	s
75.0	11	43.26	21	22.90
76.0	11	49.01	21	33.98
77.0	11	54.68	21	44.93
78.0	12	0.27	21	55.76
79.0	12	5.79	22	6.47
80.0	12	11.23	22	17.05

REFERENCE: B.L.N.Kennett (ed.), IASPEI 1991 Seismological Tables, Research School of Earth Sciences, Australian National University, Canberra.

SEISMOGRAMS SHOWING P, S, LOVE AND RAYLEIGH WAVES

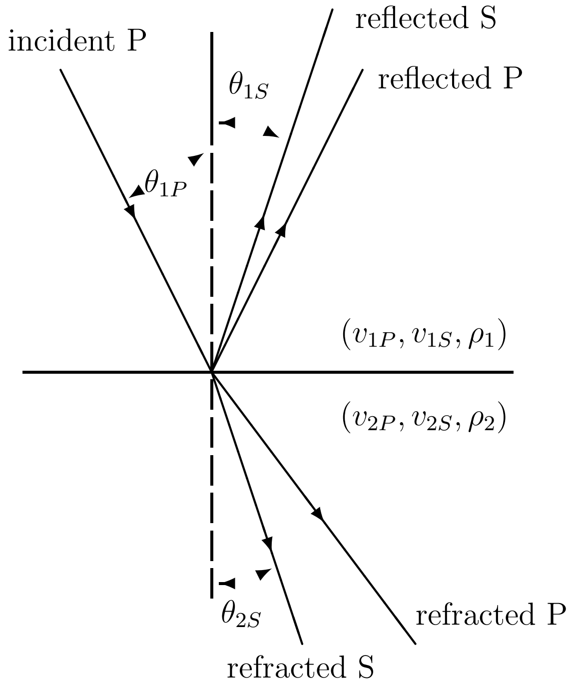
This three component seismogram from Eskdalemuir in Scotland shows beautifully clear recordings of P, S, Love and Rayleigh waves from an earthquake on the mid-Atlantic ridge.
In what direction is the earthquake from Eskdalemuir and how does this relate to ground motion shown in the recordings? .



SEISMIC RAY THEORY

Snell's law for seismic waves: mode conversion

In general a P wave incident at an interface generates both P and S waves. Similarly an incident S wave would in general generate both P and S waves. This is called *mode conversion*.



Snell's law states:

$$\frac{\sin\theta_{1P}}{v_{1P}} = \frac{\sin\theta_{1S}}{v_{1S}} = \frac{\sin\theta_{2P}}{v_{2P}} = \frac{\sin\theta_{2S}}{v_{2S}}$$

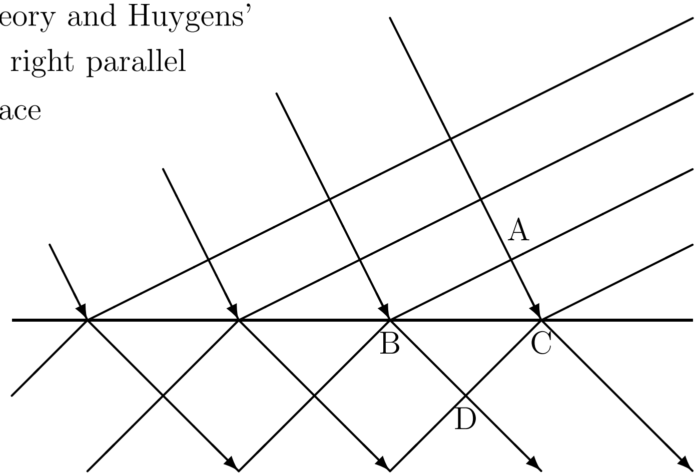
Subscript 1 denotes rock above the interface; subscript 2 rock beneath it. Reflection and refraction of a P wave at an interface generates both P and S waves, except that at normal incidence only P waves are reflected and transmitted. The normal incidence reflection coefficient for pressure waves is:

$$R_P(0) = \frac{p_{(rfl)}}{p_{(inc)}} = \frac{z_2 - z_1}{z_2 + z_1}$$

where $z_1 = \rho_1 v_{P1}$ and $z_2 = \rho_2 v_{P2}$. Whenever any $\sin\theta_2 = 1$, a critical angle is reached for θ_{1P}, beyond which the associated refraction no longer occurs.

RAYS AND WAVEFRONTS

For reflection and refraction, ray theory and Huygens' construction are equivalent. On the right parallel wavefronts are incident on an interface where the seismic wave vekocity increases. The wavefronts speed up in the faster layer causing them to bend such that the travel time from A to C equals that from B to D. The rays are perpendicular to the wavefronts and are bent in accordance with Snell's law.



The reflected wavefronts and rays are not shown in this figure.

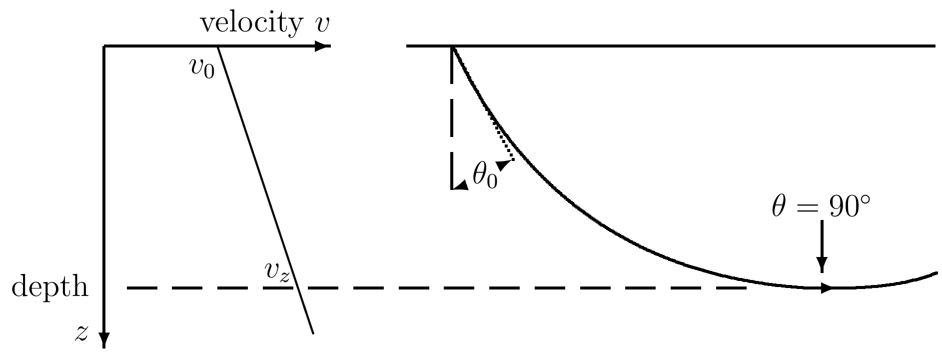
SEISMIC RAY THEORY (CTD)

Rays through a layer with a velocity gradient

Snell's law still applies.

$$\frac{\sin\theta_0}{v_0} = \frac{\sin 90^\circ}{v_z} = \frac{1}{v_z},$$

For a linear rate of increase of velocity with depth, $v = v_0 + gz$, the rays are circles of radius $v_0/(g\sin\theta_0)$.



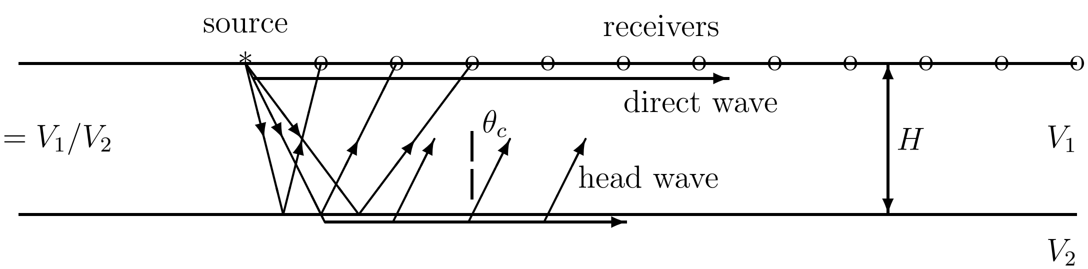
SEISMIC TRAVEL TIME CURVES (1)

A layer over a half–space

This is a simple model for a uniform crust over a uniform mantle. It illustrates some basic features of seismic travel time curves. The figure below shows the P-wave ray paths.

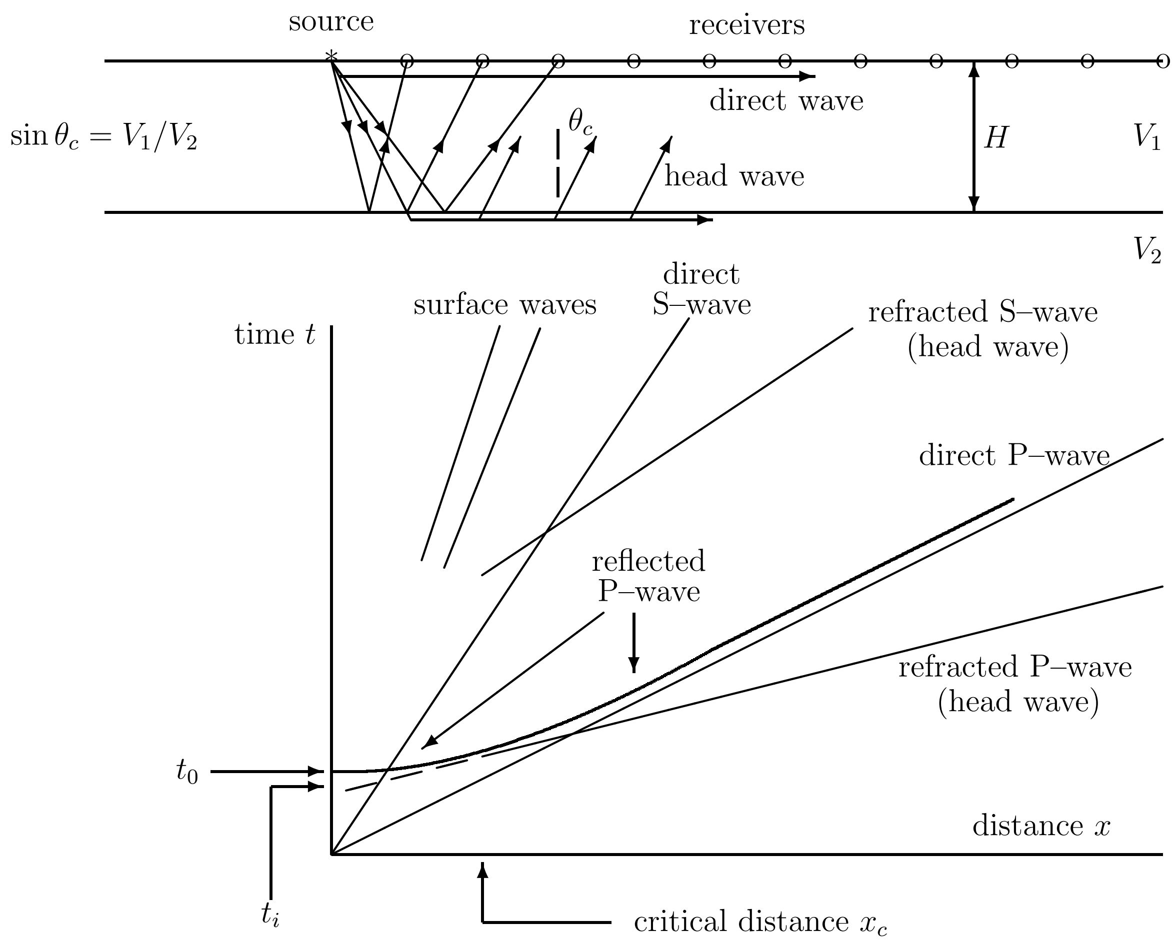
The critical angle θ_c occurs when the angle of refraction reaches 90°; i.e the refracted ray skims along the interface.

$$\sin\theta_c = V_1/V_2$$



SEISMIC TRAVEL TIME CURVES (2)

Travel times for a layer over a half–space



SEISMIC TRAVEL TIME CURVES (3)

Travel times of reflected P waves

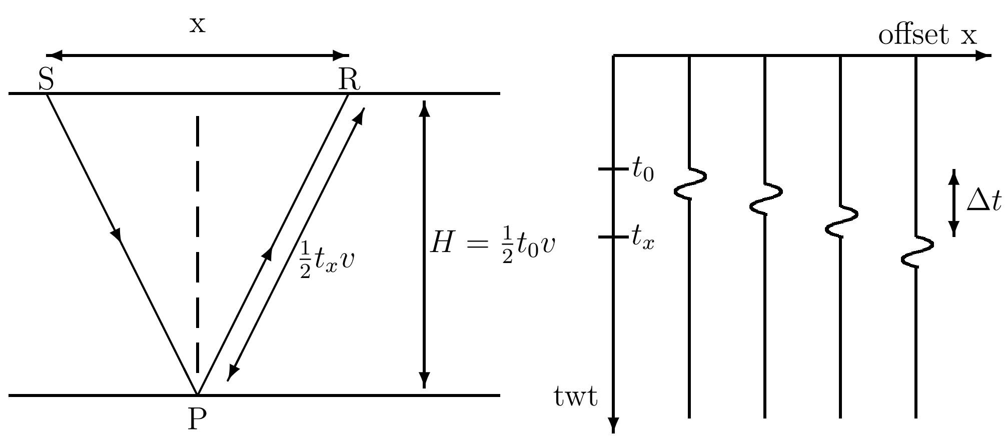
Consider a reflection from a point P on a flat reflector at depth H recorded at a receiver R, a distance or offset x from a source S, as illustrated in the following figure. The P–wave velocity between the surface and the reflector is V. For a reflection at normal incidence ($x = 0$), the reflection time is

$$t_0 = \frac{2H}{V} = \text{the normal incidence two} - \text{way time (twt)}$$

At offset x, the reflection time is $t_x =$(SP+PR)$/V$ = 2SP$/V$, since SP = RP by geometry. From Pythagoras Theorem, 2SP $= 2\sqrt{H^2 + (x/2)^2} = \sqrt{4H^2 + x^2}$. Using $2H = Vt_0$, one obtains

$$t_x^2 = t_0^2 + \frac{x^2}{V^2} \quad \text{or} \quad \text{normal moveout} = \Delta t = t_x - t_0 = \sqrt{t_0^2 + \frac{x^2}{V^2}} - t_0$$

This is the equation of a hyperbola; i.e. the recorded reflections follow a hyperbola when two–way time is plotted against offset. A plot of t_x^2 against x^2 is a straight line with slope V^{-2}. In reality the equation is an approximation, because velocity varies with depth, but it is a fairly good one.



SEISMIC TRAVEL TIME CURVES (4)

Travel time equations for a layer over a half–space

For direct P waves (P_1):

$$t = \frac{x}{V_1}$$

For reflected P waves:

$$t = \sqrt{t_0^2 + \frac{x^2}{V_1^2}} \quad \text{where} \quad t_0 = \frac{2H}{V_1}$$

For refracted P waves ($P_1P_2P_1$):

$$t = t_i + \frac{x}{V_2} \quad \text{where} \quad t_i = 2H\frac{\sqrt{V_2^2 - V_1^2}}{V_1V_2}$$

Similar equations apply to S-wave travel times. Surface wave travel times are linear in distance for a given frequency ($t = x/V_{L|R,f}$) but contain a range of frequencies. Because their velocity depends on frequency, they *disperse* as they travel; i.e. the apparent frequency of a peak or trough changes with distance.

SIMPLE HARMONIC WAVES

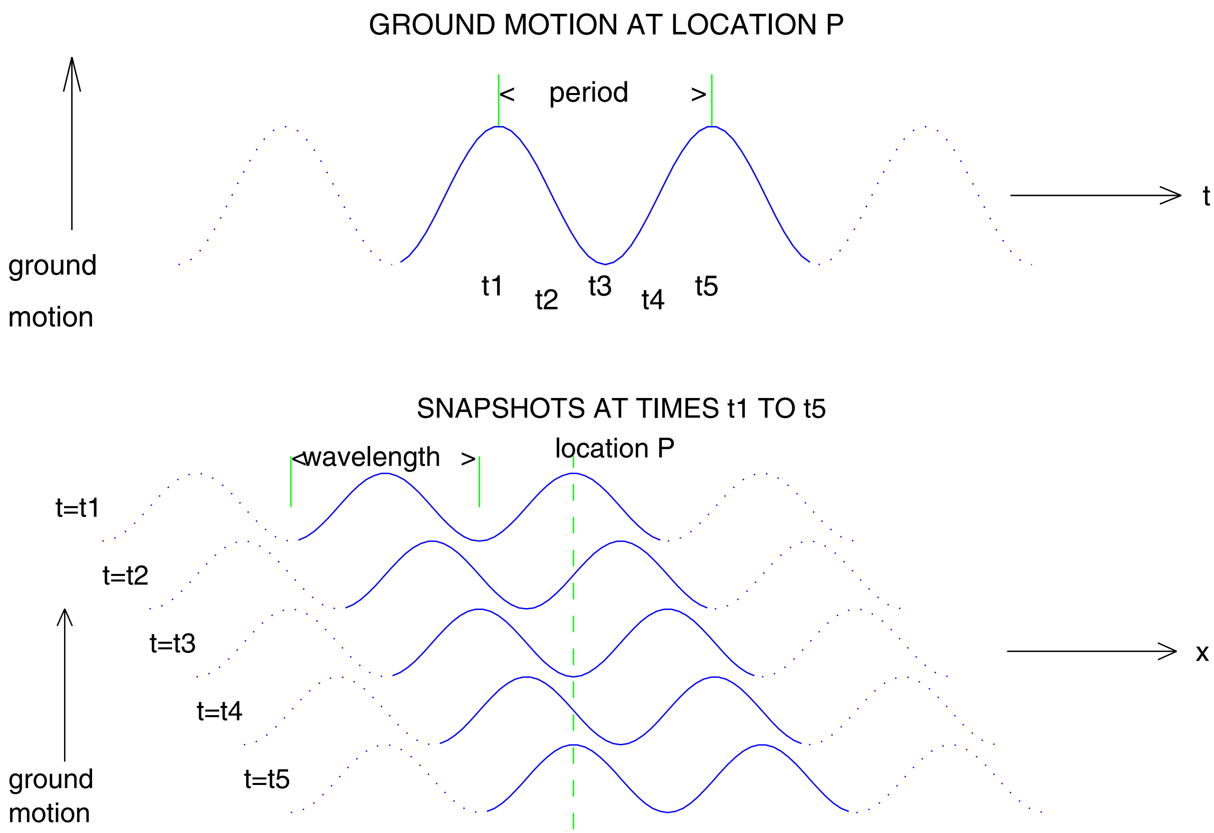
Simple harmonic waves, or sinusoidal waves, are fundamental to the analysis of waveforms of all kinds. The reason for this is that recorded waveforms can be decomposed into a sum of sine waves and reconstructed by superposing (summing) sine waves. The decomposition is called spectral analysis and is analogous to splitting white light into its pure sinusoid components which form the colours of the rainbow.

Simple harmonic waves are defined by their wavelength λ and frequency f, or period $T = 1/f$. When recorded at a fixed point P, the motion from a simple harmonic wave is just a sinusoidal oscillation. (See the figure on the next page). One complete oscillation or cycle (e.g. peak to peak) takes T seconds. There are $f = 1/T$ cycles per second (1 cycle/s = 1 Hz = 1 hertz). Similarly a snapshot of a SHM wave at any time t_1 is just a sinusoid, the distance between peaks or crests being the wavelength λ. The spatial analogue of frequency, the number of wavelengths per metre, is called the wavenumber $\nu = 1/\lambda$; its unit is simply m^{-1}. The second figure overleaf shows that a SH wave travels a distance λ in time T. Hence the wave velocity V is

$$V = \frac{\lambda}{T} = \lambda f$$

While sinusoids are unusual in seismology, it is still useful to describe waveforms by the times and distances between wave peaks (or troughs, or zero–crossings in the same sense). These measurements tell us the dominant period and wavelength of the wave.

SIMPLE HARMONIC WAVES: FIGURE

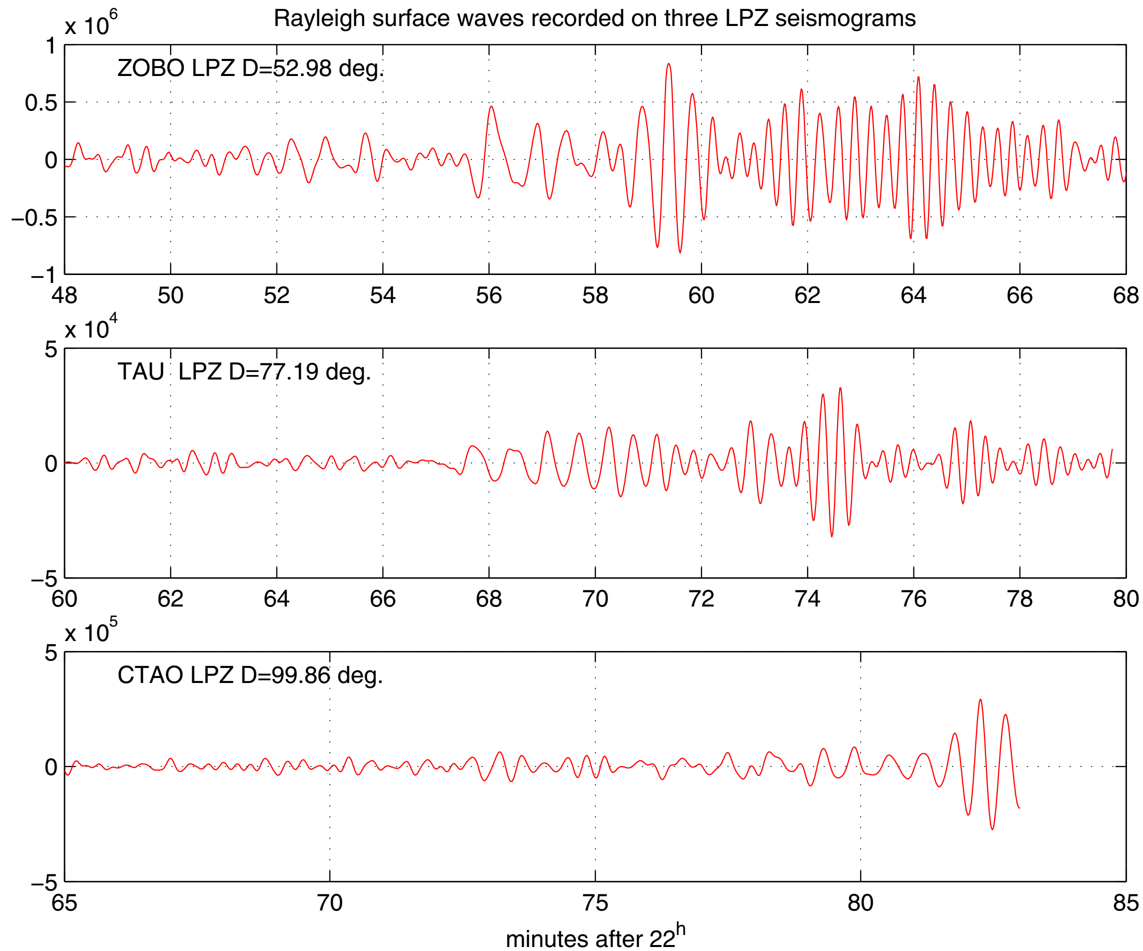


DISPERSION OF SEISMIC SURFACE WAVES

The velocity of seismic surface waves depends on their wavelength. Seismic sources generate waves covering a range of wavelengths and, because different wavelength surface waves travel at different speeds, a surface wave train disperses (i.e. spreads out) as it travels away from the source. The figure overleaf shows the Rayleigh waves recorded on the LPZ seismometers at three distances $(52.98°, 77.19°, 99.86°)$ from an earthquake. The dispersion can be seen in the increasing spread of the Rayleigh waves. The longest period waves Rayleigh waves arrive first. They are displayed approximately aligned vertically. For example, at TAU the long wavelength (long period) waves arrive at 22^h67^m and shorter wavelengths arrive later, ending with the strong *Airy* phase at about 22^h74^m, $\sim 7^m$ after the long period waves. At CTAO the Airy phase arrives about 9^m after the long period waves.

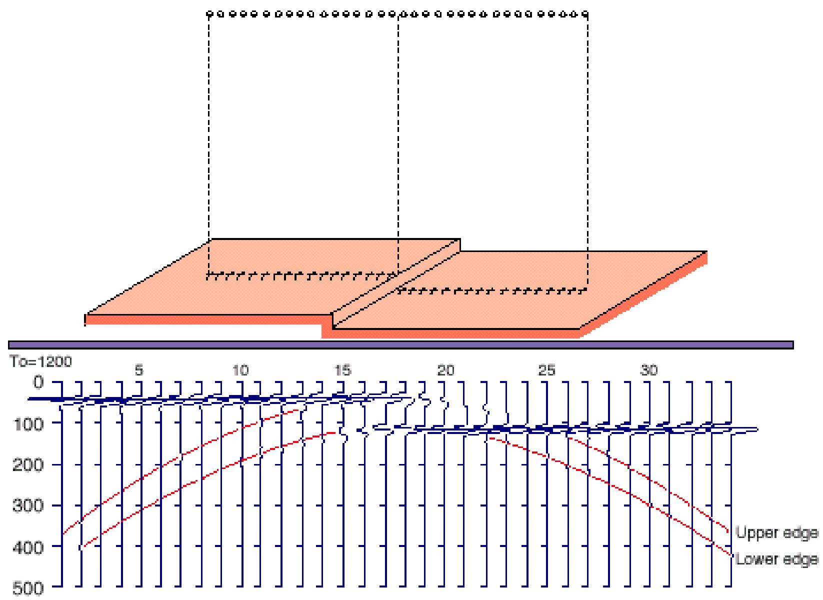
The variation of wave speed with wavelength from surface waves gives information about the variation of P and S velcities and density with depth in the earth.

EXAMPLE OF SEISMIC SURFACE WAVE DISPERSION



DIFFRACTION AND SCATTERING OF SEISMIC WAVES

Ray theory is a high-freqency (short-wavelength) approximation for seismic wave propagation. Waves are *diffracted* (bend around) and *scattered* by sharp edges and obstacles. Thus the Earth's core does not cast an absolutely sharp shadow because long-period body waves are diffracted around it. The figure below shows that seismic waves are scattered by the edge of a fault. These scattered waves appear as hyperbolic events on reflection seismic stacked sections.



EARTH OSCILLATIONS

The figures overleaf show modes of oscillation of the Earth following a very large earthquake. In the top row of figures dark and light areas denote zones oscillating with opposite displacements. The dark and light areas are separated by nodal lines on which there are no displacments. Spheroidal modes (${}_mS_n$) correspond to long-period standing Rayleigh waves. The period of the mode ${}_0S_0$ is 20.46 minutes; the period of the mode ${}_0S_2$ is 53.83 minutes. Toroidal or torsional modes (${}_mT_n$) correspond to long-period standing Love waves. Consequently their displacements are tangential to the Earth's surface. The period of the mode ${}_0T_2$ is 43.94 minutes. Very high-order sphroidal and torsional modes have periods of just a few minutes. Torsional modes involve only the mantle and crust (*Why?*).

SOME MODES OF EARTH OSCILLATION