Modelling Damping for a Pendulum

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In this investigation, two models for damping on a pendulum, dry and viscous, were tested to see which model is better by testing their correlation using experimental data of a ‘bob on a string’ oscillator. It was also investigated how much damping the pendulum undergoes was effected.

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I. INTRODUCTION

A pendulum undergoes simple harmonic motion if no external forces act on the pendulum. As a result the pendulum's angular displacement is modelled as a function of time, assuming the pendulum retains its angular amplitude and starts its oscillation at its angular amplitude as shown in Eq. (1).[1]

$$\theta = A\cos(2\pi f t)$$ (1) (angular displacement θ, angular amplitude A, frequency f, time t)

Eq. (1) can be rewritten in terms of angular frequency rather than frequency as shown in Eq. (2).[1]

$$\theta = A\cos(\omega t)$$ (2) (angular displacement θ, angular amplitude A, angular frequency ω, time t)

However if an external force acts on the pendulum, this model is not suitable because the pendulum will undergo damping. As a result, the pendulum's angular amplitude decreases over time due to the total energy of the pendulum is being transferred to the surrounding environment. Eventually the pendulum will lose all of its energy and stop oscillating.[1]

Two kinds of damping will be investigated in this investigation to see which model best suits the pendulum which undergoes damping.

Viscous damping suggests that the angular amplitude decreases exponentially over time which depends on the moment of inertia. For a bob on a string pendulum, the moment of inertia is defined in Eq. (3).[1]

$$I = ml^2$$ (3) (moment of inertia I, mass m, length of string l)

This means the amount of viscous damping depends on the mass and length of the pendulum. For a pendulum which undergoes viscous damping, its angular amplitude is modelled as shown in Eq. (4).[1]

$$\alpha = Ae^{-\frac{bt}{2I}}$$ (4) (angular amplitude α, initial angular amplitude A, viscous damping constant b, time t, moment of inertia I)

Dry damping suggests that the angular amplitude decreases linearly and modelled as shown in Eq. (5).[1]

$$\alpha = A - \frac{4\tau t}{\omega^2 IT}$$ (5) (angular amplitude α, initial angular amplitude A, dry damping constant τ, time t, angular frequency ω, moment of inertia I, time period T)

By analysing the correlation between the angular displacements with time, it can be determined which model is a better model for the pendulum. To determine which model is a better model is important because it will be useful to model further and future simple harmonic oscillators.

It was concluded that viscous damping was a better model but however it was very hard to determine if the amount of viscous damping is related to the moment of inertia as a result of a significant amount of error in the results.

II. METHOD

A 'bob on a string' pendulum was used in this investigation. The basic set-up is shown in Fig. 1.

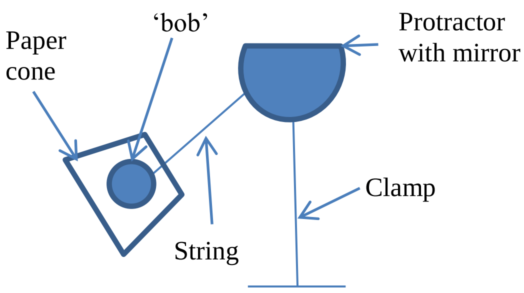


Fig. 1. Set up of bob on a string pendulum.

The string of length (60.0 ± 0.5) cm and 3 different spherical bobs were used: steel with mass (57.40 ± 0.05) g, brass with mass (69.50 ± 0.05) g and plastic with mass (10.30 ± 0.05) g.

An initial angular amplitude of 25° was used for every oscillation; this was determined by using the protractor. The paper cone was used to increase the amount of damping which should make it significant in the results.

For each type of bob, the angular amplitude for the: 1st, 2nd, 3rd, 4th, 5th, 10th and 20th period was measured using the protractor as shown in Fig. 1., using the mirror to prevent parallax error, and recorded 5 times to determine the mean and standard deviation of time and angular amplitude.

The Pearson's correlation coefficient, r, was determined for the following linear graphs as shown in Table 1, which was derived from Eq. (4) & Eq. (5).

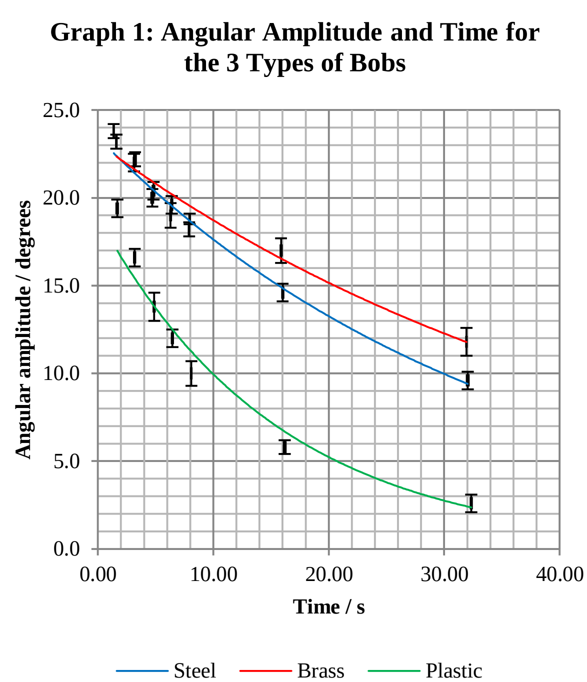
Dry damping	Viscous damping
$\alpha = -\frac{4\tau}{\omega^2 IT}t + A$	$\ln\alpha = \frac{-b}{2I}t + \ln A$

Table 1. Linear regressions for the two damping models.

Their Pearson's correlation coefficient was tested and compared to see which one of these models had the best correlation. After determining which model is the best, the model will be accepted and the constant of its damping will be calculated using least square fittings, assuming the damping constant is independent on the mass of the bob.

III. RESULTS

The angular amplitudes over time for each bob were recorded and as shown in Graph 1. Judging by eye, it could be determined that the angular amplitude for all types of bobs decreased exponentially, however the Pearson's correlation coefficient for linear regressions for the two types of damping model must be determined to confirm which model is better. The Pearson's correlation coefficient was calculated and as shown in Table 2.



Bob	Dry	Viscous	Critical value
Steel	-0.9713	-0.9941	-0.8329
Brass	-0.9771	-0.9913	-0.8329
Plastic	-0.9085	-0.9861	-0.8329

Table 2. Values of Pearson's correlation coefficient of the two damping models with the critical values at the 1% significant level.[2]

Both models have a good value of the Pearson's correlation coefficient, much bigger than the critical value at the 1% significant level in terms of magnitude, so in general both models are suitable in this case. However the viscous damping model's correlation coefficient was much bigger than the dry damping model's correlation coefficient, in terms of magnitude again, so it was concluded that the viscous damping model was a better model than dry damping model because the viscous damping had a stronger correlation than dry damping from the experiment data, as expected from our judgment from our eyes.

The viscous damping model was accepted and the viscous damping constant was calculated by creating a linear regression with the gradients of the linear regression for viscous damping from Table one and the reciprocal of twice the moment of inertia, as shown in Eq. (6).

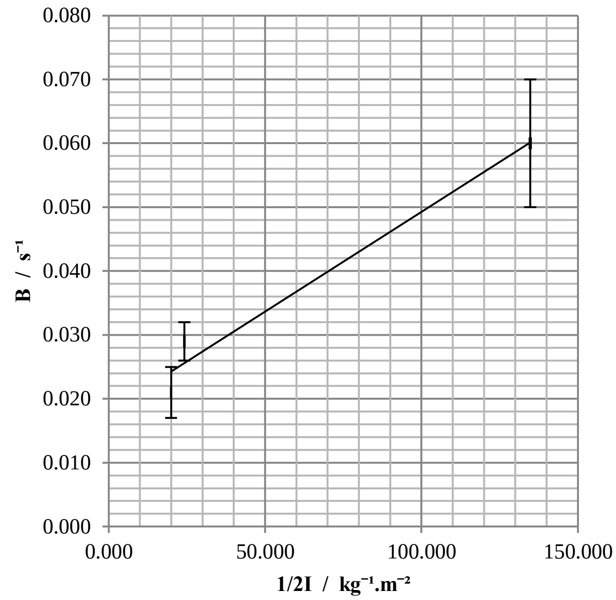
$$-B = b\frac{1}{2I}$$

(6) (gradient of linear regression for viscous damping from Table 1 B, moment of inertia I, viscous damping constant b)

The linear regression of Eq. (6) is as shown in Graph 2. The value of b was worked out to be $(3 \pm 2) \times 10^{-4}$ kg.m².s⁻¹ using Graph 2, which was very imprecise because the error corresponds to a percentage error of 67%, therefore unacceptable. As a result the correlation was tested using the Pearson's correlation coefficient as shown in Table 3.

A hypothesis test was conducted on the Pearson's correlation coefficient for Graph 2 and it was concluded that there was no relationship between the reciprocal of twice the moment of inertia with the viscous damping constant because the correlation of Graph 2 is not significant at the 5% significant level.

Graph 2 : Linear Regression to Work Out b



r	Critical value
0.9867	0.9877

Table 3. The value of Pearson's correlation coefficient for Graph 2 with the critical value at the 5% significant level.[2]

IV. CONCLUSION

It was concluded from the experiments that the angular amplitude decreased exponentially over time rather than linearly, as a result the viscous damping model was accepted.

However it was calculated that there was no relationship between the reciprocal of twice the moment of inertia with the viscous damping constant at the 5% significant level which concluded that the model was not perfect yet or the data set used was not suitable.

The Pearson's correlation coefficient was only 0.001 under the critical value so it was suggested that the viscous damping model was not completely wrong or that there wasn't enough readings and could have used more masses; 3 readings is the minimum to conduct a correlation hypothesis test.

The main cause of error in the value of b, the vicious damping constant, was due to error in finding the gradient of the linear regression from Graph 2 which was brought forward by the error in the gradient of the linear regression of viscous damping from Table 1, mainly due to error in the angular amplitude. As a result, the error in reading the angular amplitude was probably the likely cause of impreciseness and damage to the correlation of Graph 2.

The readings of the angular amplitude could be measured with less error if a capacitive angular displacement transducer was used rather than reading the angular displacement using a protractor from eye. The capacitive angular displacement transducer can read the angular displacement of the pendulum to more precision hence reduce the error in the angular displacement, as a result the error in b, the viscous damping coefficient, will reduce and should create a better correlation for Graph 2.

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[1] UCL Department of Physics and Astronomy, *Experiment NX2 Modelling Pendulum*

[2] OCR, Advanced General Certificate of Education, MEI Structured Mathematics, *Examination Formulae and Tables*, (MF2, CST251, January 2007)