GEOPHYSICS (08/430/0012)

THE EARTH - GEODESY, KINEMATICS, DYNAMICS

OUTLINE

Shape: Geodetic Measurements

Geodesy = science of the figure of the Earth; its relation to astronomy and the Earth's gravity field

The geoid (mean–sea–level surface) and the reference ellipsoid

Large–scale geodetic measurements: crustal deformation and plate movement

Kinematics (Motion)

Circular motion: angular velocity $\omega = v/r$, centripetal acceleration $\omega^2 r$

Rotation = spinning of the Earth on its axis; precession and nutation

Revolution = orbital motion of the Earth around the sun

Time: solar and sidereal, atomic clocks

Dynamics

Newton's second law: $F = ma$

Newton's law of gravitation: simplified application to circular orbits; the mass of the Earth and its mean density

Moment of inertia and precession: the moment of inertia of the Earth and its density distribution.

Background reading: Fowler §2.4 & 5.4, Kearey & Vine §5.6, Lowrie §2.1–2.4

THE FIGURE OF THE EARTH (1)

The reference ellipsoid

To a close approximation, the Earth is an ellipsoid. The *reference ellipsoid* is a best fit to the *geoid*, which is the surface defined by mean sea level (MSL) and its extension over land. The reference ellipsoid is used to define coordinate systems such as latitude and longitude, universal transverse Mercator (UTM) coordinates, and so on. Strictly speaking, an ellipsoid whose equatorial radius is constant is a spheroid.
The parameters of the Hayford international spheroid are:

$$\text{equatorial radius } a = 6378.388 \text{ km} \qquad \text{polar radius } c = 6356.912 \text{ km}$$

The ellipticity of the Earth (also called the flattening or oblateness) is

$$e = \frac{a - c}{a} \simeq \frac{1}{297}$$

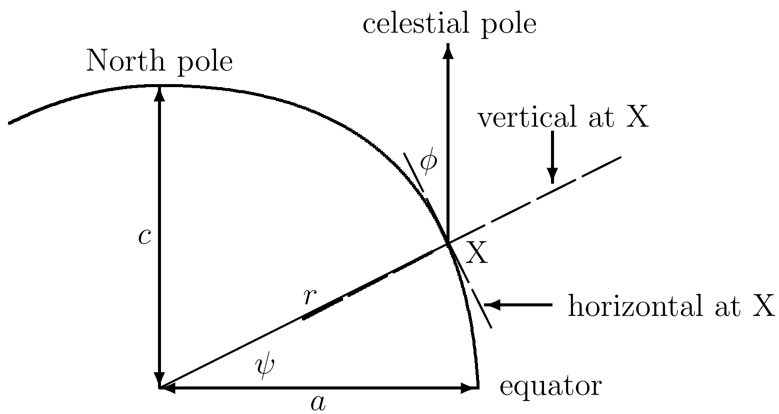
Its volume is 1.083×10^{12} km^3; the radius of a sphere of equal volume is 6371 km.

Warning: there is no **one** reference ellipsoid; the parameters of the best-fit ellipsoid depend on the method of fitting and they need to be updated as better data becomes available. Thus one finds slightly different values for the latitude and longitude of a location from different reference systems. Consult a surveyor if you need to convert from one system to another. The parameters given above are for the Hayford spheroid. More recent measurements make $e = 1/298.257$.

THE FIGURE OF THE EARTH (2)

Geographic and geocentric latitude

These are illustrated in the figure below. The usual value quoted for latitude, *geographic latitude*, ϕ is the angle between the normal to the geoidal ellipsoid and the equatorial plane. It is determined by astrogeodetic surveying. *Geocentric latitude* ψ is the angle between the radial line from the centre of the Earth and the equatorial plane. It is found from the formula in the figure.



$$\tan\psi = (1-e)^2 \tan\phi$$
$$= 0.993277 \tan\phi$$
$$r = a(1 - e\sin^2\psi)$$

Relation to astronomy and the Earth's gravity field

The figure above illustrates that the direction of the Earth's axis, and hence latitude, requires astronomical measurements. The celestial pole is the point about which the stars appear to rotate. It determines the direction of the Earth's spin axis. A telescope is pointing at the celestial pole when the stars appear to rotate about its axis (i.e. about the centre of its circle of vision). Measurements of the *1-degree meridian arc length* show that the Earth is an oblate (flattened at the poles) spheroid.

Because vertical and horizontal at any location are defined by the direction of the Earth's gravity, geodesy requires very accurate gravity measurements to determine the figure of the Earth. Also it is from measurements of the Earth's gravity that MSL is extended over land. Equally measurements of the Earth's gravity field require accurate geodetic surveying.

CRUSTAL DEFORMATION AND PLATE MOVEMENT FROM LARGE SCALE GEODETIC MEASUREMENTS

Terrestrial laser ranging

This is line-of-sight measurement of distances based on the time of flight of a laser (or microwave radio) pulse to and from a retro-reflector placed on a target. A retro-reflector, or corner-cube reflector, is a set of three mirrors arranged at right angles to form the corner of a cube. The corner-cube reflects a laser beam precisely back along its travel path. Corrections for the conditions in the intervening atmosphere are needed to achieve maximum accuracy.

Satellite Laser Ranging (SLR)

This method measures the two–way transit time of a laser beam from an accurately located site to a satellite. Thus LAGEOS, the Laser Geodynamic Satellite, was studded with over 400 corner cube retroreflectors, which return any beam of light back along its incoming path. SLR can also use reflectors on the Moon.
LAGEOS estimated relative plate velocities around the Pacific and motion of the Earth's pole. Its accuracy over a 10000 km baseline was quoted as $<\sim 1$ cm. SLR has also detected ~ 30 mm/yr shortening over a 900 km baseline across the San Andreas fault.

Satellite Radiopositioning

The Global Positioning System (GPS) uses satellites with Caesium atomic clocks transmitting 2 GHz time signals. At each ground station signals are received from several satellites. With a sufficient number of GPS recordings satellite radiopositioning approaches the accuracy of VLBI but it is much less expensive. Stations are portable (Cf. Magellan and Trimble hand–held systems). Accuracy of the order of ~ 5 mm from several days recordings are quoted.

Very Long Baseline Interferometry (VLBI)

VLBI uses 10–15 quasars observed 5–15 times over, say, a 24–hour period with (2.2, 8.4 GHz) radio telescopes. The 400 MHz bandwidth provides timing to ~ 0.3 picoseconds (10^{-12}s). VLBI can measure polar motion, secular drift of the pole, variations in the rate of rotation, and relative plate motions. Measurements include 25 ± 4 mm/yr on the San Andreas fault, and 19 ± 10 mm/yr between the USA and Europe.

LARGE SCALE GEODETIC MEASUREMENTS: REFERENCES

References

W.E.Carter & D.S.Robertson, 1986. Studying the Earth by Very–Long–Baseline Interferometry. Scientific American **255(5)**, 44–52.

S.C.Cohen & D.E.Smith, 1985. LAGEOS scientific results: Introduction. J.Geophys.Res. **90(B11)**, 9217–9220.

R. G. Gordon & S. Stein, 1992. Global tectonics and space geodesy. Science **256**, 333-342.

T.H.Jordan & J.B.Minster, 1988. Measuring crustal deformation in the American West. Scientific American **259(2)**, 48–58.

R.S.Stein, 1987. Contemporary plate motion and crustal deformation. Reviews of Geophysics **25(5)**, 855–863.

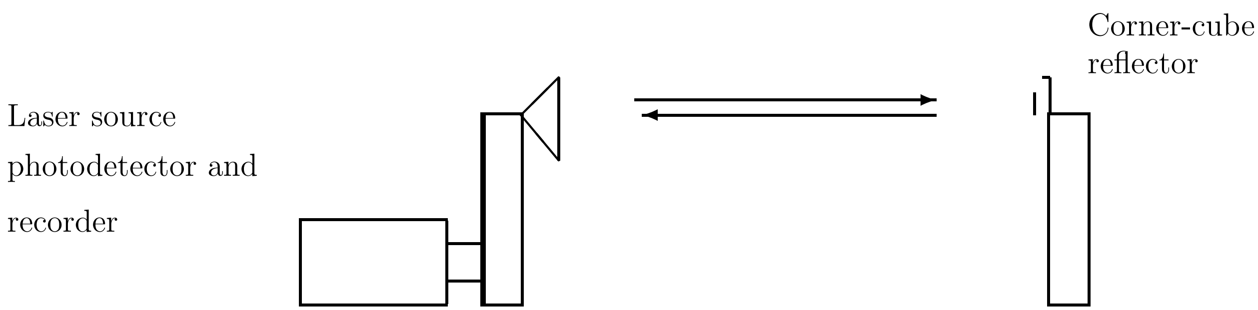
For a recent example of the use of GPS measurements to investigate smaller scale tectonic motions, try

D. Hindle, J. Kley, E. Klosko, S. Stein, T. Dixon & E. Norabuena, 2002. Consistency of geologic and geodetic displacements during Andean orogenesis, Geophys. Res. Lett. **29(8)**, 10.1029/2001GL013757.

MEASUREMENT OF DISTANCES IN THE TENS OF KILOMETRES RANGE

Terrestrial laser ranging

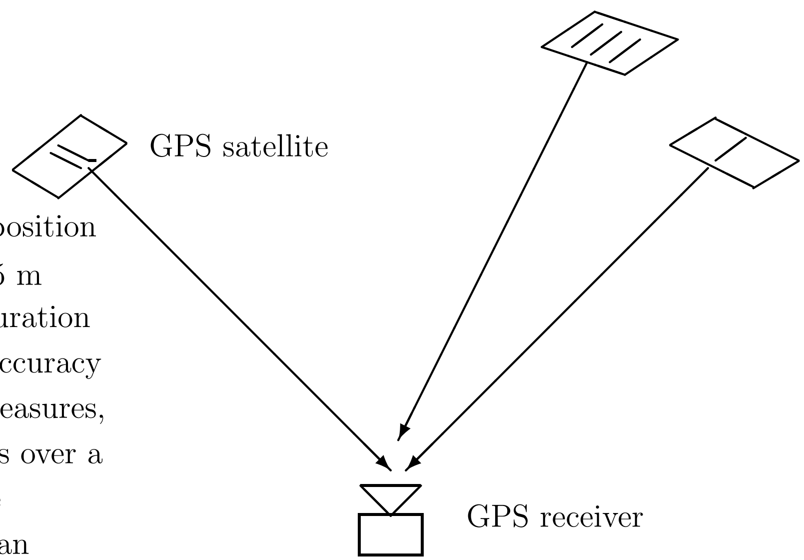
The curvature of the Earth and refraction and scattering in air limit direct measurement of distance on the Earth's surface to a maximum of some 30 to 50 km. Microwave radio and laser systems are used for this range of distances. Because the refractive index of air varies with atmospheric pressure, temperature and humidity, these parameters must be monitored. For accuracies better than 1 in a million (10^6), the air has to be sampled continuously and regularly along the length of the measured distance.



MEASUREMENT OF DISTANCES IN THE HUNDREDS OF KILOMETRES RANGE

Satellite radio positioning

Distances of 100 km or more can be measured by means of GPS (GPS = Global Positioning System) satellites. Each satellite carries a Caesium atomic clock and radiates time signals, each at its own frequency in the 2 GHz band, to a set of observing stations. The location of an observing station can be determined by recording time signals from three or more GPS satellites. The more satellites that can be recorded in any observation period, the better the accuracy of locating the station. Recordings from at least 4 satellites allow elevation to be estimated as well as location (latitude and longitude), although just 4 would not give very useful accuracy. By recording the data over several days and processing the data, locations can be determined to a precision of 0.5 cm or less. Distance between observing stations are calculated from their latitudes, longitudes and elevations.



Hand–held GPS receivers can fix position in a matter of minutes to about 25 m accuracy, depending on the configuration of the satellites recorded. Better accuracy can be achieved with additional measures, such as (i) simply averaging results over a period of time, or (ii) referring the recordings to a second receiver at an accurately known location.

MEASUREMENT OF INTER–CONTINENTAL DISTANCES (1)

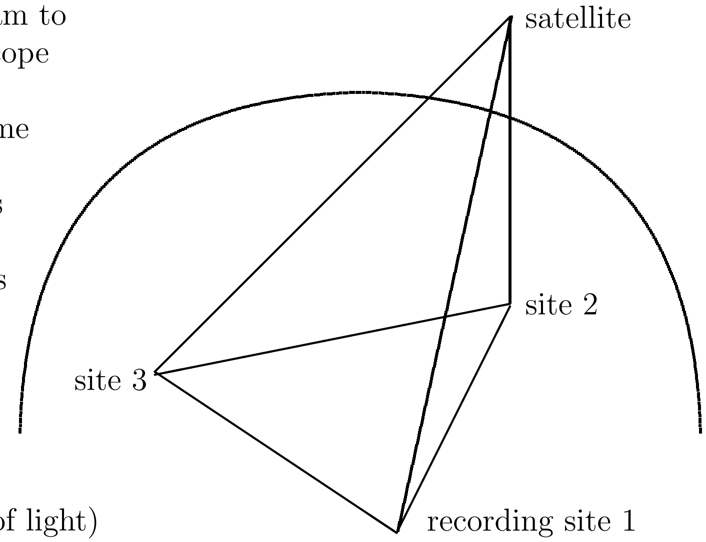
These methods employ three–dimensional geometry to triangulate the Earth from extremely accurately measured large distances.

Satellite Laser Ranging (SLR)

Laser pulses are transmitted as a confined beam to the satellite and recorded by a receiving telescope at the observing site. After correction for atmospheric refraction, the two–way travel–time is converted to distance from the site to the satellite. The correction for refraction requires that the pressure, temperature and humidity in the atmosphere is monitored. The method's accuracy is limited by the accuracy of these corrections for refraction. SLR is also very dependent on having an accurate model of the gravity field. The calculation of distances requires adaptation of the simple equation

$$\text{distance} = \tfrac{1}{2}\ (\text{two-way travel-time}) \times (\text{speed of light})$$

to allow for the variation of the speed of light in air with pressure, temperature and humidity.

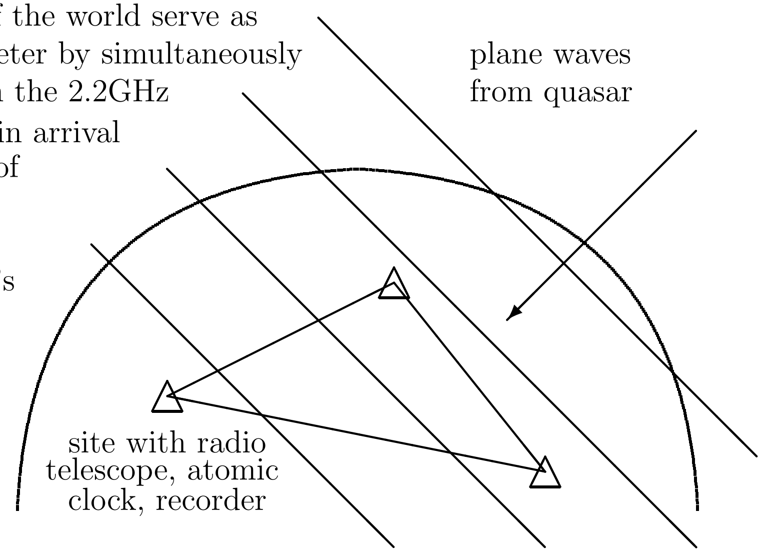


MEASUREMENT OF INTER–CONTINENTAL DISTANCES (2)

Very–long–baseline–interferometry (VLBI)

Quasars are very strong and very distant quasistellar radio sources. The wavefronts of the radio waves they emit are coherent and planar. This makes the wavefronts an ideal frame of reference for measuring large distances.

Radio telescopes in various parts of the world serve as elements of a worldwide interferometer by simultaneously recording emissions from quasars in the 2.2GHz and 8.4GHz bands. The difference in arrival time of the wavefronts at any pair of stations gives the separation of the two stations resolved along the direction to the quasar. The Earth's rotation imparts a sinusoidal daily variation to the time difference. Each quasar in a set of 10 or more is observed for about a day at a time by each of the telescopes, along with time signals from hydrogen–maser atomic clocks.



NASA claims to measure baselines of several 1000 km with a precision of < 1 cm.

CIRCULAR MOTION

Circular or radian measure

Equations that relate angles, angular velocities and angular accelerations to distances, linear velocities and accelerations generally require that angles angles are measured in radians. Recall that angle in radians is defined by

$$\theta^c = \frac{\text{arc length}}{\text{radius}}$$

Because θ^c is the ratio of two lengths, it is dimensionless.

Angular velocity

Just as linear velocity is distance travelled per unit time, so too angular velocity of a rotating body is the angle rotated per unit time. Dividing the definition of angle θ^c above by time, it follows that angular velocity ω of a body rotating on a circle of radius r with linear speed v is:

$$\omega = v/r$$

ω must be in radians per unit time for this equation to hold. Its standard (SI=Système International) unit is the radian/s.

Centripetal acceleration

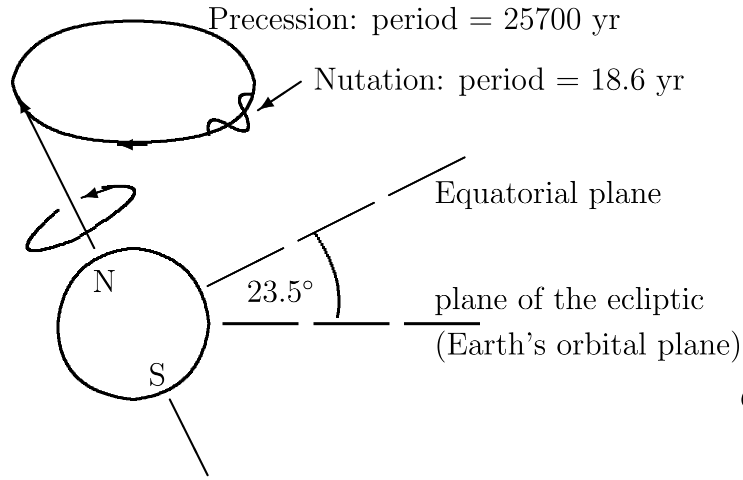
Acceleration is the rate of change of velocity per unit time. Velocity is a *vector*; that is, it has both a magnitude (speed) and a direction. A simple illustration of this is that the velocity of a boat rowed across a flowing river is the vector sum of the velocity of the water and the velocity of the boat relative to the water. A body rotating on a circle of radius r m with linear speed v m/s therefore has a continually changing velocity and is being continuously accelerated towards the centre of the circle. This acceleration is the centripetal acceleration $a_\perp$. Its magnitude is

$$a_\perp = r\omega^2 = v\omega = \frac{v^2}{r}$$

Since acceleration is the rate of change of velocity with respect to time and the SI unit of velocity is the m/s, the SI unit of accleration is the m/s^2 (metre per second per second).

MOTION OF THE EARTH: THE EARTH'S ROTATION

The axis of rotation is inclined at 23.5° to the pole to the ecliptic (i.e. to the normal to the Earth's orbital plane). Over time this angle varies between 22.0° and 24.3°. Because of the equatorial bulge, the Moon and Sun exerts *torques* on the rotating Earth that cause it to *precess* and *nutate*.
Also the Earth's rotational axis wobbles because it does not coincide exactly with the axis corresponding to the maximum *moment of inertia* of the Earth. This wobble comprises an annual variation, due to seasonal variations in the atmosphere and oceans, and the *Chandler* wobble, due to asymmetries in the internal distribution of the Earth's mass.



Period of the Chandler wobble: 435 days (1.19 yr).

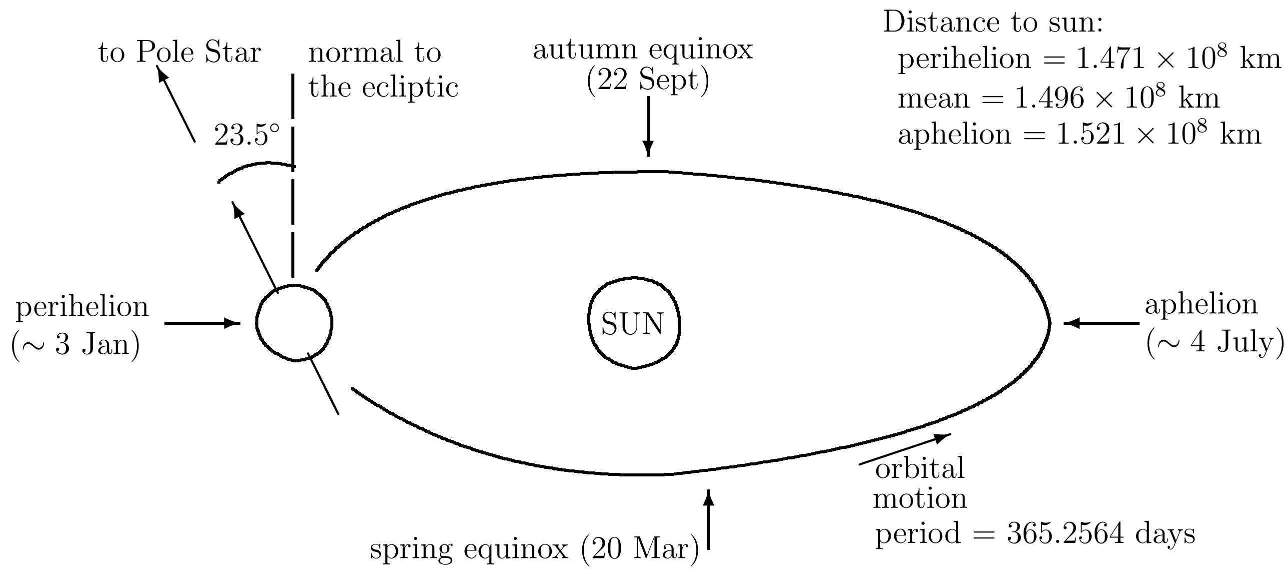
Mean solar day: 86400 s (rotation with respect to the Sun)

Mean sidereal day: 86164.1 s (rotation with respect to the stars)

Q: should one calculate the Earth's period and angular velocity using the solar day or the sidereal day?

MOTION OF THE EARTH: THE EARTH'S ORBIT

Both the eccentricity (shape) and orientation of the Earth's orbit gradually change over time. These changes, with the variations in the Earth's rotation, cause long-term climatic changes.

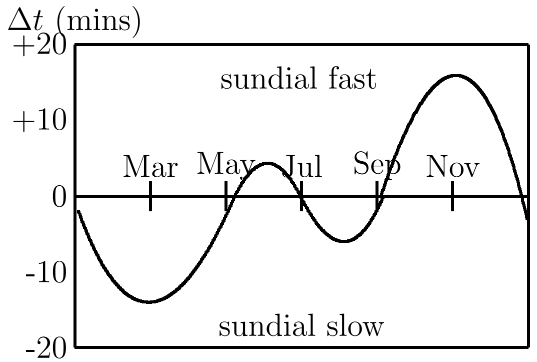


The summer and winter solstices, when the sun is furthest from the equator, occur on June 21 and December 21. (The seasons here are those of the northern hemisphere).

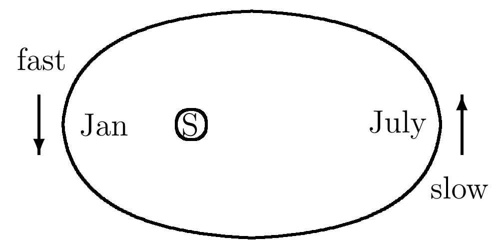
MEASUREMENT OF TIME

The equation of time

The rotation of the Earth has been found not to be a consistent enough for an accurate long term time scale. High precision pendulum clocks agreed with one another better than with sundials! The difference between clock time and sundial time, called *the equation of time*, has a six–monthly and an annual oscillation. It implies that sundial time is not uniform. It varies because of the varying orbital speed and varying inclination of the Earth.



Graph of the equation of time (Δt)



Schematic of the Earth's orbit

Mean solar time is the time kept by clocks that run at the mean rate of solar time over the whole year. When high precision pendulum clocks were checked against the stars, further variations were detected and corrections for nutation became necessary. Time based on the stars is called *mean sidereal time* . Relative to the stars, the Earth's orbit around the sun adds an extra revolution per year, i.e.

$$1 \text{ sidereal year} = 366.26 \text{ sidereal days} = 365.26 \text{ solar days}.$$

The cyclic variations in the Earth's motion from precession, tilt of its axis, and the eccentricity of its orbit are called *Milankovitch cycles*; Milankovitch postulated that they induced climatic changes. They have periods of approximately 21000, 41000, 110000 and 410000 years and these periodicities have been detected in sea level changes and the rates of sedimentary deposition.

MEASUREMENT OF TIME (CTD)

Definition of the second

In 1957 the definition of the second was changed from being based on the Earth's rotation to the Earth's revolution in orbit around the sun.

Quartz clocks then showed up variations in astronomically–based time scales due to polar variation (Chandler wobble) and fluctuations in the rate of rotation that could hardly be detected with pendulum clocks. In 1967 the definition of time was changed again to an atomic frequency standard based on vibrations of the caesium (Cs) atom at microwave frequencies: 1 second = 9192631770 Cs oscillations.

Note: In 1960 the definition of the metre was changed from the standard platinum rod to 1650763.73 times the wavelength of orange light from pure krypton 86. This allows lengths to be measured to an accuracy better than 1 in 10^9.

DYNAMICS OF THE EARTH'S MOTION (1)

The motion of the Earth, Moon and planets can be described in great detail by the laws of Newtonian mechanics. An exposition of these laws is not the job of this course but it is important to know that the application of these laws provides crucial information on the mass of the Earth and its distribution of mass with depth.

The mass of the Earth

The mass of the Earth M can be found from the orbit of the Moon or any satellite orbiting the Earth. The basis of the calculation can be understood by considering a satellite of mass m in a circular orbit of radius R around the Earth. We have seen that a body moving on a circle has a the centripetal acceleration $a_\perp$ towards the centre of the circle. Newton's second law of motion tells us that the force F needed to provide this acceleration is:

$$F = ma_\perp = mR\omega^2$$

since $a_\perp = R\omega^2$ where ω is the angular velocity of the satellite. The force F is provided by the gravitational attraction of the Earth on the satellite and, according to Newton's law of gravitation, its magnitude is:

$$F_g = \frac{GMm}{R^2}, \quad G = 6.673 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$$

G is the universal gravitational constant. Its value comes from laboratory experiments. Equating F with F_g gives, after some algebra:

$$M = \frac{R^3\omega^2}{G}$$

Lets' apply the equation for M to the Moon for example. The Moon orbits the Earth in 27.322 days and its mean distance from the Earth is 384400 km. To apply the equation we must use a consistent set of units. G above is in SI units and R and ω must also be put into SI units. So $R = 3.844 \times 10^8$ m. The period of rotation of the Moon is $27.322 \times 24 \times 60 \times 60$ s. Since it rotates 2π radians ($360°$ in this time, its angular velocity is

DYNAMICS OF THE EARTH'S MOTION (2)

The mass of the Earth (ctd)

$$\omega = \frac{2\pi}{27.322 \times 24 \times 60 \times 60} = 2.6617 \times 10^{-6} \text{ radians/s}$$

Substituting these values into the equation for M, you should find that mass of the Earth is $M = 6.030 \times 10^{24}$ kg. A more accurate value for the mass of the Earth is 5.9740×10^{24} kg. From this and its volume (1.083×10^{21} m^3), we find that its mean density is 5516 kg/m^3 (5.516 g/cc). This significantly exceeds the density of rocks commonly found in the Earth's crust, leading to the conclusion that the interior of the Earth contains high density materials.

The moment of inertia of the Earth

The mechanics of the Earth's rotation about its axis introduces a quantity called the *moment of inertia*, which is the rotational mechanical analogue of mass. The moment of inertia of a point mass Δm rotating at a distance r about an axis is Δmr^2. The moment of inertia of a body about an axis is found by integrating (summing) the contributions Δmr^2 from all the point masses that constitute the body. It generally involves a volume integral. A simple example would be a bicycle wheel with such very light spokes and axle that all its mass m was concentrated in the rim of radius r: the moment of inertia of the wheel about its axis would simply be mr^2. If all the mass were in the axle the moment of inertia would nearly zero. An intermediate case would be a wheel made as a uniform disc: its moment of inertia about the axis would be $0.5mr^2$. These idealised examples show that the moment of inertia depends on the distribution of mass around the rotational axis.

The moment of inertia of the Earth can be estimated from its rate of precession (period = 25700 yr). This is a much more complicated calculation than that for the mass of the Earth since it involves the gravitational torque acting on the Earth from the Sun and Moon due to the equatorial bulge. (It also requires the Earth's angular velocity of spin which is very accurately known). The result is that the moment of inertia of the Earth is 8.002×10^{37} kg m^2. This can be compared with the moment of inertia that the Earth would have if it were a sphere of uniform density: $0.4mr^2 = 9.699 \times 10^{37}$ kg m^2. This tells us that the Earth's densest materials are concentrated towards its centre.