Interdependent Value Auctions with Insider Information: Theory and Experiment

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We develop a model of interdependent value auctions in which two types of bidders compete: insiders, who are perfectly informed about their value, and outsiders, who are informed only about the private component of their value. Because of the mismatch of bidding strategies between insiders and outsiders, the second-price auction is inefficient. The English auction has an equilibrium in which the information outsiders infer from the history of drop-out prices enables them to bid toward attaining efficiency. The presence of insiders has positive impacts on the seller's revenue. A laboratory experiment confirms key theoretical predictions, despite evidence of naive bidding.
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I. Introduction

Most auction literature assumes that bidders hold rather equally informative information about the value of the auctioned object, while each bidder's information is privately known to him.[1] However, there are real-world auctions in which bidders are better described as *asymmetrically* informed in terms of how precisely they know about different aspects of the object's value. For instance, in art auctions, buyers with professional knowledge tend to more accurately appraise the potential value of an object than non-professional buyers (Ashenfelter and Graddy, 2003). In takeover auctions, buyers with existing shares of a target firm may have access to inside information unavailable to competitors.[2] In auctions for gas and oil leases (i.e., OCS auctions), firms owning neighboring tracts may have better information about the value of a lease—such as oil reserves or drilling conditions—than non-neighboring firms do (Hendrick and Porter, 1988).

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[1] See, for instance, Myerson (1981) and Riley and Samuelson (1981) for the environment of private value auctions, and Milgrom and Weber (1982; MW henceforth) for auctions with affiliated values.

[2] Sometimes, a current management team of the target firm participates in the bidding competition, which is a practice known as a management buyout (MBO). Shleifer and Vishny (1988) argue that the managers' special information about their company is one reason for an MBO.

While these examples illustrate that informational asymmetry among bidders is commonly observed, they also highlight that the information held by a better-informed bidder helps other bidders, in particular less-informed ones, to evaluate the object.

Motivated by informational asymmetry in these examples, we study its implication on auction outcomes such as the efficiency of allocation and the seller's revenue. To do so, we introduce an auction environment with insiders and outsiders in the interdependent value setup, which we term an *asymmetric information structure*. In this environment, bidders' values are composed of a private value component and a common value component.[3] Our key assumption is that the set of bidders is partitioned into two types: insiders, who are perfectly informed of their values, and outsiders, who are informed only of the private value component and thus imperfectly informed of their values. Outsiders know the identity of insiders. Our model is general in that we allow for any arbitrary number of insiders and outsiders. This feature enables us to understand the impact of varying the number of insiders on the auction outcomes. Under the asymmetric information structure, we study the performance of two standard auction formats, second-price and English, in terms of their revenue and efficiency.

When only one insider and one outsider exist, the second-price and English auctions are equivalent and achieve an efficient allocation. With more than two bidders and at least one insider, however, the second-price auction does not permit an efficient equilibrium. This inefficiency arises from the mismatch between insiders' and outsiders' bidding strategy, because the former depends on both private and common components of values, whereas the latter depends on only the private component. The issue underlying this problem is that as a sealed-bid format, the second-price auction offers no way of eliminating informational asymmetry between outsiders and insiders.

In contrast, the English auction provides opportunities for outsiders to learn about others'—both insiders and outsiders'—private information through the history of prices at which they drop out. In an environment with a symmetric information structure (i.e., with no insider), each bidder's private signal is precisely revealed in equilibrium because his equilibrium drop-out price is monotonically related to his private signal. However, the complexity of the inference problem can increase in the presence of insiders who employ a (weakly) dominant strategy of dropping out at their values that reflect both private and common components. We characterize an English auction equilibrium where outsiders overcome this inference problem to get the object allocated efficiently. We do so by extending the equilibrium constructions in MW and in Krishna (2003) to our setup, which involves finding a system of equations that yields *break-even signals* at any given price in the presence of insiders. The outsider's equilibrium strategy is then

[3] To be more precise, with n bidders and an n-dimensional signal profile s, each bidder i's value is given as a function $h_i(s_i)+g(s)$, where h_i corresponds to the private value component and g corresponds to the common value one.

characterized by the cutoff strategy in which he drops out at such a price that the break-even signal becomes equal to his true signal. The key to the equilibrium construction lies in showing that each outsider's break-even signal is strictly monotone as a function of the price. The outsiders' drop-out strategy then implies that all outsiders drop out in order of their values and before the price reaches their values. An immediate consequence of this strategy and insiders' dominant strategy is that in the case that an insider has the highest value, he becomes a winner. In the case where an outsider has the highest value, the allocation is also efficient because all other outsiders drop out earlier, revealing their precise signals. Thus, the informational gap between the highest-value outsider and any remaining insiders is reduced to an extent that the efficient allocation is achieved. Our results highlight the efficiency-achieving role of the dynamic nature of the English auction in an interdependent value setup with an asymmetric information structure.

We also explore the revenue implications of the asymmetric information structure in the English auction. To investigate this, we consider two English auctions that differ by only one bidder who switches from an outsider in one auction to an insider in the other. With our constructed equilibrium of English auction, we show that for any signal profile, the latter auction with the switched insider yields a (weakly) higher revenue than the former one. This result is based on two effects of turning an outsider into an insider on bidders' bidding behavior. First, the switched insider drops out at a higher price than when he is an outsider. Second, the higher drop-out price of the switched insider in turn causes active outsiders to drop out at higher prices. This revenue prediction is reminiscent of the linkage principle of MW in that the switch of an outsider into an insider increases information available to bidders. However, there are important differences between our revenue result and MW's prediction. Unlike the case of MW where the extra information is made available to all bidders using a public announcement, only one bidder entertains better information in our setup. Furthermore, our revenue prediction holds *ex-post*—that is, for every realization of the signal profile—and thus does not depend on the assumption that signals are affiliated.

While our theory offers a benchmark for comparing the efficiency and revenues of two standard auction formats in an environment with an asymmetric information structure, its predictions are based on the nontrivial inference process in the English auction. Whether individual bidders can rationally process the information revealed through the auctions and whether the theoretical predictions are thus valid are ultimately empirical questions. To test the validity of the theory, we design a simple experiment by varying the auction format—between English and second-price—and the composition of insiders and outsiders. Specifically, we employ three-bidder auctions of each format with three outsiders, two outsiders and one insider, or one outsider and two insiders. Each combination of auction format and insider-outsider composition serves as a single treatment. In order to make the outsiders' inference problem as transparent as possible, we let the computer play the role of insider who follows the dominant strategy of dropping out at its own value. This was public information to all human subjects.

Our experiment presents several findings. First, the English auction achieves a higher level of efficiency than the second-price auction does when at least one insider is present. Furthermore, there is no significant difference in efficiency between the two auctions when there is no insider. This finding on efficiency is consistent with our theory. Second, average revenues tend to deviate upward from the equilibrium benchmark, particularly in both auction formats with no insider. Despite this, the increase in the number of insiders has a positive impact on revenues in the English auction, as the theory predicts. We also find a similar pattern of the increase of revenues in the second-price auction with respect to the number of insiders. Third, there is evidence of naive bidding in both auction formats in the sense that subjects responded to their own signal less sensitively than predicted by the equilibrium theory, while those with low signals tend to overbid. This behavioral pattern is consistent with common findings of overbidding and the resulting winner's curse in the experimental literature. In addition, the degree of naive bidding in the data declines in both auction formats as the number of insiders increases. In particular, this pattern is statistically significant in the English auction. We conjecture that the presence of insiders—who have an informational advantage—makes the outsider more cautious in bidding and creates a behavioral incentive for the outsider to hedge against an informational disadvantage. This may work toward the correction of naive bidding and thus of the winner's curse in our setup.

Literature. — This paper contributes to the literature of auctions with an asymmetric information structure. Engelbrecht-Wiggans et al. (1982) study a first-price common-value auction in which a single "insider" has proprietary information about the common value of the object while other bidders have public information. Hendrick and Porter (1988) and Hendrick et al. (1994) extend the analysis of Engelbrecht-Wiggans et al. (1982) and study oil and gas drainage lease auctions. In first-price common-value auctions, Campbell and Levin (2000) and Kim (2008) theoretically examine the effects of an insider on revenues, whereas Kagel and Levin (1999) experimentally study the effects of an insider on revenues and bidding behavior. Our paper differs from these in some important manners. First, we study the implications of insider information on efficiency as well as revenues in interdependent value auctions, whereas the existing literature focuses on revenue implications owing to the pure common-value assumption. In addition, we study English auction and second-price auctions rather than first-price auction. Lastly, we provide an equilibrium analysis for any arbitrary number of insiders and offer the revenue implication of introducing an extra insider, whereas existing literature allows only a single insider.

Our model brings new insights to the literature concerning allocative efficiency in an interdependent value environment. Dasgupta and Maskin (2000) and Perry and Reny (2002) design some original mechanisms that implement the efficient allocation under a fairly weak condition (single crossing property). Krishna (2003) studies the English auction with interdependent values and asymmetric bidders in valuation and adapts the equilibrium characterization of MW. We extend Krishna (2003)'s framework to accommodate the asymmetric information structure and study how the English auction can overcome the informational gap among bidders and achieve an efficient allocation (as opposed to the inefficiency of the sealed bid auction). This extension yields novel insights, because we have a natural way of varying insider information by switching an outsider to an insider and establish the linkage principle connecting insider information to the seller's revenues.

Our experimental findings contribute to the experimental literature of auctions that investigates the effects of auction formats on outcomes and bidding behavior. Most experimental literature has focused on either private value auctions or pure common value auctions. For excellent surveys, see Kagel (1995) and Kagel and Levin (2011). There are a handful of experimental studies on interdependent value auctions. Goeree and Offerman (2002) report an experiment on a first-price auction with interdependent values composed of private and common values and measure the degree of inefficiency by varying the level of competition and the degree of uncertainty on common value information. Kirchkamp and Moldovanu (2004) experimentally study an interdependent value environment with asymmetric bidders in valuation in the English and second-price auctions. They find that the English auction yields higher efficiency than the second-price auction does, consistent with equilibrium predictions. Boone et al. (2009) study an auction environment with a restricted structure of interdependent values and a single insider in which both English and second-price auctions are inefficient, and report an experimental evidence that the English auction performs better in both efficiency and revenues than the second-price auction. We add novel evidence on the efficiency and revenue performance of the two standard auctions in a general environment with interdependent values and insider information.

The rest of the paper is organized as follows. Section 2 develops an interdependent value auction model with an asymmetric information structure and provides the theoretical results for the second-price and English auctions. Section 3 describes the experimental design and procedures. Section 4 summarizes experimental findings and Section 5 concludes. All theoretical proofs are contained in the Appendix.[4]

[4]The paper is supplemented by four Online Appendices that provide sample instructions of the experiment and other technical results. These are available at https://sites.google.com/site/jikim72/home/working_papers.

II. Theory

A. Setup

A seller has a single, indivisible object to sell to n bidders. Let $N = \{1, \cdots, n\}$ denote the set of bidders. The value of the object to each bidder is determined by n-dimensional information $s \in [0, 1]^n$, which we call a signal profile. At this point, we do not specify who observes what signals, which is a central part of our asymmetric information structure and is discussed shortly. However, we adopt the convention of calling the i^{th} signal, s_i, as bidder i's signal. To denote signal profiles, let $s = (s_j)_{j \in N}$, $s_{-i} = (s_j)_{j \neq i}$, and let $s_B = (s_j)_{j \in B}$ for any subset of bidders $B \subset N$. It is assumed that the distribution of the signal profile has full support on $[0, 1]^n$. Each bidder i's value, denoted by $v_i(s)$, is assumed to be additively separable into two parts, a private value component $h_i : \mathbb{R} \to \mathbb{R}$ and a common value component $g : \mathbb{R}^n \to \mathbb{R}$, that is,

$$v_i(s) = h_i(s_i) + g(s), \tag{1}$$

where $\frac{dh_i}{ds_i} > 0$ and $\frac{\partial g}{\partial s_j} \geq 0$ for all $j \in N$. According to this function, each bidder i's private value component depends only on his own signal s_i, whereas the common value component, which is the same across all bidders, depends on the entire signal profile s. We adopt this functional form in part because it provides a natural model of the asymmetric information structure in that some bidders often have superior information than other do about the common value aspect of the object.[5] This functional form is also used later for our experimental study.[6] We assume that h_i and g are twice continuously differentiable and that $h_i(0) = g(0) = 0$ for normalization. We further assume that for each i, $\lim_{s_i \to \infty} v_i(s_i, s_{-i}) = \infty$ for any $s_{-i} \in [0, 1]^{n-1}$.[7] The value function defined in (1) satisfies the *single crossing property*: For all s and $i \neq j$, $\frac{\partial v_i}{\partial s_i} > \frac{\partial v_j}{\partial s_i}$. Moreover, for any s, $v_i(s) > v_j(s)$ if and only if $h_i(s_i) > h_j(s_j)$, that is, bidders with higher private component have higher values. The allocation that grants the object to the bidder with the highest value—or the highest private component—for every realization of the signal profile in the support is termed (ex-post) *efficient*.

The asymmetric information structure is modeled by partitioning N into I, a set of *insiders*, and O, a set of *outsiders*: Each outsider $i \in O$ knows only the private value component $h_i(s_i)$, whereas each insider $i \in I$ knows both the private and common value components, $h_i(s_i)$ and $g(s)$, respectively. This assumption is tantamount to having each outsider i be informed of his own signal s_i (because

[5] Some studies adopt this value function in a standard information setup. For instance, see Wilson (1998) and Hong and Shum (2003).

[6] Our efficiency and revenue results continue to hold in more general conditions for the value function. They are available upon request.

[7] This can always be satisfied by, if necessary, redefining the function v_i for signal profiles that are not in the support $[0, 1]^n$.

of the monotonicity of h_i). Then, if $N = O$ (i.e., there is no insider), the model reduces to the *standard information case* where every bidder i is informed only of his own signal s_i. We make a parsimonious assumption on insiders' information by specifying only that they know the values of $h_i(s_i)$ and $g(s)$ for each s, and being silent on the precise signals that they are informed of.[8] We impose no restriction on the number of insiders or outsiders except that there are at least one insider and one outsider, that is, $|I| \geq 1$ and $|O| \geq 1$. The information structure described thus far is assumed to be common knowledge among bidders In particular, outsiders know who insiders are. This assumption is reasonable in the examples mentioned in the introduction, because it is commonly known who owns neighboring tract in an OCS auction, who are existing shareholders or current management trying to buy the target firm in an takeover auction, and who are expert bidders in an artwork auction.

For the auction format, we focus on the sealed-bid second-price (SBSP) auction and the English auction.[9] In the second-price auction, the highest bidder wins the object and pays the second highest bid. For the English auction, we consider the Japanese format, where bidders drop out of the auction as the price continuously rises starting from zero until only one bidder remains and is awarded the object at the last drop-out price (ties are broken uniform randomly in both the second-price and English auctions). Despite the similarity in the pricing rules, the two auction formats are different in that bidders in the English auction, especially outsiders, are given the opportunity to observe others' drop-out prices and update their information while this does not occur in the second-price auction. In both auction formats, we assume that each insider, who knows his value precisely, employs the weakly dominant strategy of bidding (or dropping out at) his value.

B. Second-Price Auction

To begin, we provide the efficiency result with two bidders, one insider and one outsider. In this case, the second-price and English auctions are (strategically) equivalent because the English auction ends as soon as one bidder drops out; thus, bidders have no chance to update their information as in the second-price auction. With two bidders, both auction formats yield the efficient allocation as shown by the following result, whose proof is rather straightforward and omitted:[10]

Theorem 1 (Efficiency with Two Bidders)**.** *With $n = 2$, there is an ex-post equilibrium of the second-price or English auction that achieves the efficient allocation.*

[8] In the first-price auction, however, what insiders know beyond their values can be important, because it provides useful information about their opponents' bids.

[9] We do not consider the first-price auction mainly because of its analytical intractability under the asymmetric information structure. In the case of one insider and one outsider, however, an inefficiency result can be established. Refer to Kim (2003) for this result.

[10] In fact, there is a unique Bayesian equilibrium if the insider is restricted to use undominated strategy. Refer to Kim (2003) for the proof of this result.

With more than two bidders, however, the second-price auction ceases to be efficient, as shown in the following result. The proofs of this theorem and all subsequent results are contained in the Appendix.

Theorem 2 (Inefficiency of Second-Price Auction)**.** *Suppose that* $n \geq 3$ *and* $\frac{\partial g}{\partial s_i} > 0$ *for all* i*. Suppose also that the efficient allocation requires insiders to obtain the object with a positive probability less than one. Then, there exists no efficient equilibrium for the second-price auction.*

To illustrate the cause of this inefficiency, consider a symmetric case with three bidders where $h_i = h$ for some h and all i; then, a bidder with the highest signal has the highest value. Suppose that bidder 3 is an insider and bidder 1 is an outsider. Fix bidder 1's signal at $\bar{s}_1$. In Figure 1, E_i denotes a set of signal profiles (s_2, s_3) for which bidder i has to be a winner according to efficient allocation.

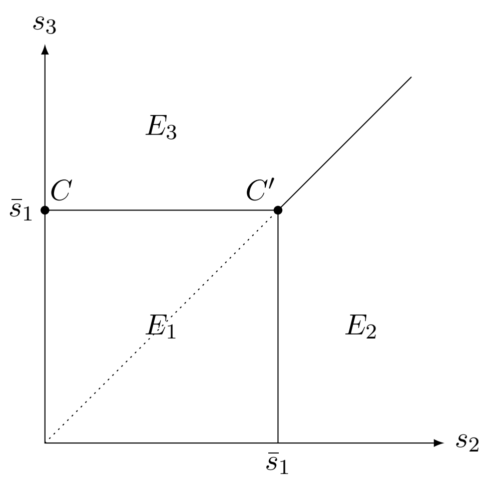


Figure 1

FIGURE 1. INEFFICIENCY OF SECOND-PRICE AUCTION

Along the line $\overline{CC'}$, bidder 1's bid remains unchanged. The same has to be true for bidder 3's bid in order for bidder 1 (resp. bidder 3) to be a winner below (resp. above) the line, as efficiency requires. This is not possible, however, because the value bidding strategy of bidder 3 depends on s_2 as well as on s_1 or s_3 so it varies along $\overline{CC'}$.

Intuitively, this inefficiency is caused by a mismatch between the bidding strategies of outsider and insider in that the former's bid depends only on his own signal whereas the latter's depends on the entire signal profile. To understand why the same intuition does not apply to the two bidder case where efficiency is always obtained (as shown in Theorem 1), we need to consider what extra information reflected by insider's bid, compared to the standard case. In the two bidder case, extra information is a single outsider's signal that is also known to that outsider. In the case of three or more bidders, however, extra information includes signals unknown to outsiders. For instance, in the previous example, bidder 3's bid reflects s_1 and s_2 in addition to s_3, and s_2 is not known to outsider bidder 1. The inability of bidder 1 to adjust his bid depending on s_2 (in contrast to bidder 3's ability to do so) leads to an inefficient allocation in the second-price auction.

The equilibrium bidding strategy for outsiders in the second-price auction is difficult to characterize because of the asymmetry between outsider's and insider's strategies. In the following example, we provide partial characterizations of equilibrium strategies with linear value functions that are used in our experimental study.

Example 1. Suppose that there are three bidders each of whom has a signal uniformly distributed on the interval $[0, 1]$. For each $i \in N = \{1, 2, 3\}$, $v_i(s) = as_i + \sum_{j \neq i} s_j$ with $a > 1$ (for the experimental study, we will set $a = 2$). We consider three information structures, $I = \emptyset$, $I = \{3\}$, and $I = \{2, 3\}$. Assuming that insiders use the dominant strategy of bidding their values, we aim to find symmetric Bayesian Nash equilibrium bidding strategy for outsiders, denoted as $B : [0, 1] \to \mathbb{R}_+$. In the case of $I = \emptyset$, Milgrom and Weber (1982) provides a symmetric equilibrium bidding strategy as follows:

$$B(s_i) = \mathbb{E}_{s_{-i}}[v_i(s_i, s_{-i}) | \max_{j \neq i} s_j = s_i] = (a + \tfrac{3}{2})s_i.$$

In the case of $I = \{2, 3\}$, the equilibrium bidding strategy for bidder 1, which is the best response to value-bidding by bidders 2 and 3, is given as

$$B(s_1) = \begin{cases} \frac{2a^2+4a+1}{2a+1} s_1 & \text{if } s_1 \in [0, \frac{2a+1}{2a+2}] \\ (2a+3)s_1 - a - 1 & \text{otherwise.} \end{cases} \tag{2}$$

Detailed analysis for this result is provided in Online Appendix I. In Figure 2 below, we reproduce Figure 1 with $\bar{s}_1 = 0.7$ and illustrate how the object is allocated according to the equilibrium bidding strategy given in (2). The kinked, dashed line corresponds to the signals (s_2, s_3) for which the equilibrium bid of bidder 1 with $\bar{s}_1 = 0.7$ is equal to the highest bid between bidder 2 and 3. Thus, bidder 1 is a winning (losing) bidder below (above) that line. This implies that in the shaded area A_i, the object is allocated to bidder i, although his value is not the highest.

In the case of $I = \{3\}$, we have only partial characterization of a monotonic equilibrium bidding strategy that is symmetric for the two outsiders:[11]

[11] We are *not* claiming here that any symmetric equilibrium must be monotonic. We note that the presence of insider prevents us from establishing the necessity of monotonicity by using the usual argument based on incentive compatibility.

Proposition 1. *For the second-price auction with $I = \{3\}$, any symmetric, strictly increasing equilibrium strategy for outsiders must satisfy the following properties: for $i = 1, 2$, (i) $B(s_i) \in [(a+1)s_i, (a+2)s_i], \forall s_i \in [0,1]$; and (ii) $B(1) = v_i(1,1,1) = a+2$.*[12]

Online Appendix I provides the proof of this result along with a numerical analysis of the equilibrium bidding strategy.

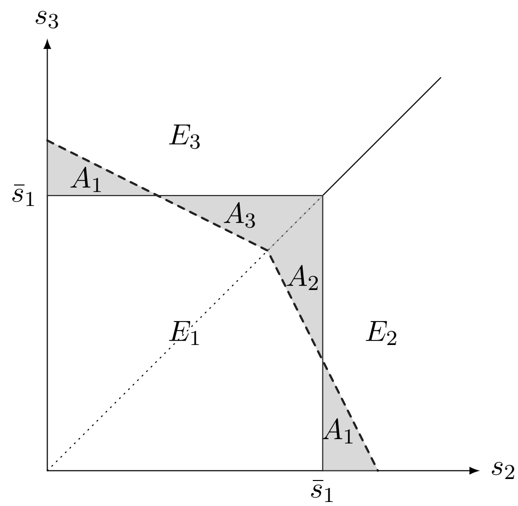


Figure 2. Inefficiency of Equilibrium Allocation in Second-Price Auction

C. English Auction

In this section, we provide the English auction equilibrium that achieves the efficient allocation. Using this equilibrium construction, we also establish comparative statics for the English auction revenue with respect to the number of insiders. As mentioned earlier, the English auction has an advantage over the second-price auction in terms of the additional information bidders can acquire during the dynamic bidding process. We show that this feature enables English auction to overcome the asymmetric information between insiders and outsiders and to achieve an efficient allocation. It is also shown that a larger number of insider bidders increases the seller's revenue.

[12]This means that an outsider bidder i with $s_i = 1$ bids the highest possible value, which may seems surprising at first. The argument for this result is simple, however. Suppose that $B(1) < a+2$ and that an outsider i with signals $s_i = 1$ deviates to some bid greater than $B(1)$. This deviation only increases his chance of winning against bidder 3, in which case his payoff increases by $v_i(1, s_{-i}) - v_3(1, s_{-i}) = (a-1)(1-s_3) > 0$ (unless $s_3 = 1$). Thus, the deviation is profitable.

To this end, we construct the English auction equilibrium following MW and Krishna (2003). A key feature of our equilibrium construction consists of describing how outsiders infer others' signals from their drop-out prices and how to use this information to determine their own drop-out prices. Assume for the moment that each outsider's drop-out price at any point in the auction is strictly increasing in his signal, so his signal is revealed as he drops out. Moreover, insiders' drop-out prices are equal to their values. Using this information, active outsiders who have yet to drop out calculate the *break-even signals* at each current price, which is defined as the signal profile that makes all active bidders break even if they acquire the object at the current price. Then, each active outsider stays in (exits) the auction if his break-even signal at the current price is smaller (greater) than his true signal. The assumed monotonicity of each outsider's drop-out price with respect to his signal is ensured if his break-even signal is strictly increasing as a function of the current price.

To formalize this idea, we introduce a few notations. Let A denote the set of active bidders. Then, $N\backslash A$, $O\backslash A$, and $I\backslash A$ denote the set of inactive bidders, inactive outsiders, and inactive insiders, respectively. With p_i denoting the drop-out price of bidder i, let $p_B = (p_i)_{i\in B}$ for a subset of bidders $B \in N$. Then, a price profile, $p_{N\backslash A}$, corresponds to the history of the game at the point where only bidders in A are active. Next, suppose that the current price is equal to p with a history $p_{N\backslash A}$. Suppose also that the signals of inactive outsiders have been revealed to be $s_{O\backslash A}$. Given these revealed signals, we define the break-even signal profile, denoted as $(s_A(p; p_{N\backslash A}), s_{I\backslash A}(p; p_{N\backslash A}))$, to solve the following system of equations:

$$v_i(s_{O\backslash A}, s_A(p; p_{N\backslash A}), s_{I\backslash A}(p; p_{N\backslash A})) = \begin{cases} p_i & \text{for } i \in I\backslash A & \text{(3a)} \\ p & \text{for } i \in A & \text{(3b)} \end{cases}$$

The equations in (3a) says that with the signal profile $(s_{O\backslash A}, s_A(p; p_{N\backslash A}), s_{I\backslash A}(p; p_{N\backslash A}))$, the value of each inactive insider i is equal to his drop-out price p_i, which is consistent with the insiders' value bidding strategy. The equations in (3b) says that if the active bidders acquire the object at the current price p, then they would break even, which is why the solution of (3) is called break-even signals. Then, the outsiders' equilibrium strategy is simple: After any history $p_{N\backslash A}$, each outsider i drops out (stays in) at price p if and only if $s_i \leq (>) s_i(p; p_{N\backslash A})$. Thus, if $s_i(p; p_{N\backslash A})$ is strictly increasing and continuous with p, an outsider i with signal s_i drops out at price p at which $s_i = s_i(p; p_{N\backslash A})$. This means that each outsider's signal is revealed via his drop-out price, as we assumed in order to set up the system of equations (3). The following theorem proves that the value function in (1) admits a strictly monotonic solution for the break-even signals, thereby establishing the existence of an efficient equilibrium:

Theorem 3 (Efficiency of English Auction)**.** *Consider the English auction with* $n \geq 3$*.*

(i) There exists an ex-post Nash equilibrium where each outsider $i \in O \backslash A$ drops out at price p after history $p_{N \backslash A}$ if and only if $s_i < s_i(p; p_{N \backslash A})$, where $s_i(p; p_{N \backslash A})$ solves (3).

(ii) The equilibrium bidding strategy described in (i) leads to the efficient allocation.

An intuitive understanding of the efficiency of the English auction can be gained through a comparison with the second-price auction. For this, we revisit the three bidder example. Recall from Figure 1 (or Figure 2) that the source of the inefficiency result with the second-price auction is that insider bidder 3 adjusts his bid according to s_2 (in particular, along the threshold $\overline{CC'}$), whereas outsider bidder 1 fails to do so. This problem disappears in the English auction. To see this, suppose that bidder 3 is the only insider, and that bidder 2 drops out first and reveals his true signal, which bidder 1 learns and subsequently reflects in his drop-out strategy. Then, there is no longer informational asymmetry between bidder 1 and 3 regarding s_2. Furthermore, from this point on, the bidding competition is reduced to the two bidder case with one outsider and one insider, as in Theorem 1, where efficiency is easily obtained. In the current example with three bidders and one insider, the argument for the efficiency result based on this intuition can be completed by observing that the outsiders' drop-out strategy implies the following: (i) they drop out before the price reaches their values; (ii) they drop out in order of their values. Because of (i), an insider with the highest value always becomes a winner. In case an outsider has the highest value, efficiency is also achieved because, given (ii), another outsider drops out and reveals his signal before the highest-value outsider does so. The proof of Theorem 3 generalizes this argument to cases with more insiders or outsiders. The difficult part of the proof lies in proving the strict monotonicity of the break-even signals, which follows from the fact that the value functions in (1) satisfy a sufficient condition provided by Krishna (2003).

Turning to the revenue property, we ask how the presence of more insiders affects the seller's revenue. To do so, we consider two English auctions, E and E', which differ by only one bidder who switches from an outsider in E to an insider in E'. Given a signal profile s, let $P(s)$ and $P'(s)$ denote the seller's revenue in E and E', respectively, under the equilibrium described earlier.

Theorem 4. *For any signal profile $s \in [0,1]^n$, $P(s) \leq P'(s)$.*

This result follows from establishing two facts: (i') the switched insider drops out at a higher price in E' than in E; (ii') the higher drop-out price of the switched insider causes (active) outsiders to also drop out at higher prices in E'. To provide some intuition behind (ii'), we revisit the three bidder example. Suppose that bidder 1 is a winner and pays bidder 2's drop-out price p_2 after bidder 3, the only insider, first dropped out at p_3. Then, the break-even signals

$s_1(p_2; p_3)$ and $s_3(p_2; p_3)$ satisfy

$$v_3(s_1(p_2; p_3), s_2, s_3(p_2; p_3)) = p_2 > p_3 = v_3(s_1, s_2, s_3). \tag{4}$$

Because bidder 1 is active at p_2, we have $s_1 > s_1(p_2; p_3)$, which implies $s_3 < s_3(p_2; p_3)$ by (4). The fact that $s_1 > s_1(p_2; p_3)$ means the break-even signal of the active outsider underestimates his true signal, which has the effect of lowering the selling price p_2. However, this effect is mitigated by the fact that $s_3 < s_3(p_2; p_3)$, that is, the insider's signal is overestimated. The underestimation of active outsiders' signals results from their attempt to avoid the winner's curse by bidding as if the currently unknown signals are just high enough to make them break even at the current price. This is why an outsider drops out before the price reaches his value, as (i') above suggests. Thus, an outsider becoming a better-informed insider alleviates the detrimental effect of the winner's curse on both his and other outsiders' drop-out prices.

The revenue result in Theorem 4 is reminiscent of the linkage principle of MW in that the shift from E to E' increases the information available to the bidders, There are some crucial differences, however: Unlike MW's result, the additional information here is not public because only the switched insider gets better informed. Furthermore, the revenue increase in Theorem 4 holds for every realization of the signal profile and thus does not rely on the assumption that the signals are affiliated.

Example 2. Let us consider the same linear example as in Example 1. In the case of $I = \emptyset$ (i.e., no insider), the equilibrium strategy in Theorem 3 takes the same form as in MW: If no one has dropped out, the break-even signal for each bidder i is given as $\frac{p}{(a+1)}$, meaning that bidder i drops out at price equal to $(a+2)s_i$; if one bidder j has already dropped out at p_j and thereby revealed his signal s_j, the break-even signal for each remaining bidder i is given as $\frac{1}{a+1}(p - s_j)$, meaning that bidder i drops out at a price equal to $(a+1)s_i + s_j$.

Let us turn to the case where $I \neq \emptyset$. If no one has dropped out or one outsider has dropped out, an (active) outsider's drop-out strategy remains the same as before. After an insider $j \in I$ has dropped out at price p_j, the condition in (3) becomes

$$as_i(p; p_j) + \sum_{k \neq i} s_k(p; p_j) = p \text{ for each } i \neq j$$
$$as_j(p; p_j) + \sum_{k \neq j} s_j(p; p_j) = p_j,$$

which yields the break-even signal $s_i(p; p_j)$ for each $i \neq j$ that solves

$$(a+1)s_i(p; p_j) + \tfrac{1}{a}(p_j - 2s_i(p; p_j)) = p.$$

Given the equilibrium strategy that calls for each outsider i to drop out at price p where $s_i(p; p_j) = s_i$, this equation implies that an outsider i drops out at a price equal to $(a+1)s_i + \frac{1}{a}(v_j(s) - 2s_i)$.

Tables 1 and 2 summarize the drop-out prices that result from the equilibrium strategy described in the case of $I = \{3\}$ and $I = \{2, 3\}$, respectively. We restrict our attention to the cases of $s_1 > s_2$ in Table 1 and $s_2 > s_3$ in Table 2, so the value ranking within outsiders/insiders, and thus the order of their drop-out prices, is fixed. The drop-out prices are for bidders who do not have the highest value, meaning that the bidder with the highest value wins. The rightmost columns of the two tables show the sale prices (or the second drop-out prices) with one less insider, which are lower than the prices in the second column from the right. Thus, the seller's revenue becomes (weakly) higher for each realized signals as an outsider switches to an insider.

Table 1—Drop-out prices for the English auction in the case where $I = \{3\}$ and $s_1 > s_2$

	1st drop-out price	2nd drop-out price (= sale price)	sale price with $I = \emptyset$
(1-i) $s_3 > s_1 > s_2$	$p_2 = (a+2)s_2$	$p_1 = (a+1)s_1 + s_2$	$(a+1)s_1 + s_2$
(1-ii) $s_1 > s_3 > s_2$	$p_2 = (a+2)s_2$	$p_3 = v_3(s)$	$(a+1)s_3 + s_2$
(1-iii) $s_1 > s_2 > s_3$ and $(a+2)s_2 > v_3(s)$	$p_3 = v_3(s)$	$p_2 = (a+1)s_2 + \frac{1}{a}(v_3(s) - 2s_2)$	$(a+1)s_2 + s_3$
(1-iv) $s_1 > s_2 > s_3$ and $(a+2)s_2 < v_3(s)$	$p_2 = (a+2)s_2$	$p_3 = v_3(s)$	$(a+1)s_2 + s_3$

Table 2—Drop-out prices for the English auction in the case where $I = \{2, 3\}$ and $s_2 > s_3$

	1st drop-out price	2nd drop-out price (= sale price)	sale price with $I = \{3\}$
(2-i) $s_1 > s_2 > s_3$	$p_3 = v_3(s)$	$p_2 = v_2(s)$	$\max\{v_3(s), (a+1)s_2 + \frac{1}{a}(v_3(s) - 2s_2)\}$
(2-ii) $s_2 > s_1 > s_3$ and $(a+2)s_1 > v_3(s)$	$p_3 = v_3(s)$	$p_1 = (a+1)s_1 + \frac{1}{a}(v_3(s) - 2s_1)$	$(a+1)s_1 + \frac{1}{a}(v_3(s) - 2s_1)$
(2-iii) $s_2 > s_1 > s_3$ and $(a+2)s_1 < v_3(s)$	$p_1 = (a+2)s_1$	$p_3 = v_3(s)$	$v_3(s)$
(2-iv) $s_2 > s_3 > s_1$	$p_1 = (a+2)s_1$	$p_3 = v_3(s)$	$v_3(s)$

III. Experimental Design and Procedures

The experiment was run at the Experimental Laboratory of the Centre for Economic Learning and Social Evolution (ELSE) at University College London (UCL) between December 2011 and March 2012. The subjects in this experiment were recruited from an ELSE pool of UCL undergraduate students across all disciplines. Each subject participated in only one of the experimental sessions. After subjects read the instructions, the instructions were read aloud by an experimental administrator. Each experimental session lasted around 2 hours. The experiment was computerized and conducted using experimental software z-Tree developed by Fischbacher (2007). Sample instructions are reported in Online Appendix II.

In the design, we use a variety of auction games with three bidders, $i = 1, 2, 3$. Each bidder i receives a private signal, s_i, which is randomly drawn from the uniform distribution over the set of integer numbers, $\{0, 1, 2, ..., 100\}$. Given a realization of signal profile $s = (s_1, s_2, s_3)$, the object valuation for each bidder i is

$$v_i(s) = 2 \times s_i + \sum_{j \neq i} s_j.$$

There are two auction formats in the experiment—the SBSP auction and the English auction. In each auction format, three distinct games are conducted where the number of insiders varies from zero to two, $k = 0, 1, 2$. Thus, there are six treatments in total in terms of the auction format and the number of insiders. A single treatment consisting of one auction format and a single value of k is used for each session. We conduct 12 sessions in total with two sessions for each auction game treatment. Each session consists of 17 independent rounds of auction games, where the first two rounds were practice rounds in which auction outcomes were not counted for actual payoffs. Throughout the paper, we use data generated only after the first two rounds. The following table summarizes the experimental design and the amount of experimental data. The first number in each cell is the number of subjects and the second is the number of group observations in each treatment. In total, 233 subjects participated in the experiment.

Auction format	# of insiders (k)	Session 1	Session 2	Total
English	0	21 / 105	21 / 105	42 / 210
	1	18 / 135	20 / 150	38 / 285
	2	23 / 345	19 / 285	42 / 630
Second-price	0	21 / 105	21 / 105	42 / 210
	1	20 / 150	16 / 120	36 / 270
	2	17 / 255	16 / 240	33 / 495

We use an irrevocable-exit, ascending clock version of the English and the second-price auctions (Kagel et al., 1987; Kirchkamp and Moldovanu, 2004). In the English auction treatments, each subject sees three digital clocks representing the bidding process on his or her computer screen, one for each bidder in the group. In the second-price auction treatments, there is only one clock presented to each subject. In the English auction, the computer screen clearly indicates which clock belongs to each subject and, if any, which belongs to an insider.

At the start of an auction round, subjects are randomly formed into three-bidder groups. In treatments with insider(s), each insider bidder is played by the computer, whereas outside bidders are played by human subjects. The groups formed in each round depend solely upon chance and are independent of the groups formed in any of the other rounds. Once assigned to a three-bidder group, each subject observes his or her private signal and the valuation formula. Other bidders' signals in the formula are hidden to indicate that subjects cannot observe others' signals.

Clocks simultaneously start at -4 and synchronously increase by 1 unit per half second. In the second-price auction, if the subject drops out by stopping his or her clock, it turns red at the price at which it stops. If the other two participants have not yet dropped out, their clocks continue to ascend at the same speed. Once all three bidders have stopped their own clocks, the auction is over. The remaining bidder who chooses the highest price wins the auction and pays the price at which the second bidder drops out. In the English auction, in the next bid increment after one bidder stops his or her clock, the remaining bidders observe that the bidder's clock has stopped and has turned red. Then, there is a 3 seconds of time pause before the two remaining clocks synchronously increase by 1 unit per second. If one more participant drops out, the auction is over. The remaining bidder wins the auction and pays the price at which the second bidder drops out. If all remaining bidders dropped out at the same price level or if the price level reaches 500 (the maximum bid allowed), the winner is selected at random from the set of those participants and pay the price at which this event occurs.

Each subject simply move the mouse over his or her own clock and clicks on it when the price on the clock reaches the level he or she wants to drop out at. This makes the subject drop out of the bidding (i.e., stops his or her own clock). Once subjects have dropped out, they are not allowed to re-enter the auction. Subjects cannot stop their clocks before it reaches 0, which is the minimum bid allowed.

In treatments with at least one insider, the insider is played by the computer. This (computer-generated) insider uses a simple drop-out decision rule: it drops out at a price equal to its own valuation. The computer participant always abides by this rule, and this information is common knowledge to subjects.

When an auction round ends, the computer informs each subject of the results of that round, which include bids at which bidders dropped out, signals bidders received, auction object values, payments and earnings in the round. Once every subject has confirmed the results, the next round starts with the computer randomly forming new groups of three bidders and selecting signals for them. This process was repeated until all 17 rounds were completed.

Earnings were calculated in terms of tokens and exchanged into pounds at the rate of 40 tokens to £1. The earnings in each round are determined using the difference between winning revenue and cost. The winning revenue is the valuation assigned to the subject if he or she won the auction and zero otherwise. The winning cost for subjects that did not win the auction is simply equal to zero. In our experiment, subjects may accumulate losses, exhaust their balance, and go bankrupt during the experiment. If this event occurs, subjects are no longer allowed to participate in the experiment. In order to avoid potentially adverse impacts of limited liability for losses on bidding behavior,[13] we provided each subject a relatively large amount of money, £10, as the initial balance in the experiment. None of the subjects experienced bankruptcy during the experiment. The total payment to a subject was the sum of his or her earnings over the 15 rounds after the first two practice rounds, plus the initial balance £10 and an extra £5 participation fee. The average payment was about £18.32. Subjects received their payments privately at the end of the session.

IV. Experimental Findings

A. Efficiency

We analyze the experimental data by comparing efficiency performance across auction treatments. Table 2 reports the frequency of efficient allocation as well as the average efficiency ratio by measuring the economic magnitude of inefficient outcomes. The efficiency ratio is defined as the actual surplus improvement over random allocation as a percentage of the first-best surplus improvement over random assignment.[14] This ratio equals one if the allocation is efficient, that is, if the highest-value bidder wins the object, and less than one otherwise. For auction treatments with one insider (resp. two insiders), we group the data with respect to the ranking of the insider's value (resp. the outsider's). For each treatment of the number of insiders, the last two columns on the right side report the p-values from the one-sided t-test of the null hypothesis that the efficiency outcome of the English auction is less than or equal to that of the second-price auction.

- Table 2 here -

The level of efficiency is high in all treatments. The frequencies of efficient allocation range from 76% (in the second-price auction with $k = 1$) to almost 90% (in the English auction with $k = 2$). The efficiency ratio shows similar patterns: subjects achieved, on average, between 74% (in the second-price auction with $k = 1$) and 93% (in the English auction with $k = 2$) of the first-best surplus over random surplus. On the other hand, we find that the English auction exhibits better performance in efficiency than the second-price auction, when predicted by theory. In the symmetric information structure with no insider ($k = 0$), there is no significant difference in efficiency between the two auction formats. In the presence of an insider, however, the efficiency outcomes are significantly

[13]Hansen and Lott (1991) argued that aggressive bidding behavior in a common value auction experiment conducted by Kagel and Levin (1986) may be a rational response to limited liability rather than a result of the winner's curse. Lind and Plott (1991) designed an experiment eliminating the limited-liability problem and found that this problem does not account for the aggressive bidding patterns in the experiment of Kagel and Levin.

[14]Our measure of the efficiency ratio normalizes the realized surplus by both the best-case scenario (efficiency) and the worst-case one (random assignment). This double-normalization renders a more robust measure against the rescaling of the value support than an alternative measure such as the percentage of the first-best surplus realized.

higher in the English auction than in the second-price auction. These results are qualitatively consistent with the theoretical predictions that when at least one insider is present in auction, an efficient equilibrium exists in the English auction but not in the second-price auction. We further divide the data with respect to the value ranking of bidders. In the auctions with one insider ($k = 1$), the efficiency ratio of the English auction is significantly higher than that of the second-price auction, regardless of the value ranking. In the auctions with two insiders ($k = 2$), the English auction attains higher efficiency than the second-price auction in each case of value ranking except for the case where the outsider has the second-highest value.

To further examine the English auction's superior efficiency performance, we divide the English auction treatment data into two subsamples with respect to whether the second-price auction, if it had been used, would have produced an efficient or inefficient allocation according to the theoretical prediction. That is, at each English auction treatment data point, we check if the equilibrium allocation of the second-price auction is efficient or inefficient. Because of the small sample size of the inefficient equilibrium allocations, we allow for a five-token margin to classify the case of inefficiency such that a single data point is treated as *inefficient* if the equilibrium of the second-price auction is either inefficient or efficient but the difference between its two high bids is less than five tokens. We then check how often subjects are able to achieve an efficient allocation in each of these data points. We conduct the same analysis for the second-price auction treatments. Table 3 presents the frequencies of efficient allocation in each case in the treatments with at least one insider with the number of observations in parentheses.[15] The last column reports p-values from the one-sided t-test of the null hypothesis that the frequency of efficient allocations in the English auction is less than or equal to that in the second-price auction.

- *Table 3 here* -

Table 3 offers further insights on the efficiency performances across treatments with insiders. In the case where the second-price auction format predicts an efficient allocation, actual efficiency frequencies are quite high, ranging from 81% (in the second-price auction treatment with $k = 1$) to 92% (in the English auction treatment with $k = 2$). In contrast, the level of efficiency becomes much lower in the case where the equilibrium allocation under the second-price auction format is inefficient: the frequencies of efficiency range between 33% (in the second-price auction with $k = 1$) and 66% (in the English auction with $k = 2$). In each treatment, the frequency difference of efficient outcomes between these two cases is statistically significant at usual significance levels. Thus, when it is predicted that the bidding mismatch problem between outsiders and insiders would arise, subjects are more likely to fail to attain an efficient allocation. Furthermore, in

[15]The results of Table 3 remain basically the same either whether we use no margin or small one in classifying the case where the second-price auction format predicts an inefficient outcome.

the case where the equilibrium of the second-price auction predicts an inefficient allocation, the English action performs significantly better than the second-price auction when there is a single insider, that is, $k = 1$: 33% in the second-price auction *versus* 62% in the English auction. In the case of $k = 2$, we find no difference between the two auction formats.

We summarize the efficiency outcomes as follows.

Finding 1 (efficiency) *The English auction exhibits higher efficiency performance than the second-price auction does in the presence of insiders, as theory predicts. In the symmetric information structure where there is no insider, there is no difference of efficiency performance between the two auction formats.*

B. Revenue

Next, we compare revenue performance across treatments. Table 4 presents the average percentage deviations of observed revenues from their theoretically predicted values across treatments, along with their standard errors and p-value from the t-tests for the null hypothesis that the sample mean is equal to zero. For auction treatments with one insider (resp. two insiders), we divide the data with respect to the ranking of the value of the insider (resp. the outsider).

- *Table 4 here* -

When all bidders are outsiders, observed revenues are significantly higher than theoretically predicted ones: 22% higher in the English auction and 26% higher in the SBSP auction. This tendency becomes weaker in auctions with at least one insider: 3% (0%) higher in the English auction with $k = 1$ ($k = 2$) and 14% (1%) higher in the SBSP auction with $k = 1$ ($k = 2$). This may not be unexpected, because insiders in the experiment are computer-generated and play the equilibrium strategy of bidding their own values. Despite this consideration, observed revenues in the SBSP auction with $k = 1$ are significantly above the theoretical prediction. A closer look at the data by the ranking of values reveals that in auctions with $k = 1$, the tendency of revenues to be higher than theoretically predicted strengthens when both outsiders have lower or higher values than the insider. This apparently results from overbidding (relative to the BNE) by outsiders. We investigate the bidding behavior of subjects more thoroughly in the next subsection. In auctions with $k = 2$, observed revenues appear to concentrate around the theoretically predicted values. The magnitude of the departures of observed revenues from theoretical ones is not large, although some of the departures remain significant.

Our theory establishes the linkage principle of the English auction that for any signal profile, the switch of an outsider to an insider weakly increases revenue. In order to examine the linkage principle, we run regressions of observed revenues (resp. theoretical revenues) on signal profiles and dummies for the number of insiders in each auction format. The results are reported in Table 5.[16]

- Table 5 here -

Controlling for the signal profile, switching an additional outsider to an insider improves revenues significantly in the English auction. The observed revenues increase on average by 10 from $k = 0$ to $k = 1$ and by 8 from $k = 1$ to $k = 2$ in the English auction, both of which are statistically significant at usual significant levels. The magnitudes of revenue improving with an extra insider in the data are also consistent with those predicted by theory. In regressions with theoretical revenues, we similarly observe an increase in revenues by 12 from $k = 0$ to $k = 1$ and by 9 from $k = 1$ to $k = 2$ in the English auction. Analogously, we observe the revenue-improving outcomes in the SBSP auction data, despite that we are unable to prove it theoretically. Observed revenues in the SBSP auction increase by 10 from $k = 0$ to $k = 1$, whereas they remain unchanged from $k = 1$ to $k = 2$. In the current experimental setup, theory predicts the linkage principle in that the theoretical revenues of the SBSP auction increase by 14 from $k = 0$ to $k = 1$ and by 8 from $k = 1$ to $k = 2$.

We summarize our revenue findings as follows.

Finding 2 (revenue) *Revenues in the data tend to deviate above theoretically predicted values, in particular in auctions with no insider. Despite this tendency, the increase in the number of insiders has a positive impact on revenues in the English auction, consistent with the linkage principle of the English auction. We find similar revenue-improving patterns with extra insiders in the SBSP auction.*

C. Bidding Behavior

We have found that auction treatments (in terms of both auction formats and the number of insiders) have significant impacts on efficiency and revenues. Concurrently, we have identified some quantitative departures from the BNE predictions. In this section, we examine several features of the data more closely to better understand the subjects' bidding behavior. We begin this analysis with the SBSP auction.

SBSP Auction. — We first overview the general bidding behavior patterns in the SBSP auction by drawing scatter plots between subjects' bids and their private signals across insider treatments; then, we match them with the BNE strategy.

[16]As a robustness check of Table 5, we conduct regression analysis with more flexible functional specifications of quadratic forms of signals or dummies for insider/outsider and their interactions with signals. These results are reported in Online Appendix III. In essence, the empirical findings on the linkage principle remain unchanged.

The scatter plots are based only on human participants in the experiment who are outsiders. This is presented in Figure 3.

- Figure 3 here -

This simple graphical representation of bidding behavior already reveals some useful information on the nature of the bidding behavior. There is a notable departure of bidding from the BNE strategy when the value of private signal is low: low-signal bidders bid substantially higher than the BNE strategy dictates. This overbidding pattern in low values of signal appears present in all insider treatments, although this pattern appears to be weaker in the treatment with two insiders. On the other hand, subjects' bids appear less responsive to their own private signals than the BNE predicts. As a consequence, many observed bids tend to lie below the BNE strategy values when the value of private signal is high.

In order to examine the bidding behavior of subjects more closely, we run linear regressions of subjects' bids on their private signals. Our theory predicts that the BNE strategy has a kink at the signal value equal to 82.109 when $k = 1$ and that equal to 500/6 when $k = 2$. We thus use the regression specifications with and without these kinks.[17] Table 6 reports the results of the regressions with robust standard errors clustered by individual subjects. We also present p-values of the F-test for the null hypothesis that observed bids follow the equilibrium strategy.

- Table 6 here -

The regression analysis confirms the information extracted from the scatter plots in Figure 2. The subjects respond less sensitively to their own private signals than theory predicts. The estimated coefficients on signals are in the range between 2.4 and 2.9, and significantly lower than the theoretical prediction of approximately 3.5 in each insider treatment. Furthermore, the constant term in the regression is significantly positive in each treatment. In summary, the regression results confirm that subjects tend to overbid relative to the equilibrium when signals are low, and that this overbidding tendency diminishes when signals are high. The joint test based on the F-statistic indicates that subjects' behavior differ significantly from the equilibrium strategy at the usual significance levels. Overall, the overbidding pattern in our data is consistent with findings in the experimental literature of auctions (see Kagel, 1995; Kagel and Levin, 2002).

English Auction. — We now turn to the subjects' behavior in the English auction. We again overview the general patterns of bidding in this auction format by drawing scatter plots with subjects' bids. If the subject is the first drop-out bidder, we relate his drop-out price to his own private signal. If the subject is the

[17] As shown in Proposition 1, there is only a partial characterization of the BNE strategy when $k = 1$. Nonetheless, we are able to numerically derive the kink of the BNE strategy when $k = 1$.

second drop-out bidder, we associate his drop-out price with the BNE strategy we constructed in Theorem 3 and showed in Table 1. The second drop-out bidder's equilibrium strategy contains information on her private signal as well as on the first drop-out price (and the identity of the first bidder dropping out when $k \geq 1$). Thus, it is a convenient, visual way of summarizing the behavior of the second drop-out prices and comparing them with the BNE strategy. The scatter plots are presented in Figure 4.

- Figure 4 here -

Analogous to the overbidding pattern in the SBSP auction, we observe that the first drop-out prices tend to deviate upward from the equilibrium strategy when the value of the private signal is low. However, this pattern appears much less notable in the treatment with two insiders, where there is a cluster of observed bids along the equilibrium strategy line even when the signal value is quite low. It also appears that the subjects who drop out first tend to respond less sensitively to their private signals. Regarding the subjects who drop out second, they appear to bid less responsively to the combination of their own signal and the first drop-out price than the BNE dictates. It is not clear from the simple scatter plot whether this behavioral departure results from insensitivity to their private signal or to the first drop-out price or some other combination. This partly motivates the following regression analysis.

We run censored regressions of the first drop-out and second drop-out prices with the sample of outsiders. The censored regression method is required because we observe only the first drop-out price for the lowest bidder and the bids of two remaining bidders are right-censored, and because the second drop-out price is left-censored by the first drop-out price. In the treatment with one insider ($k = 1$), as the BNE strategy predicts, the regression specification for the second drop-out price interacts with the signal of the second drop-out bidder; furthermore, the first drop-out price interacts with the dummy indicating whether it is an insider who drops out first. We further include an alternative specification by adding this dummy in the regression equation to capture the potential empirical impact of this dummy on the constant term. Table 7 reports the regression results and p-values from the F-test for the joint null hypothesis that observed bids follow the equilibrium strategy.[18] Robust standard errors clustered by individual subjects are reported in parentheses.

- Table 7 here -

The regression analysis of the first drop-out prices reveals that the subjects respond less sensitively to their private signal than the equilibrium strategy across insider treatments predicts. The estimated coefficients on private signals are 2.88,

[18] The censored regression approach, using the maximum likelihood estimation method, is given in detail in Online Appendix IV.

3.26, and 3.48 in the treatments with no insider ($k = 0$), one insider ($k = 1$), and two insiders ($k = 2$), respectively, whereas the equilibrium behavior responds to the private signal by a factor of 4. We also found that the constant term is significantly positive in all insider treatments, 77.89 when $k = 0$, 65.67 when $k = 1$, and 32.09 when $k = 2$. Given these results, the null hypothesis that the first drop-out prices follow the BNE strategy in each insider treatment is rejected at usual significance levels. Intriguingly, the estimated coefficient on the private signal increases and the constant term declines as the number of insiders increases. Thus, the overbidding pattern and its resulting winner's curse diminish when the number of insiders in the experiment increases. The presence of an insider who knows the value of the object may make outsiders more wary of hedging against informational asymmetry between insider and outsider. This need to hedge against informational asymmetry may help correct the winner's curse.

We turn to the regression analysis of the second drop-out prices, which is quite revealing. Similar to the first drop-out bidders, the second drop-out bidders respond less sensitively to their own private signal than the equilibrium analysis predicts. However, they respond excessively to the first drop-out prices. According to our theory, the equilibrium bid would respond to the first drop-out price by a factor of 0.25 in the treatments with no insider and with one insider when an outsider dropped out first, and by a factor of 0.5 in the treatment with one insider when the first drop-out bidder is an insider as well as in the treatment with two insiders. In the experiment, the subjects responded to the first drop-out price by about 0.40 on average when $k = 0$, 0.34 when $k = 1$ and an outsider dropped out first, 0.65 when $k = 1$ and the insider dropped out first, and 0.70 in the treatment with two insiders.[19] Despite the excessive responsiveness to the first drop-out price, as suggested in Figure 3, the combined behavioral response of the private signal and first drop-out price appears less sensitive than the equilibrium strategy dictates. The joint null hypothesis that the experimental behavior is equivalent to the equilibrium strategy is rejected at the usual significance levels in each treatment. Finally, in the auction treatment with one insider ($k = 1$), the subjects differentially responded to the identity of the first drop-out bidder: they tend to place more weight on the first drop-out price and less on their private signal when the insider drops out first. This is qualitatively consistent with the BNE prediction.

Quantifying naive bidding. — The overbidding pattern in our interdependent value environment with insider information is closely related to the findings of the winner's curse in the experimental literature of common value auctions (see Kagel and Levin, 2002). Winning against other bidders implies that the outsider's value estimate happens to be the highest among outsiders *as well as* higher than

[19]The p-values of the t-test for the null hypothesis that the coefficient of $p/4$ is equivalent to the equilibrium prediction are 0.000 in each case.

each insider's value. Thus, failure to account for this adverse selection problem results in overbidding and can make the outsider fall prey to the winner's curse. Indeed, the winners in our data ended up with negative surplus more frequently than the theory predicts. These frequencies range between 25% and 32% in the SBSP auctions, and between 17% and 22% in the English auctions.[20]

We employ a simple strategy of quantifying the extent to which subjects in our experiment fail to account properly for the adverse selection problem and thus bid naively. We define naive bidding as bidding based on the unconditional expected value (by completely ignoring the adverse selection problem). For the second-price auction and the first drop-out bidder in the English auction, the naive bidding strategy takes the form $b^{naive}(s_i) = 2s_i + 100$. We also assume the same naive bidding strategy form after observing a first drop-out price in the English auction, which means that the naive bidder ignores any information from the first drop-out price. We then consider a convex combination of the naive bidding strategy and the BNE one. For all bidders in the second-price auction and the first drop-out bidders in the English auction, this combined bidding strategy is represented by

$$b(s_i;\alpha) = \alpha \times b^{naive}(s_i) + (1-\alpha)\, b^{BNE}(s_i).$$

The equation for the second drop-out bidders in the English auction with first drop-out price p_j can be written as

$$b(s_i, p_j;\alpha) = \alpha \times b^{naive}(s_i) + (1-\alpha)\, b^{BNE}(s_i, p_j).$$

We estimate α by matching this form to the data for each treatment. α measures the degree to which the subjects' behavior departs from the BNE and is close to the naive bidding strategy. In our setup, α is well identified. For instance, for the first drop-out bidder in the English auction with each k, this convex combination can be rewritten as $b(s_i;\alpha) = 100 \times \alpha + (2\alpha + 4(1-\alpha))\, s_i$; then α is identified by matching the constant term and the slope of this equation to the data.

It is of interest to compare α estimates across situations where the equilibrium drop-out strategy is the same. In particular, the equilibrium drop-out strategies for the first drop-out bidder are the same in all English auction insider treatments. The equilibrium drop-out strategies for the second drop-out bidder are the same in the English auction with no insider ($k = 0$) and in that with one insider ($k = 1$) when the first drop-out bidder is an outsider. Moreover, they are the same in the English auction with one insider ($k = 1$) when the first drop-out bidder is an insider and in the English auction with two insiders ($k = 2$). We focus on comparing α in such situations.

Table 8 reports the regression results of α estimates across auction treatments.

[20]Online Appendix V reports empirical and theoretical frequencies of the winner getting negative surplus and average surplus across treatments.

Robust standard errors clustered by individual subjects are reported in parentheses. We also report the t-test for the equivalence of α between two insider treatments in a given auction format.

- Table 8 here -

There is substantial evidence of naive bidding in all treatments: an estimated parameter α is statistically significant at the usual significance level in all treatments of the English and second-price auctions. On the other hand, we observe notable variations of $\widehat{\alpha}$ across English auction insider treatments. For the case of first drop-out prices (where the equilibrium strategies are common across insider treatments), $\widehat{\alpha}$ decreases in the number of insiders: $\widehat{\alpha} = 0.66$ when $k = 0$; $\widehat{\alpha} = 0.53$ when $k = 1$; and $\widehat{\alpha} = 0.18$ when $k = 2$. The reduction of the degree of naive bidding is statistically significant relative to the treatment with two insiders. A similar pattern is established for the second drop-out prices. Comparing $\widehat{\alpha}$'s between the treatment with $k = 0$ and that with $k = 1$ when the first drop-out bidder is an outsider, we again find a significant drop in the degree of naive bidding: $\widehat{\alpha} = 0.70$ when $k = 0$ and $\widehat{\alpha} = 0.32$ when $k = 1$ and the first drop-out bidder is an outsider. We do not find a statistical difference between the two estimates in the treatments with $k = 1$ and 2 where the first drop-out bidder is an insider. In the second-price auction treatments, we do not find similar monotonic patterns of $\widehat{\alpha}$: $\widehat{\alpha} = 0.49$ when $k = 0$; $\widehat{\alpha} = 0.73$ when $k = 1$; and $\widehat{\alpha} = 0.40$ when $k = 2$.

The declining pattern of the degree of naive bidding in the English auction is quite intriguing. We conjecture that the presence of insiders—who have an informational advantage—makes the outsider more wary about information asymmetry and thus motivates the outsider to hedge against such asymmetry. This may work toward the correction of naive bidding and thus the winner's curse in our setup. We summarize the bidding behavior in the experiment as follows.

Result 3 (bidding behavior) (i) *There is evidence of naive bidding in both the second-price auction and the English auction, that is, overbidding relative to the equilibrium prediction.* (ii) *The degree of naive bidding declines significantly as the number of insiders in the English auction increases.*

V. Conclusion

In this paper, we have proposed a model of interdependent value auctions with ex ante information asymmetry and examined key predictions of the model via a laboratory experiment. We study two standard auction formats—the SBSP auction and the English auction. In each auction we allow any composition of insiders, who are perfectly informed of their value, and outsiders, who are only informed about the private component of their value. The information asymmetry between insiders and outsiders gives rise to potential mismatch of bidding strategies between them.

Our model is distinct from the existing auction literature with insider information in a couple of important respects (see, e.g., Engelbrecht-Wiggans et al., 1982; Hendrick and Porter, 1988; Hendrick et al., 1994). First, unlike the literature where a common value is typically assumed, we adopt the interdependent value setup and study the effects of insider information in standard auctions—the SBSP and English auctions—on their efficiency and revenue. The English auction has an efficient equilibrium, whereas the second-price auction suffers inefficiency caused by the presence of the insider. Second, our theory is general in that we provide an equilibrium characterization for any number of insiders, in contrast to most studies, where only one insider is introduced. This enables us to provide novel results—both theoretically and experimentally—regarding the effects of varying the number of insiders on the performance of the two auction formats. Most importantly, the increase in insider information by turning an outsider into an insider positively affects the seller's revenue in the English auction.

The experimental evidence supports the theoretical predictions on efficiency and revenues. We observe that subjects achieve an efficient allocation more frequently in the English auction than in the second-price auction when insider information is present. This is consistent with the theory that the English auction has an efficient equilibrium, whereas the second-price auction does not. Controlling for realized signals, average revenues of the English auction increase in the number of insiders. Although we do not have revenue predictions for the second-price auction, we find similar patterns showing that second-price auction revenues increase with the number of insiders. Despite the accordance of experimental data with our theory, there is substantial evidence of naive bidding, as typical in experimental auction literature (see Kagel, 1995; Kagel and Levin, 2011). Intriguingly, we find that in the English auction, the degree of naive bidding declines as the number of insiders increases. We conjecture that the more insiders are present, the more wary outsiders are in their bidding behavior. This may reduce naive bidding and, as a result, the winner's curse. This offers an interesting phenomenon that warrants further theoretical investigation such as developing an alternative—perhaps behavioral—model.

Appendix

Proof of Theorem 2. Suppose to the contrary that there exists an efficient equilibrium of the second-price auction. For a given bidder i, we define $E_i := \{s \in [0,1]^n \mid v_i(s) \geq v_j(s) \text{ for all } j \neq i\}$, that is the set of signals for which bidder i wins the object at the efficient equilibrium. Because of the assumption that insiders obtain the good with some positive probability less than one, there must exist an outsider i, an insider j, and a signal profile s in the interior such that $v_i(s) = v_j(s) > \max_{k \neq i,j} v_k(s)$ (or $h_i(s_i) = h_j(s_j) > \max_{k \neq i,j} h_k(s_k)$). Fix any such profile s and let $E_{ij}(s_i) := \{s' | s'_i = s_i \text{ and } s' \in E_i \cap E_j\}$. Then, we can find another profile $\tilde{s} \in E_{ij}(s_i)$ such that $\tilde{s}_i = s_i$, $\tilde{s}_j = s_j$, and $\tilde{s}_k < s_k, \forall k \neq i, j$.

Next, given the efficient allocation and value bidding of bidder j, bid $b_i(s_i)$ of bidder i with s_i has to satisfy

$$\max_{\{s' \in E_i | s'_i = s_i\}} v_j(s') \leq b_i(s_i) \leq \min_{\{s' \in E_j | s'_i = s_i\}} v_j(s'). \tag{5}$$

If the first inequality were violated, bidder i with signal s_i would lose to bidder j when the former has a higher value. If the second inequality were violated, bidder j would lose to bidder i with signal s'_i when the former has a higher value. From (5),

$$\max_{s' \in E_{ij}(s_i)} v_j(s') \leq \max_{\{s' \in E_i | s'_i = s_i\}} v_j(s') \leq b_i(s_i) \leq \min_{\{s' \in E_j | s'_i = s_i\}} v_j(s') \leq \min_{s' \in E_{ij}(s_i)} v_j(s').$$

Thus, $v_j(\cdot)$ has to be constant on $E_{ij}(s_i)$. This implies that for some constant k, $v_j(s') = k, \forall s' \in E_{ij}(s_i)$, which in turn implies that $v_i(s') = k, \forall s' \in E_{ij}(s_i)$ because $v_i(s') = v_j(s')$ for any $s' \in E_{ij}(s_i)$. Thus, for any $s' \in E_{ij}(s_i)$, we must have $h_i(s_i) = h_i(s'_i) = k - g(s')$; this cannot be true, because given our assumption, we have $g(s) > g(\tilde{s})$ even though $s, \tilde{s} \in E_{ij}(s_i)$. □

We provide the proofs of Theorem 3 and 4 in Section II.C. To simplify the notation, we let $s(p) := (s_{O\setminus A}, s_A(p; p_{N\setminus A}), s_{I\setminus A}(p; p_{N\setminus A}))$ and $s_i(p) = s_i(p; p_{N\setminus A})$ for $i \in I \cup A$, by omitting the price history $p_{N\setminus A}$. We first establish a couple of preliminary results in Lemma 2 and 3. To do so, let $\overline{v} = \max_{i\in N} \max_{s\in[0,1]^n} v_i(s)$. Given the assumption that $\lim_{s_i\to\infty} v_i(s_i, s_{-i}) \to \infty$ for any $s_{-i} \in [0,1]^{n-1}$, we can find some $\overline{s}_i$ for each i such that $v_i(\overline{s}_i, s_{-i}) \geq \overline{v}$ for any $s_{-i} \in [0,1]^{n-1}$.

The following result from Krishna (2003) is used to prove the existence of solution of (3):

Lemma 1. *Suppose that $R = (r_{ij})$ is an $m \times m$ matrix that satisfies the dominant average condition:*

$$\frac{1}{m}\sum_{k=1}^{m} r_{kj} > \quad r_{ij}, \forall i \neq j \quad \text{and} \quad \sum_{k=1}^{m} r_{kj} > 0, \forall j. \tag{6}$$

Then, A is invertible. Also, there exists a unique $x \gg 0$ such that $Ax = 1$, where 1 is a column vector of m 1's.

Lemma 2. *For any $s_{O\setminus A}$ and $p_{N\setminus A}$, there exists a solution $(s_A, s_{I\setminus A}) : [\max_{i\in N\setminus A} p_i, \overline{v}] \to \times_{i\in I\cup A}[0, \overline{s}_i]$ of (3) such that for each $i \in A$, $s_i(\cdot)$ is strictly increasing.*

Proof. Let $v'_{A\cdot B}(s)$ denote a $|A| \times |B|$ matrix, where its ij element is $\frac{\partial v_i}{\partial s_j}(s)$ for $i \in A$ and $j \in B$. We denote $v'_{A\cdot B}$ for convenience. Let 0_A and 1_A respectively denote column vectors of 0's and 1's with dimension $|A|$.

To obtain a solution to (3) recursively, suppose that the set of active bidders is A and that the unique solution of (3) exists up to price $\overline{p} = \max_{k\in N\setminus A} p_k$.

Let $(\underline{s}_A, \underline{s}_{I\setminus A})$ denote this solution at $\overline{p}$. We extend the solution beyond $\overline{p}$ to all $p \in [\overline{p}, \overline{v}]$. To do so, we differentiate both sides of (3) with p to obtain the following differential equation:

$$\begin{pmatrix} v'_{A\cdot A} & v'_{A\cdot I\setminus A} \\ v'_{I\setminus A\cdot A} & v'_{I\setminus A\cdot I\setminus A} \end{pmatrix} \begin{pmatrix} s'_A(p) \\ s'_{I\setminus A}(p) \end{pmatrix} = \begin{pmatrix} 1_A \\ 0_{I\setminus A} \end{pmatrix} \tag{7}$$
$$(s_A(\overline{p}), s_{I\setminus A}(\overline{p})) = (\underline{s}_A, \underline{s}_{I\setminus A}).$$

The first matrix on the left-hand side can be written as $v'_{I\cup A\cdot I\cup A}$. Assume for the moment that $v'_{N\cdot N}$ is invertible, which implies that its principal minors $v'_{I\cup A\cdot I\cup A}$ and $v'_{I\setminus A\cdot I\setminus A}$ are also invertible. Then, by Peano's theorem, a unique solution of (7) exists since the value functions are twice continuously differentiable. We next show that $v'_{N\cdot N}$ is invertible and that $s_A(p)' \gg 0$. To do so, we rewrite the last $|I\setminus A|$ lines of (7) as $s'_{I\setminus A} = -(v'_{I\setminus A\cdot I\setminus A})^{-1} v'_{I\setminus A\cdot A} s'_A$. Substituting this into the first $|A|$ lines of (7) yields $V s'_A = 1_A$ after rearrangement, where

$$V := v'_{A\cdot A} - v'_{A\cdot I\setminus A}(v'_{I\setminus A\cdot I\setminus A})^{-1} v'_{I\setminus A\cdot A}.$$

If V satisfies the dominant average condition for any A, then, with $A = N$, $V = v'_{N\cdot N}$ is invertible by Lemma 1. Moreover, by Lemma 1, $s'_A(p) \gg 0$.

To prove that V satisfies the dominant average condition, let $g'_k = \frac{\partial g}{\partial s_k}$ and $g'_A = (g'_k)_{k\in A}$, where g'_A is considered a column vector. Let D_A denote the diagonal matrix whose diagonal entry is $h'_k = \frac{dh_k}{ds_k}$ for $k \in A$. Then, for any $A, B \subset N$,

$$v'_{A\cdot B} = \begin{cases} D_A + 1_A (g'_A)^t & \text{if } A = B \\ 1_A (g'_B)^t & \text{if } A \cap B = \emptyset, \end{cases}$$

where $(\cdot)^t$ denotes the transpose of the matrix. Using this, we can rewrite V as

$$\begin{aligned} V &= D_A + 1_A(g'_A)^t - 1_A(g'_{I\setminus A})^t \left(D_{I\setminus A} + 1_{I\setminus A}(g'_{I\setminus A})^t\right)^{-1} 1_{I\setminus A}(g'_A)^t \\ &= D_A + (1-x)1_A(g'_A)^t, \end{aligned} \tag{8}$$

where $x = (g'_{I\setminus A})^t \left(D_{I\setminus A} + 1_{I\setminus A}(g'_{I\setminus A})^t\right)^{-1} 1_{I\setminus A}$. Because all entries in any given column of the matrix $1_A(g'_A)^t$ are identical and because the diagonal entries of D_A are positive, the first inequality of (6) is easily verified. The proof is complete if the second inequality of (6) is shown to hold, for which it suffices to show $x < 1$:

$$\begin{aligned}
x &= (g'_{I\setminus A})^t \left(D_{I\setminus A} + 1_{I\setminus A}(g'_{I\setminus A})^t\right)^{-1} 1_{I\setminus A} \\
&= (g'_{I\setminus A})^t \left(D^{-1}_{I\setminus A} - \left(\frac{1}{1+(g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}}\right) D^{-1}_{I\setminus A} 1_{I\setminus A} (g'_{I\setminus A})^t D^{-1}_{I\setminus A}\right) 1_{I\setminus A} \\
&= (g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A} - \frac{\left((g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}\right)^2}{1+(g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}} \\
&= \frac{(g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}}{1+(g'_{I\setminus A})^t D^{-1}_{I\setminus A} 1_{I\setminus A}} = \frac{\sum_{k\in I\setminus A} g'_k/h'_k}{1+\sum_{k\in I\setminus A} g'_k/h'_k} < 1,
\end{aligned}$$

where the second equality is derived using the formula for an inverse matrix,

$$\left(A + bc^t\right)^{-1} = A^{-1} - \left(\frac{1}{1+c^t A^{-1} b}\right) A^{-1} b c^t A^{-1},$$

with $A = D_{I\setminus A}$, $b = 1_{I\setminus A}$, and $c = g'_{I\setminus A}$. □

Given the break-even signals obtained in Lemma 2, we consider each outsider i's strategy of dropping out (staying in) at p if and only if $s_i < s_i(p)$ after any history $p_{N\setminus A}$. Along with the insiders' value-bidding strategy, we refer to this strategy profile as β^*.

Lemma 3. *Given the strategy profile β^*, for any signal profile $s \in [0,1]^n$, (i) outsiders drop out in order of their values; (ii) for each outsider i, $p_i \le v_i(s)$; and (iii) at any outsider i's drop-out price p_i, $s_j(p_i) \ge s_j$ for each insider j who is inactive at p_i.*

Proof. Consider two outsiders i and j with $p_i \le p_j$. Then, at price p_i at which bidder i drops out, we have

$$h_i(s_i) = h_i(s_i(p_i)) = p_i - g(s(p_i)) = h_j(s_j(p_i)) \le h_j(s_j),$$

where the first equality and the inequality follow from the drop-out strategy of outsiders i and j, respectively, whereas the second and third equalities follow from the break-even condition at p_i. This proves (i) because $h_i(s_i) \le h_j(s_j)$ implies that $v_i(s) \le v_j(s)$.

To prove (ii), suppose to the contrary that $p_i > v_i(s)$. Because $s_i(p_i) = s_i$, this implies that $h_i(s_i) + g(s(p_i)) = p_i > h_i(s_i) + g(s)$, so $g(s(p_i)) > g(s)$. Given this, for each insider $j \in I$ (whether active or not), we must have

$$h_j(s_j) - h_j(s_j(p_i)) \ge g(s(p_i)) - g(s) > 0, \tag{9}$$

where the first inequality holds because the break-even condition implies that for an inactive insider j, $h_j(s_j)+g(s) = v_j(s) = p_j = v_j(s(p_i)) = h_j(s_j(p_i))+g(s(p_i))$ and because, for an active insider j, $h_j(s_j) + g(s) = v_j(s) \geq p_i = v_j(s(p_i)) = h_j(s_j(p_i)) + g(s(p_i))$. The inequality (9) implies that $s_j > s_j(p_i)$ for each insider j. Furthermore, for each active outsider $j \in O \cap A$, we have $s_j \geq s_j(p_i)$. Thus, $s \geq s(p_i)$, and thus $v_i(s) \geq v_i(s(p_i)) = p_i$, which a contradiction.

To prove (iii), note first that we have $h_i(s_i) + g(s) = v_i(s) \geq p_i = h_i(s_i(p_i)) + g(s(p_i)) = h_i(s_i) + g(s(p_i))$ because of (ii) and the fact that $s_i(p_i) = s_i$. This inequality implies that $g(s) \geq g(s(p_i))$. If an insider j is inactive at p_i, we must have $v_j(s) = p_j = v_j(s(p_i))$ or $h_j(s_j(p_i)) - h_j(s_j) = g(s) - g(s(p_i)) \geq 0$, which yields $s_j(p_i) \geq s_j$. □

Proof of Theorem 3. We first prove Part (ii) by showing that the strategy profile β^*, if followed by all bidders, leads to the efficient allocation. Then, we show that it constitutes an ex-post equilibrium.

Given that outsiders drop out in order of their values (according to (i) of Lemma 3), the efficiency result follows if an outsider i with the highest value among outsiders drops out before (after) an insider j with the highest value among insiders if and only if $v_i(s) < (>) v_j(s)$. In case $v_i(s) < v_j(s)$, outsider i dropping out at some $p_i \leq v_i(s)$ (from (ii) of Lemma 3) means that the insider j is a winner, because $p_i \leq v_i(s) < v_j(s)$ is lower than insider j's drop-out price $v_j(s)$. Assume now that $v_i(s) > v_j(s)$ and suppose to the contrary that the outsider i drops out at some price $p_i < v_j(s)$ at which only insiders, including j, are active.[21] Then, the break-even condition at p_i implies that $h_i(s_i) = p_i - g(s(p_i)) = h_k(s_k(p_i))$ for each $k \in I \cap A$. Because $h_i(s_i) > h_k(s_k)$ for all those k, this means that $h_k(s_k(p_i)) > h_k(s_k)$ or $s_k(p_i) > s_k$. Thus, due to (iii) of Lemma 3, we have $s(p_i) = (s_O, s_{I\cap A}(p_i), s_{I\setminus A}(p_i)) \geq s$ with $s_k(p_i) > s_k$, which implies that $p_i = v_k(s(p_i)) > v_k(s)$ for all $k \in I \cap A$. This contradicts the value bidding strategy of insiders.

Turning to the proof of Part (i), we show that β^* constitutes an ex-post equilibrium, by focusing on an arbitrary outsider i. If i has the highest value and follows the equilibrium strategy to become a winner, his payoff is $v_i(s) - \max_{k\neq i} p_k \geq v_i(s) - \max_{k\neq i} v_k(s) \geq 0$.[22] Then, any nontrivial deviation by i cannot be profitable, because it results in losing and earning a zero payoff. Suppose now that there is some j with $v_j(s) > v_i(s)$. If j is an insider, any nontrivial deviation by i to become a winner makes him pay at least $v_j(s)$, that is, more than his value. Let us therefore focus on the case where j is an outsider with the highest value. Any nontrivial deviation by i would require him to wait beyond some price p such that $s_i(p) = s_i$, and then become a winner after j drops out last at some $p_j > p$.[23] Then, we must have $s_i(p_j) > s_i$ and $s_j(p_j) = s_j$. Combining this with

[21] This holds because bidder i is the last to drop out among outsiders, according to (i) of Lemma 3.

[22] The first inequality holds because each insider drops out at his value and each outsider drops out below his value according to (ii) of Lemma 3.

[23] An argument similar to that in the proof of the efficiency can be used to show that because j has

$s_I(p_j) \geq s_I$ (from (iii) of Lemma 3), we have $s(p_j) = (s_{O\setminus\{i\}}, s_i(p_j), s_I(p_j)) \geq s$ with $s_i(p_j) > s_i$, so $v_i(s(p_j)) = p_j > v_i(s)$, implying that the deviation incurs a loss to i. $\square$

Proof of Theorem 4: Throughout the proof, for any variable x in E, we let x' denote its counterpart in E'. For instance, p'_k denotes the drop-out price of bidder k in E'. Let $O = \{1, 2, \cdots, l\}$ and thus $I = \{l+1, \cdots, n\}$, and assume that $v_1(s) \leq v_2(s) \leq \cdots \leq v_l(s)$, without loss of generality. Then, $O' = O\setminus\{i\}$ and $I' = I \cup \{i\}$.

First, according to (ii) of Lemma 3, switched insider i drops out at a (weakly) higher price in E' than in E. The proof is complete if we show that all other outsiders drop out at (weakly) higher prices in E' as well. Next, suppose by contradiction that some outsider drops out at a lower price in E' than in E. For all outsiders $k < i$, we have $p'_k = p_k$, because the history of drop-out prices is the same across E and E' until p_i is reached. Using this and our assumption, we define $j = \min\{k \,|\, p'_k < p_k, \text{ and } i < k \leq l\}$. We first make a few of observations: (i) a signal s_k for each $k < j$ with $k \neq i$ is revealed in both E and E' by the time the price clock reaches p'_j, because $p_k \leq p'_k \leq p'_j$ for all such k[24]; (ii) $s'_j(p'_j) = s_j = s_j(p_j) > s_j(p'_j)$, because $p_j > p'_j$; and (iii) $s'_i(p'_j) \geq s_i$. (iii) here follows from (iii) of Lemma 3 if i is inactive at p'_j in E'. If i is active at p'_j, the monotonicity of $s'_i(\cdot)$ implies that $s'_i(p'_j) \geq s'_i(p_i) = s_i(p_i) = s_i$, because $p'_j \geq p_i$.[25] We next show that

$$s'_k(p'_j) \geq s_k(p'_j) \text{ for all } k \in \{j+1, \cdots, n\}, \tag{10}$$

which, given (i), (ii), and (iii) above, implies that $s'(p'_j) \geq s(p'_j)$[26] with $s'_j(p'_j) > s_j(p'_j)$, so $p'_j = v_j(s'(p'_j)) > v_j(s(p'_j)) = p'_j$, yielding the desired contradiction. To prove (10), observe first that the break-even conditions at price p'_j in E and E' yield

$$g(s(p'_j)) = p'_j - h_j(s_j(p'_j)) > p'_j - h_j(s'_j(p'_j)) = g(s'(p'_j)),$$

where the inequality holds because of (ii). We then prove (10) by considering two cases depending on whether $k \in \{j+1, \cdots, n\}$ or not is active at p'_j in E'. Because each outsider $k \in \{j+1, \cdots, l\}$ is active at p'_j in E', an inactive bidder $k \in \{j+1, \cdots, n\}$ must be an insider. For such k, we obtain (10) because the break-even conditions at price p'_j in E and E' yield

$$h_k(s'_k(p'_j)) = p_k - g(s'(p'_j)) > p_k - g(s(p'_j)) = h_k(s_k(p'_j)), \tag{11}$$

the highest value, j drops out last (except for i) even under deviation by i.

[24] The second inequality here holds, because outsiders drop out in order of their values in E'.

[25] The equality $s'_i(p_i) = s_i(p_i)$ follows from the fact that the price history is the same across E and E' until p_i is reached.

[26] The i-th component of $s(p_j)$ is equal to s_i, so this inequality follows from (iii).

where the inequality follows from (11).Turning to the case in which bidder $k \in \{j+1, \cdots, n\}$ is active at p'_j in E', he must also be active at p'_j in E. This is because if k is an outsider, $p'_j < p_j$ and he drops out no sooner than j in E (because of (i) of Lemma 3) and because if k is an insider, he drops out at the same price (i.e., his value) in E and E'. Thus, we obtain (10), because the break-even conditions at p'_j in E and E' yield

$$h_k(s'_k(p'_j)) = p'_j - g(s'(p'_j)) > p'_j - g(s(p'_j)) = h_k(s_k(p'_j)),$$

where the inequality follows again from (11). □

REFERENCES

ASHENFELTER, O., and K. GRADDY (2003), "Auctions and the Price of Art," *Journal of Economic Literature*, **41**, 763-787.

BOONE, J., R. CHEN, J. K. GOEREE, and A. POLYDORE (2009), "Risky Procurement with an Insider Bidder," *Experimental Economics*, **12**, 417-436.

CAMPBELL, C., and D. LEVIN (2000), "Can the Seller Benefit from an Insider in Common- Value Auctions?," *Journal of Economic Theory*, **91**, 106-120.

DASGUPTA, P., and E. MASKIN (2000), "Efficient Auctions," *Quarterly Journal of Economics*, **115**, 341-388.

ENGELBRECHT-WIGGANS, R., P. MILGROM, and R. WEBER (1982), "The Value of Information in a Sealed-Bid Auction," *Journal of Mathematical Economics*, **10**, 105-114.

GOEREE, J. K., and T. OFFERMAN (2002), "Efficiency in Auctions with Private and Common Values: An Experimental Study," *American Economic Review*, **92**, 625-643.

HANSEN, R. G., and J. R. Jr. LOTT (1991), "The Winner's Curse and Public Information in Common Value Auctions: Comment," *American Economic Review*, **81**, 347-361.

HENDRICKS, K., R. PORTER, and C. WILSON (1994), "Auctions for Oil and Gas Leases with an Informed Bidder and a Random Reservation Price," *Econometrica*, **62**, 1414-1444.

HONG, H., and M. SHUM (2003), "Econometric Models of Ascending Auctions," *Journal of Econometrics*, **112**, 327-358.

KAGEL, J. H. (1995), "Auctions: A Survey of Experimental Research," In: Kagel, J. H., Roth, A. E. (Eds.), *Handbook of Experimental Economics*. Princeton University Press, 501-585.

KAGEL, J. H., R. M. HARSTAD, and D. LEVIN (1987), "Informational Impact and Allocation Rules in Auctions with Affiliated Private Values: A Laboratory Study," *Econometrica*, **55**, 1275-1304.

KAGEL, J. H., and D. LEVIN (1986), "The Winner's Curse and Public Information in Common Value Auctions," *American Economic Review*, **76**, 894-920.

KAGEL, J. H., and D. LEVIN (1999), "Common Value Auctions with Insider Information," *Econometrica*, **67**, 1219-1238.

KAGEL, J. H., and D. LEVIN (2002), Common Value Auctions and the Winners Curse, *Princeton University Press.*

KAGEL, J. H., and D. LEVIN (2011), "Auctions: A Survey of Experimental Research, 1995 – 2010," Forthcoming in the *Handbook of Experimental Economics, Vol.2.*.

KIM, J. (2003), "Interdependent Value Auctions with Insider Bidders," mimeo.

KIM, J. (2008), "The Value of an Informed Bidder in Common Value Auctions," *Journal of Economic Theory*, **143**, 585-595.

KIRCHKAMP, O., and B. MOLDOVANU (2004), "An Experimental Analysis of Auctions with Interdependent Valuations," *Games and Economic Behavior*, **48**, 54-85.

KRISHNA, V. (2003), "Asymmetric English Auctions," *Journal of Economic Theory*, **112**, 261-288.

LIND, B., and C. R. PLOTT (1991), "The Winner's Curse: Experiments with Buyers and with Sellers," *American Economic Review*, **81**, 335-346.

MASKIN, E. (1992), "Auctions and Privatization," in H. Siebert (ed.) *Privatization* (Institute fur Weltwirschaften der Universät Kiel) 115-136.

MILGROM, P., and R. WEBER (1982), "A Theory of Auctions and Competitive Bidding," *Econometrica*, **50**, 1089-1122.

MYERSON, R. B. (1981), "Optimal Auction Design," *Mathematics of Operations Research*, **6**, 58-73.

PERRY, M., and P. RENY (2002), "An Efficient Auction," *Econometrica*, **70**, 1199-1212.

RILEY, J. G., and W. SAMUELSON (1981), "Optimal Auctions," *American Economic Review*, **71**, 381-392.

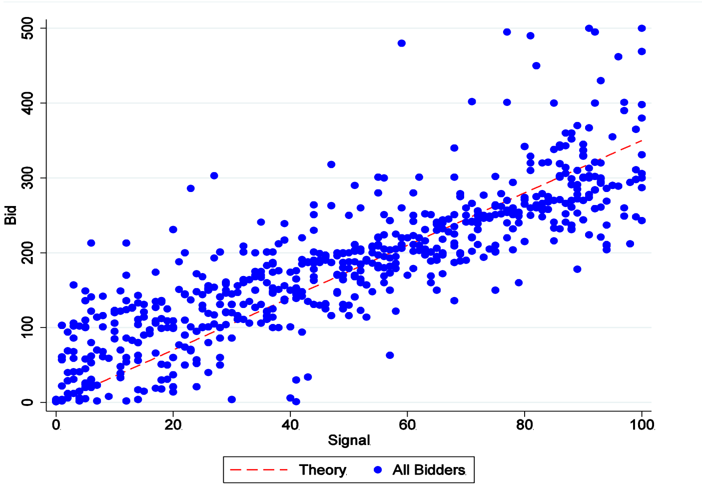
SHLEIFER, A., and R. VISHNY (1988), "Management Buyout as a Response to Market Pressure," in Auerbach (ed.) *Mergers and Acquisitions* (Chicago University Press) 87-102.

WILSON, R. (1998), "Sequential Equilibria of Asymmetric Ascending Auctions: The Case of Log-Normal Distributions," *Economic Theory*, **12**, 433-440.

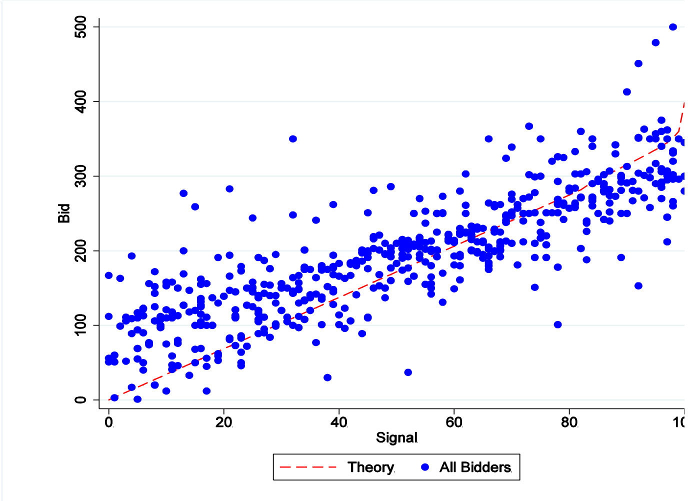
WOOLDRIDGE, J. M. (2003), Econometric Analysis of Cross Section and Panel Data, *MIT Press.*

Figure 3. Scatter plots of bids and signals: Second-price auctions

A. (# of insiders) = 0



B. (# of insiders) = 1



C. (# of insiders) = 2

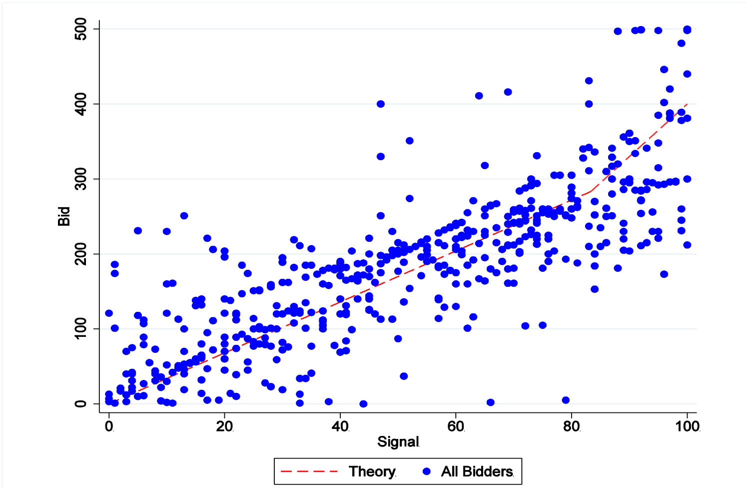
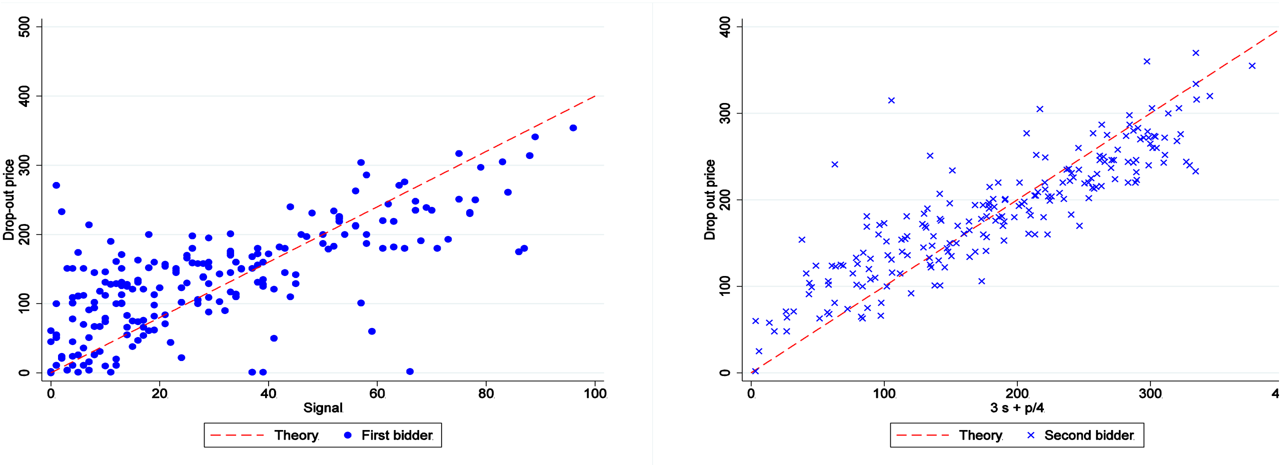
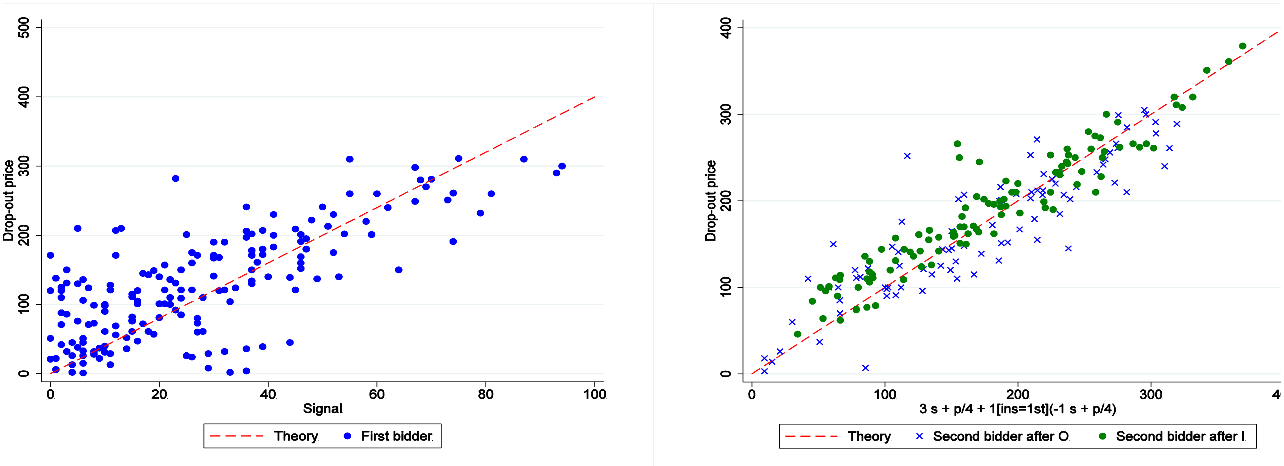


Figure 4. Scatter plots of bids and signals: English auctions

A. (# of insiders) = 0



B. (# of insiders) = 1



C. (# of insiders) = 2

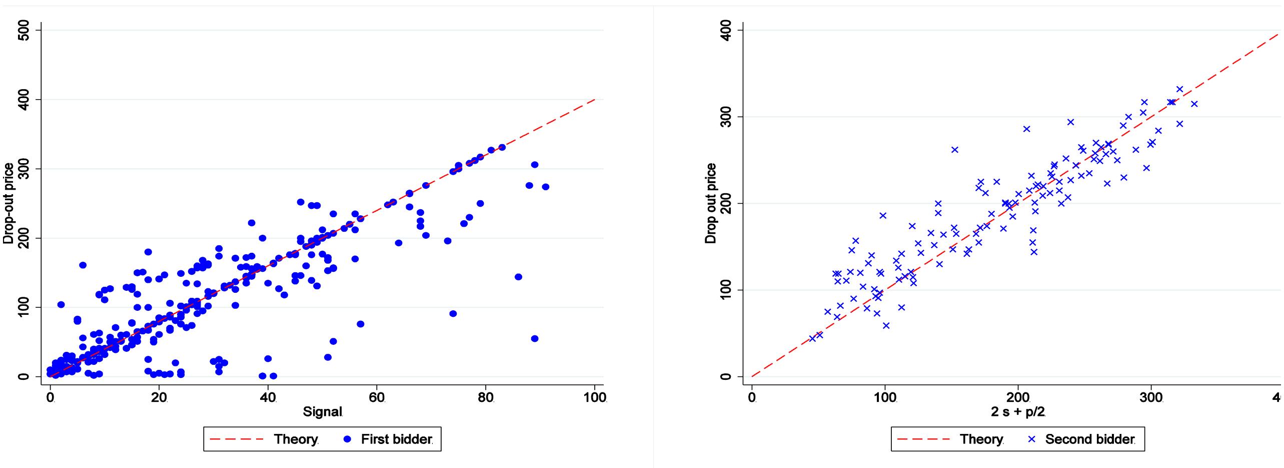


Table 2. Frequencies and ratios of efficient allocation

# of insiders	Ranking of values (I or O)	Second-price auction (1) Freq.	Second-price auction (2) Ratio	English auction (3) Freq.	English auction (4) Ratio	H_0: (1) = (3)	H_0: (2) = (4)
0	All	0.78	0.80	0.83	0.84	0.17	0.39
		(210)		(210)			
1	All	0.76	0.74	0.83	0.89	0.05	0.00
		(270)		(285)			
	I = (highest-value)	0.70	0.65	0.81	0.83	0.08	0.03
		(83)		(100)			
	I = (second highest-value)	0.73	0.79	0.88	0.93	0.02	0.01
		(93)		(82)			
	I = (lowest-value)	0.83	0.75	0.80	0.90	0.55	0.03
		(94)		(103)			
2	All	0.83	0.86	0.89	0.93	0.01	0.00
		(495)		(630)			
	O = (highest-value)	0.73	0.80	0.87	0.92	0.00	0.00
		(169)		(206)			
	O = (second highest-value)	0.83	0.91	0.79	0.90	0.37	0.58
		(168)		(205)			
	O = (lowest-value)	0.94	0.87	0.99	0.97	0.01	0.02
		(158)		(219)			

Note. The efficiency ratio is defined as (realized surplus minus random surplus) divided by (first-best surplus minus random surplus). The two columns on the right side report the p-value from the t-test for the null hypothesis that outcomes between the second-price auction and the English auction are equivalent. I denotes an insider and O an outsider. The number of observations is in parentheses

Table 3. The decomposition of efficiency outcomes against theoretical predictions of the second-price auction

A. (# of insiders) = 1

Second-price auction: Theoretical allocation Second-price auction	Observed Allocation (1): Inefficient	Observed Allocation (1): Efficient	English auction: Theoretical allocation Second-price auction	Observed Allocation (2): Inefficient	Observed Allocation (2): Efficient	H_0: (1) ≥ (2)
Inefficient	0.67	0.33	Inefficient	0.39	0.62	0.017
	(20)	(10)		(10)	(16)	
Efficient	0.19	0.81	Efficient	0.15	0.85	0.136
	(46)	(194)		(40)	(219)	
Total	0.24	0.76	Total	0.18	0.82	
	(66)	(204)		(50)	(235)	
	Ho: Ineff-Eff=0	0.000		Ho: Ineff-Eff=0	0.003	

B. (# of insiders) = 2

Second-price auction: Theoretical allocation Second-price auction	Observed Allocation (1): Inefficient	Observed Allocation (1): Efficient	English auction: Theoretical allocation Second-price auction	Observed Allocation (2): Inefficient	Observed Allocation (2): Efficient	H_0: (1) ≥ (2)
Inefficient	0.36	0.64	Inefficient	0.34	0.66	0.408
	(21)	(37)		(25)	(48)	
Efficient	0.15	0.85	Efficient	0.08	0.92	0.001
	(64)	(373)		(47)	(510)	
Total	0.17	0.83	Total	0.11	0.89	
	(85)	(410)		(72)	(558)	
	Ho: Ineff-Eff=0	0.000		Ho: Ineff-Eff=0	0.000	

Note. In discerning if the second-price auction format predicts an inefficient allocation for each sample, we allow a 5-token margin with which a sample is treated inefficient if the equilibrium of the second-price auction is either inefficient or efficient but the difference of its two high bids is less than 5 tokens.The last column on the right side reports the *p* -value from the *t* -test for the null hypothesis that the efficiency outcome of the second-price auction is no lower than that of the English auction. The bottom row reports the p-value from the t-test for the nully hypothesis that the frequencies of efficient allocation is the same between when the second-price auction format predicts an efficient allocation and when not. The number of observations is in parentheses.

Table 4. Average percentage deviations of observed revenues from theoretically predicted revenues

# of insiders	Ranking of values (I or O)	English auction	SBSP auction
0	All	0.22 (0.096, 0.023)	0.26 (0.045, 0.000)
1	All	0.03 (0.014, 0.047)	0.14 (0.024, 0.000)
	I = (highest-value)	0.06 (0.036, 0.101)	0.28 (0.058, 0.000)
	I = (second highest-value)	-0.02 (0.012, 0.112)	0.03 (0.019, 0.165)
	I = (lowest-value)	0.03 (0.012, 0.005)	0.13 (0.036, 0.001)
2	All	0.00 (0.002, 0.453)	0.01 (0.004, 0.063)
	O = (highest-value)	-0.01 (0.003, 0.000)	-0.03 (0.005, 0.000)
	O = (second highest-value)	0.02 (0.006, 0.008)	0.03 (0.010, 0.001)
	O = (lowest-value)	0.00 (0.001, 0.024)	0.02 (0.006, 0.005)

Notes. I stands for an insider and O represents an outsider. The first number in parentheses is a standard error of sample mean and the second number is p-value from t- test for the null hypothesis that the mean is equal to zero.

Table 5. Regression analysis of observed and theoretical revenues

Variables	Observed revenues		Theoretical revenues	
	English	**SBSP**	**English**	**SBSP**
k = 1	10.466***	10.491***	12.424***	13.554***
	(1.983)	(2.477)	(.996)	(1.374)
k = 2	18.716***	9.838***	21.897***	21.635***
	(1.738)	(2.218)	(.873)	(1.23)
$s_{(1)}$	0.889***	0.766***	1.027***	0.638***
	(.039)	(.055)	(.02)	(.03)
$s_{(2)}$	2.087***	1.837***	2.330***	2.515***
	(.038)	(.051)	(.019)	(.029)
$s_{(3)}$	0.796***	0.761***	0.661***	0.629***
	(.04)	(.054)	(.02)	(.03)
constant	-7.739***	18.436***	-16.151***	-14.889***
	(2.98)	(3.883)	(1.497)	(2.154)
# of obs.	1125	975	1125	975
R^2	0.907	0.837	0.978	0.960
p-value H_0: $(k=0) = (k=1)$	0.000	0.000	0.000	0.000
p-value H_0: $(k=1) = (k=2)$	0.000	0.749	0.000	0.000

Note. Standard errors are reported in parentheses. *, **, and *** represent 10%, 5%, and 1% significance level, respectively. $s_{(1)} = \min[\mathbf{s}]$, $s_{(3)}=\max[\mathbf{s}]$, $s_{(2)}=\text{med}[\mathbf{s}]$

<u>Table 6. Regressions of bids on signals in the SBSP auctions</u>

Variables	***(k = 0)***	***(k = 1)***		***(k = 2)***	
s_i	2.754***	2.309	2.309***	2.756	2.756***
	(0.16)	(0.130)***	(0.13)	(0.219)***	(0.22)
$1[s_i>82.109]$			-90.933		
			(79.09)		
$1[s_i>82.109] \times s_i$			1.125		
			(0.89)		
$1[s_i>500/6]$					-352.69
					(226.80)
$1[s_i>500/6] \times s_i$					4.006
					(2.50)
Constant	48.797***	75.876	75.876***	39.836	39.836***
	(8.57)	(7.273)***	(7.29)	(12.048)***	(12.07)
R^2	0.71	0.6	0.7	0.58	0.65
# of obs.	630	447§	540	412§	495
*Ho:(**b**,c)=(**b,o**)*	(s=3.5)	(s=3.43578)	n/a	(s=17/5)	(s=17/5, 1[.]=-300, s+1[.]s = 7)
F test	16.21	54.56		5.5	3.64
p value	0.00	0.00		0.00	0.02

Notes: Robust standard errors clustered by individual subjects are reported in parentheses. *, **, and *** represent 10%, 5%, and 1% significance level, respectively. § Restricts sample to $s_i \leq 82.109$. § Restricts sample to $s_i \leq 500/6$.

Table 7. Censored regressions of bids in the English auctions

Variables	(k = 0) first drop-out (1)	(k = 0) second drop-out (2)	(k = 1) first drop-out (3)	(k = 1) (4)	(k = 1) (5)	(k = 2) first drop-out (6)	(k = 2) second drop-out (7)
s_i	2.878***	1.893***	3.258***	2.394***	2.456***	3.48***	1.268***
	(0.14)	(0.10)	(0.16)	(0.15)	(0.16)	(0.14)	(0.10)
p/4		1.591***		1.369***	1.409***		2.812***
		(0.12)		(0.21)	(0.21)		(0.19)
1[1st drop-out = insider]× s_i				-1.010***	-1.076***		
				(0.21)	(0.22)		
1[1st drop-out = insider] ×p/4				1.240***	1.093***		
				(0.33)	(0.35)		
1[1st drop-out = insider]					10.775		
					(8.45)		
Constant	77.893***	46.28***	65.669***	22.705***	17.285***	32.089***	21.836***
	(9.61)	(5.60)	(10.13)	(3.82)	(6.28)	(8.04)	(6.79)
sigma	61.947	30.677	58.226	26.358	26.378	56.556	23.396
	(8.365)***	(2.896)***	(6.230)***	(1.794)***	(1.810)***	(8.449)***	(1.785)***
pseudo-R^2	0.13	0.20	0.16	0.23	0.23	0.16	0.24
# of obs.	630	420	570	388	388	630	343
Ho:(**b**,*c)=(* ***b*,*o*)**	(s=4)	(s=3,p/4=1)	(s=4)	(s=3,p/4=1, 1[.]s+s=2, 1[.]p/4+p/4=2)	(s=3,p/4=1, 1[.]s+s=2, 1[.]p/4+p/4=2, 1[.]=0)	(s=4)	(s=2,p/4=2)
F test	40.17	45.63	21.11	36.65	30.63	9.43	48.41
p value	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Robust standard errors clustered by individual subjects are reported in parentheses. *, **, and *** represent 10%, 5%, and 1% significance level, respectively.

Table 8. Nonlinear least squares of naïve bidding

English auction: first drop-out prices

	k=0	*k=1*	*k=2*
α	0.657	0.525	0.179
	(.080)***	(.096)***	(0.060)***
R^2	0.87	0.87	0.87
N	210	179	281
H_0: $\alpha_k = \alpha_{k'}$	$\alpha_0 = \alpha_2$	$\alpha_0 = \alpha_1$	$\alpha_1 = \alpha_2$
F test:	23.38	1.14	9.48
p value	0.00	0.29	0.00

English auction: second drop-out prices

	k=0	*k=1*		*k=2*
		1st drop-out = outsider	1st drop-out = insider	
α	0.696	0.319	0.627	0.680
	(0.093)***	(0.111)***	(0.089)***	(0.136)***
R^2	0.96	0.95	0.97	0.96
N	211	88	109	123
H_0: $\alpha_k = \alpha_{k'}$	$\alpha_0 = \alpha_{1,O}$		$\alpha_{1,I} = \alpha_2$	
F test:	6.84		0.745	
p value	0.01		0.11	

Second-price auction

	k=0	*k=1*	*k=2*
α	0.488	0.729	0.402
	(.086)***	(.079)***	(.122)***
R^2	0.94	0.95	0.92
N	630	540	495
H_0: $\alpha_k = \alpha_{k'}$	$\alpha_0 = \alpha_2$	$\alpha_0 = \alpha_1$	$\alpha_1 = \alpha_2$
F test:	0.34	4.32	5.15
p value	0.56	0.04	0.03

Note. Robust standard errors clustered by individual subjects are reported in parentheses. *,**, and *** represent 10%, 5%, and 1% significance level, respectively.