



# Scattering, Clouds & Climate: A Short Workshop of Exploration

*Mathematical Institute, University of Oxford*

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## Post-workshop report

[www.maths.ox.ac.uk/~hewett/scatteringcloudsclimateworkshop.htm](http://www.maths.ox.ac.uk/~hewett/scatteringcloudsclimateworkshop.htm)



## **Scattering, Clouds & Climate: A Short Workshop of Exploration**

### **Aim:**

This one-and-a-half-day workshop brought together twenty-six mathematicians and atmospheric physicists working on problems related to scattering of electromagnetic radiation by atmospheric ice crystals<sup>1</sup>. The aim of the workshop was to identify areas where advances in mathematical methods may be useful in climate modelling, and, in so doing, nucleate future collaborative efforts.

### **Organisers:**

David Hewett, John Ockendon and John Wettlaufer (Oxford)

### **Participants:**

David Abrahams (Manchester), Anthony Baran (Met Office), David Bebbington (Essex), Timo Betcke (UCL), Simon Chandler-Wilde (Reading), Paul Connolly (Manchester), Steven Dobbie (Leeds), Cathryn Fox (Imperial), Andrew Gibbs (Reading), Ivan Graham (Bath), Samuel Groth (Reading), John Hannay (Bristol), Evelyn Hesse (Hertfordshire), David Hewett (Oxford), Robin Hogan (Reading), Stephen Langdon (Reading), John Ockendon (Oxford), Caroline Poulsen (RAL), Anthony Rawlins (Brunel), Helen Smith (Manchester), Valery Smyshlyaev (UCL), Thorwald Stein (Reading), Joseph Ulanowski (Hertfordshire), Chris Westbrook (Reading), John Wettlaufer (Oxford).

### **Acknowledgements:**

The organisers are grateful to the Oxford Centre for Collaborative Applied Mathematics and the Oxford Mathematical Institute for funding this workshop, to Chris Westbrook, Robin Hogan and Stephen Langdon (University of Reading) and Anthony Baran (Met Office) for their assistance in planning the workshop, and to Anthony Baran (Met Office) for supplying the image of the ice crystal rosette on the front cover of this report.

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<sup>1</sup> We note that scattering by cirrus ice crystals forms just one part of a wider effort in the meteorological community to understand scattering by more general atmospheric particles, including aerosols (dust, smoke, volcanic ash etc.). However, to sharpen the focus of the workshop, aerosols were not considered in this meeting.

## **Report:**

The workshop was built around 10 short talks, with time scheduled for structured discussions and more informal networking. Participants included mathematicians wanting to understand the key issues involved in the meteorological application, and atmospheric physicists keen to learn about current developments in mathematical scattering theory.

The main motivation for studying scattering by ice crystals in cirrus clouds is to understand the effect they have on the short-wave and long-wave radiative balance of the earth-atmosphere system. Cirrus reflects/transmits/scatters the short-wave radiation reaching the earth from the sun, and also the long-wave radiation emitted from the surface of the earth and the atmosphere. Key questions for climate modelling are: How can the influence of cirrus be better parametrized in climate models? Does cirrus have a net cooling or net warming effect?

In this introductory talk, **Chris Westbrook** (Reading) set the scene for the workshop by giving an overview of the key aspects of the problem from the point of view of the meteorological application, including the formation, aggregation and basic properties of cirrus ice crystals, the use of radiative transfer models to predict the cooling/warming effect of cirrus clouds (which comprise a vast number of individual ice crystals of different sizes, shapes and orientations), and the remote sensing (e.g. active radar and lidar, passive radiometry) methods used to measure the scattering properties of clouds. He also drew our attention to the beautiful (and in some cases currently unexplained) optical phenomena (e.g. arcs, halos) that can arise from the scattering of sunlight by cirrus clouds.

Chris explained that because the ice crystals in a cloud are generally quite well-separated, it is standard practice to compute the net effect of the complicated multiple scattering between crystals using a radiative transfer model, the inputs to which are single scattering properties of the constituent ice crystals. He emphasized the huge range of size parameters<sup>2</sup> involved (a factor of  $10^6$  between smallest and largest), the importance of polarization (so we really need to study Maxwell's equations not just the scalar Helmholtz equation), and the complexity and variation of ice crystal geometries. However, he pointed out that, in applications, single scattering calculations are usually done "offline", and the results stored in look-up tables for use in radiative transfer models. So one does not need "online" scattering calculations to be computed in real time. However, one still wants single scattering calculations to be relatively fast because of the large number of different such calculations that are required to explore a representative range of ice crystal geometries and wavelengths.

Chris went on to describe the main tools currently used for single scattering calculations. For relatively small size parameters, i.e. in the "low frequency" regime, these include (i) Mie

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<sup>2</sup> Here "size parameter" stands for the ratio  $L/\lambda$ , where  $\lambda$  is the wavelength and  $L$  is a typical lengthscale of the scatterer. Different choices of  $L$  are used by different authors; some use  $L$ =diameter or  $L$ =radius, others take  $L$  to be a geometry-specific quantity, e.g. the height of a hexagonal column. Some authors use  $2\pi$  times one of these choices (equivalently, they define "size parameter" to be  $kL$ , where  $k=2\pi/\lambda$  is the wavenumber). Note that for non-spherical scatterers "diameter" means the largest distance between any two points on the scatterer, and "radius" is then defined as half the diameter.

theory (modelling the scatterer as a sphere and evaluating the exact closed-form solution for scattering by a sphere); (ii) The T-matrix method, which essentially involves expressing the incident and scattered fields as eigenfunction expansions in spherical coordinates, the T-matrix being the linear operator which maps incident to scattered coefficients (in a numerical implementation the expansions have to be truncated); and (iii) the Discrete Dipole Approximation (DDA), which is essentially a discretised domain integral equation approach. Other less commonly used approaches include the Rayleigh-Gans (Born) approximation, in some sense a first-order approximation to DDA, the Fredholm Integral method, and the generalized multi-particle Mie-solution approach. For large size parameters, i.e. in the high frequency regime, the main tools are ray-based methods like geometrical optics (GO), which are often implemented in a Monte Carlo framework, summing intensity contributions (phase is usually neglected) from a large number of incident rays. Finite Difference Time Domain (FDTD) methods are also used, but have high computational cost for larger size parameters.

**Dave Hewett** (Oxford) concluded the introductory talk by outlining some of the areas currently being investigated by the mathematicians present. In terms of analytical methods, one area of particular interest is in high frequency approximations such as GO (with phase information retained) and the Geometrical Theory of Diffraction, a generalisation of GO which includes diffraction effects in a systematic (and physically intuitive) way by the introduction of diffracted ray fields emanating from edges, vertices and points/curves of ray tangency. Application of these methods requires the solution of so-called “canonical problems” which encapsulate the key scattering mechanisms; these include scattering by infinite half-planes, wedges, cones, parabolas etc. There are still a number of important unsolved canonical problems, the most relevant of which for this workshop is that of scattering by a transmission (dielectric) wedge. Crucially, the lack of a simple closed-form solution for this problem severely hampers any attempts to apply GTD to scattering by ice crystals. Dave explained that another area of interest/expertise in analytical methods was in determining effective scattering properties of inhomogeneous media (e.g. composite materials) using techniques such as mathematical homogenisation. John Wettlaufer has also worked on the problem of scattering at an ice-water interface (e.g. between two ice adjoining ice crystals in an aggregate).

The other main area of expertise represented by the mathematicians present was in numerical methods for wave equations (Helmholtz, Maxwell, linear elasticity). Specifically the focus was on Finite Element Methods (FEM) and Boundary Element Methods (BEM) (discussed in more detail in the talks by Ivan Graham, Timo Betcke, and Dave Hewett). Current research is on the development of new FEM/BEM formulations with desirable mathematical and numerical properties, fast implementations (e.g. using matrix compression, preconditioning, domain decomposition), and the development of novel “hybrid numerical-asymptotic” BEM approximation spaces for high frequency problems.

The first research talk was given by **Anthony Baran** (Met Office), who gave a broad overview of the various components that make up a climate model, including the measurement and modelling of (i) the scattering properties (e.g. refractive index) of ice, (ii) the variation and complexity in ice crystal geometry and size distribution, and (iii) the state-of-the-art in computational methods for single scattering calculations. With regard to (iii), results from the Invariant Imbedding T-Matrix method (II-TM) were presented from the

group of Yang, which significantly advances the applicability of the T-matrix method to larger size parameters than considered previously. Anthony's talk focussed especially on the importance of good (i.e. representative of reality) models for ice crystal geometries, which must satisfy observed area ratios, and for the particle size distribution (PSD) of ice crystals in a cloud, his results illustrating strikingly how the choice of PSD can have a significant impact on the predictions of climate models in terms of global temperature, as computed using radiative transfer models. In particular, Anthony demonstrated that by constraining the PSD by observation, the global zonally averaged tropospheric temperature could quite easily be changed from warming to cooling by merely changing the cirrus habit mixture model. He then went on to show that by constraining the PSD and the habit mixture model by remote sensing, a climate model could be improved relative to observation by using consistent microphysics in the cloud and radiation schemes of the climate model.

The next talk was given by **Helen Smith** (Manchester), one of Anthony's CASE PhD students, who shared some fascinating images of ice crystals created artificially in the University of Manchester's ice cloud chamber. Helen explained how the cloud chamber can simulate a range of atmospheric conditions to create ice crystals of varied size and shape, whose optical scattering properties are investigated in a laser scattering chamber. Images of the particles passing through the scattering chamber can be obtained using a Cloud Particle Imager (CPI), and formvar (plastic) replicas are also produced to allow closer scrutiny of the internal structure of the crystals. One key finding was that the complexity of the crystal geometry depends strongly on temperature – in particular, at colder temperatures one sees a significant number of crystals with hollow (concave) ends, often characterised by a hexagonal step-like concavity. Helen argued that to simulate the complicated scattering by such crystals one needed to go beyond GO and include diffraction effects, e.g. using the ray tracing with diffraction on facets (RTDF) method of Hesse/Ulanowksi. She also touched on the difficulty of dealing with (both experimentally and computationally) crystals with rough surfaces, an area picked up on in more detail by Joseph Ulanowski on day two.

**David Bebbington** (Essex) then gave a stimulating talk on the use of polarisation measurements in the radar imaging of clouds. He explained how the spatial anisotropy of hydrometeors (e.g. raindrops or ice crystals), both in shape and orientation, gives rise to polarisation sensitivity which can be exploited to help with identification and estimation of drop size distribution, for example. David then presented an outline of the Fredholm Integral Method, a technique for calculating single scattering amplitudes based on the solution of a system of coupled integral equations in Fourier space. One attraction of the method is that it appears to be adaptable to inhomogeneous scatterers such as melting snowflakes, which are extremely difficult to model using other methods.

In the final talk of day one, **Cathryn Fox** (Imperial) took us on a journey to the skies above Western Scotland, as she described the Cirrus Coupled Cloud Radiation Experiment (CIRCCREX) campaign currently being undertaken by her and her collaborators. The campaign involves flying an airborne laboratory through cirrus clouds to obtain a comprehensive new dataset combining precision measurements of broadband radiances, cloud microphysics and atmospheric states. A key instrument in the campaign is the Tropospheric Airborne Fourier Transform Spectrometer (TAFTS) for measuring radiances (looking both up and down). Using the results of the experiments already conducted, Cathryn has been able to infer measured particle size distributions (PSDs - see Anthony

Baran's talk) for use in radiance simulations, for comparison with measured radiances from TAFTS. A key difficulty in obtaining accurate measurements of PSDs is that larger ice crystals shattering on the measurement apparatus can cause an artificial bias towards small ice crystals. The current experiment is designed to suffer less from this effect compared to previous experiments, but the effect is still believed to be a major source of error that needs to be addressed if more representative PSDs are to be obtained.

The scientific programme on day one ended with a structured discussion lead by **John Wettlaufer** (Oxford). The discussion centred mostly on cloud microphysics, specifically on surface roughening, and on the formation of the “hollow” ice crystals mentioned by Helen Smith in her talk earlier in the day. John showed a number of figures which illustrated the complex dependence of crystal growth on atmospheric conditions. Just a small change in temperature or humidity can have a dramatic effect on crystal morphology – in some conditions crystals grow outwards like a pancake, in others they grow like long columns, and in others they develop hollows (concavities) or surface roughness. Two different mechanisms were cited for the development of concavities: humidity gradients in the surrounding air, and dislocations on the crystal surface. A secondary discussion followed on whether or not it was necessary to take into account the birefringent nature of ice when performing scattering calculations. It appeared that no-one in the meteorological community is including this effect at present, and many participants argued that it was probably a high order effect of less importance than e.g. surface roughness. But others maintained that at optical wavelengths birefringence may play a role, especially in crystal aggregates where there is an ice-water interface.

After a delicious meal at Somerville College and a good night's sleep the workshop restarted on day two with two talks focussing on the complexity of ice crystal geometries in cirrus clouds. First **Steven Dobbie** (Leeds) provided evidence to suggest that the commonly assumed six-fold symmetry of ice and snow particle shapes is not the full story, and that under certain atmospheric conditions one finds that a significant proportion of ice crystals are “trigonal”, i.e. they exhibit a three-fold symmetry. Steven presented a sequence of intriguing images of crystal geometries ranging from simple triangular prisms, through scalene hexagonal prisms to regular hexagonal prisms, and highlighted evidence for the existence of scalene particles in the atmosphere based on observed optical phenomena (e.g. arcs), and laboratory studies of Chris Westbrook. Using simple estimates of the optical scattering properties of such crystals (using the Anomalous Diffraction Approximation (ADA)) Steven illustrated that these trigonal ice particles may play an important role in the optical and radiative properties of ice clouds.

Second, **Joseph Ulanowski** (Hertfordshire) focussed on the measurement and modelling of surface roughness of ice crystals, an issue raised by Helen Smith on the first day in the context of cloud chamber experiments. Joseph began by describing the Small Ice Detector 3 (SID-3) apparatus developed at the University of Hertfordshire, which records the far-field scattering intensity of ice particles over a range of forward scattering angles (between 6 and 25 degrees). Both airborne and lab-based versions of SID-3 have been built, the airborne version being an integral part of the CIRCCREX campaign described by Cathryn Fox on day one. SID-3 output has been used to validate the Ray Tracing with Diffraction on Facets (RTDF) method for single scattering computations, with very encouraging results. It also offers a way to characterise the scattering effect of surface roughness, in terms of the

nature of the “speckling” observed in the measured scattering pattern. The apparent prevalence of crystals with significant surface roughness observed using SID-3 could explain why optical phenomena like arcs and halos are so rare, because such phenomena rely on “clean” scattering by smooth crystals aligned in some special way. More importantly from the perspective of climate modelling, Joseph emphasized that surface roughness can have a dramatic effect on the amount of shortwave radiation reflected by an ice crystal, and suggested that the ice scattering community should view surface roughness as being at least as important (if not more important) than crystal habit (i.e. the size and shape of crystals).

After coffee **Robin Hogan** (Reading) explained to us the importance of wave coherency in the context of backscatter in active and passive radar/lidar remote sensing of cloud properties. In scattering problems one often thinks of decomposing the wave field into constituent components (e.g. separate wave fields propagating along geometrical rays). One then has a choice about how to combine the constituent fields to estimate a particular property of the total field. For energy/intensity calculations with a very complicated scatterer geometry, or with a very large number of particles in a multiple scattering situation, it is often appropriate to assume that the interference effects between the different constituent fields all cancel each other out (i.e. average to zero), so that the energy/intensity can be calculated “incoherently”, i.e. by summing energies/intensities of the constituent fields (the path length along a ray, and hence the phase of the associated wave field, is assumed to be random). However, for some transmitter/receiver locations symmetry effects invalidate this assumption, and “coherent” summation must instead be used, i.e. summing the fields rather than their energies/intensities, which can lead to significant differences in the calculated energy/intensity for the total field. Robin cited the “Seeliger effect” as a striking illustration of this phenomenon; Saturn's rings appear brighter when the sun is in opposition. Robin suggested that it is crucial that this effect be taken into account correctly when simulating scattering by complex ice aggregates or when interpreting radar/lidar retrievals. Robin used the Rayleigh-Gans (or Born) approximation to derive an equation for the radar backscatter cross-section of an ensemble of snow aggregates that exploited a new statistical description of the internal structure of the particles.

The final three talks of the workshop were from mathematicians, giving a flavour of the techniques currently being developed in the mathematical scattering community. Two of the talks concerned the boundary element method (BEM), and one the Finite Element Method (FEM). These methods do not appear to be used currently in meteorological applications, but are used extensively in engineering and acoustics, and are widely studied by the mathematical community because of their generality, flexibility, the systematic nature of the approximations they are built on, their error-controllability, and the existence of well-established frameworks in which to perform error analysis. In their “Galerkin” forms, FEM and BEM are special cases of the same general abstract theory: write down a “weak” formulation, posed on an infinite dimensional vector space  $V$ , then turn this into a “discrete” problem by replacing  $V$  by a finite-dimensional approximation space  $V_N$ . The discrete problem is equivalent to solving a linear system of equations for the coefficients of the approximate solution in terms of a basis of the space  $V_N$ . Constructing the finite-dimensional approximation space  $V_N$  for FEM involves discretising (meshing) the propagation domain (in fact a truncated version of it), whereas applying BEM involves first

reformulating the problem as an integral equation on the scatterer boundary, then discretising (meshing) this boundary. The general result is that FEM matrices are usually larger but sparser and BEM matrices are usually smaller but denser (because the boundary integral operators involved are “non-local”). Another difference between the two is that BEM can only be applied for homogeneous media with constant (or piecewise-constant) material parameters, whereas FEM applies more generally (e.g. to inhomogeneous media with non-constant wave propagation speed).

In a change to the advertised programme, due to illness (possibly brought on by David Bebbington's dastardly use of the “S-band” terminology the day before?), **Timo Betcke** (UCL) spoke first of the three, presenting the BEM++ software package he is currently developing with colleagues from UCL and beyond. BEM++ is an open source C++ library (with a Python interface) for solving 3D Laplace, Helmholtz and Maxwell boundary value problems using the boundary element method. The user provides the scatterer geometry (with a suitable mesh, computed e.g. using Gmsh) and the boundary conditions, and chooses from a palette of piecewise-polynomial approximation spaces. BEM++ then solves the resulting problem efficiently and stably, taking advantage of a number of optimised implementations of matrix assembly/storage and linear algebra routines. Timo showed numerical results for the full electromagnetic transmission problem of scattering by a hexagonal ice crystal column, providing details of computation times both for the matrix assembly and the solution of the linear system, which suggest that with some further optimisation (currently being implemented) BEM++ could be competitive with other “exact” electromagnetic methods being used in the meteorological community such as the T-matrix method.

After lunch **Ivan Graham** (Bath) presented recent work into the solution of the Helmholtz equation in inhomogeneous media (e.g. in seismic inversion applications in oil/gas exploration) using FEM. Standard implementations for large propagation domains use iterative methods (e.g. GMRES) for solving the large (and sparse) system of linear equations. But this system is generally very badly conditioned (i.e. “hard to invert”) and so usually requires a prohibitive number of GMRES iterations to converge to an acceptable accuracy. Ivan explained how this could be avoided by “preconditioning” the problem using an (approximate) solution of a modified problem, that of the Helmholtz equation with absorption (i.e. introducing a positive imaginary part in the wavenumber). Ivan described clearly and concisely how the amount of absorption should be chosen in order to achieve both effective preconditioning (resulting in a much reduced iteration count in GMRES) and also an acceptable solution time for the modified problem, which can be solved efficiently using domain decomposition methods.

The final talk of the workshop was given by **Dave Hewett** (Oxford), who described some recent developments in the theory and application of “hybrid numerical-asymptotic” BEMs for high frequency scattering problems. Conventional BEMs use approximation spaces  $V_N$  (see above) based on piecewise polynomial functions on a discretisation/mesh of the boundary. These methods are prohibitively expensive at large size parameters because capturing the rapid spatial oscillations of the solution using such piecewise polynomials requires a very fine mesh. Hybrid numerical-asymptotic methods aim to significantly reduce the computational cost by using approximation spaces based on *oscillatory* functions, carefully chosen to capture the high frequency asymptotic behaviour. Dave showed that



these methods can be extremely effective for relatively simple scatterer geometries (e.g. impenetrable (perfectly conducting) convex 2D scatterers), and in some cases a full frequency-explicit numerical error analysis is possible. Recent work has shown that generalising the method to nonconvex, penetrable (dielectric) and 3D problems is possible in principle. While the methodology is not yet sufficiently well-developed to be applied to the complex scatterers arising in meteorological applications, it may provide a means of validating other high frequency approximate methods in simple model geometries.

The workshop closed with a second structured discussion session lead by **Simon Chandler-Wilde** (Reading). After surveying the themes covered in the talks, Simon encouraged the participants to focus on some possible areas for future collaboration. A lively discussion ensued, and overall it was agreed that the workshop had been a good “first step” in bringing together two disparate communities with quite different perspectives on wave scattering.

Some of the main points arising from the discussion are detailed below:

- One obvious barrier to communication which arose during the workshop was the **difference in terminology** uses by the different communities present. For example, by “ray-tracing methods” and “Geometrical Optics”, meteorologists usually mean methods when phase is neglected and only energy/intensities are considered, whereas to mathematicians this same terminology is used when phase is retained and wave fields are summed with interference effects included. The parameters required by radiative transfer models (extinction, albedo, asymmetry) and quantities such as the phase function were unfamiliar to many of the mathematicians present, as was the formalism of the FEM and BEM to many of the meteorologists. It was agreed that an immediate next step to counter these difficulties should be the production of a **basic “glossary” of terminology** used by both communities, with clear definitions of the quantities studied and explanations of how they relate to each other (with references to accessible sources in the literature as appropriate). (But see the “Notes” section below.)
- One slightly contentious question that arose was whether high accuracy in scattering calculations was actually important or not. That is, **how close to the “true” electromagnetic wave solution do our simulations need to be?** 1% accurate? 5% accurate? 50% accurate? The consensus seemed to be that the answer to this question depends on the application – when comparing the output of scattering calculations to measurements of scattering patterns for individual ice crystals (e.g. from SID-3 data) it may be important to have high accuracy. On the other hand, when interpreting remote sensing retrievals for scattering by an ensemble of particles in a cloud, other uncertainties in the model (e.g. knowledge of the PSD) might mean that high accuracy is not worth chasing.
- There was also a debate about whether future work should focus on **stochastic or deterministic modelling**. Some argued that it would be very beneficial to be able to invert the scattering pattern (measured e.g. using SID-3) to recover geometrical information about individual ice crystals. Others argued that because of the prevalence (and dominance in terms of scattering, because of their relatively large

size) of crystals with complex geometries and rough surfaces, it might be more fruitful to adopt a stochastic approach, which would consider only the statistics of the crystal geometry and surface roughness rather than dealing with individual realisations.

- A number of different methods (T-Matrix, DDA, GO, RTDF, ADA, Fredholm Integral Method, Rayleigh-Gans (Born) approximation, FEM, BEM) for single scattering were discussed during the course of the workshop. It was agreed that a medium-term collaborative output could be a review/survey paper (e.g. in the Journal of Quantitative Spectroscopy and Radiative Transfer (JQSRT)) **comparing the capabilities of different methods for solving a range of model/challenge scattering problems** relevant for the meteorological application. (But see the “Notes” section below.)
- Another specific area for collaboration could be the development of a **user interface for BEM++ specifically targeted at meteorological applications**. This is something that Timo and the BEM++ team (including Chris Westbrook) already have in mind but have not yet embarked on.
- It was suggested that a **comparison with the literature on elastic wave propagation** should be made – for example a comparison between the quantities of interest for EM radiative transfer models, and the “Stokes vector” of elasticity. It was pointed out that there is a wide literature on homogenisation and effective medium properties in elasticity, and that some of this literature might be relevant to the meteorological application.
- The complicated problem of **how atmospheric conditions affect crystal size, shape and surface roughness** was one which interested some of the mathematicians present. This could possibly be an interesting spin-off collaboration, rather different from scattering problems, focussing on crystal growth/free boundary problems, in the light of the experimental data being obtained e.g. in the Manchester ice cloud chamber.

A number of other mechanisms for ongoing interaction were proposed, including a follow-up meeting (or an annual series of such meetings), an online forum, and the formation of an “ice scattering club”. Simon remarked that, for example, the recently-awarded Reading-Imperial EPSRC Maths of Planet Earth CDT could provide an opportunity for funding collaborative research projects in this area. He also suggested that a longer-term outcome, if collaboration appeared to be fruitful, could be the preparation of a large consortium grant application to either EPSRC or NERC, with the Met Office as a partner. It was agreed that the post-workshop report, and details of any future meetings, should be drawn to the attention of the Met Office Academic Partnership (MOAP).

## Notes:

Slides from the talks are available on the workshop website at [www.maths.ox.ac.uk/~hewett/scatteringcloudsclimateworkshop.htm](http://www.maths.ox.ac.uk/~hewett/scatteringcloudsclimateworkshop.htm)

Anthony Baran pointed out that there is a **comprehensive glossary of radiative transfer terminology** at <http://www.tpdsci.com>.

Anthony Baran suggested that participants might like to join the Electromagnetic and Light Scattering newsletter, for updates about forthcoming conferences/meetings. Details at <http://www.astro.umd.edu/~elsnews/>.

Chris Westbrook recommended the book “A First Course in Atmospheric Radiation” by G.W. Petty as a very accessible introduction to how radiative transfer and remote sensing work, definitions of various quantities of interest, etc.

Caroline Poulsen (RAL) recommended the book “Inverse methods for atmospheric sounding” by Clive Rogers (AOPP Oxford) as a good mathematical introduction to the subject of atmospheric inverse modelling.

Caroline Poulsen noted that the **radiative transfer package** many atmospheric scientists use (at least at infrared wavelengths) for predicting satellite radiances and assimilating satellite observations into models is RTTOV. For information see

[http://research.metoffice.gov.uk/research/interproj/nwpsaf/rtm/rttov\\_description.html](http://research.metoffice.gov.uk/research/interproj/nwpsaf/rtm/rttov_description.html)).

(Dave Hewett was amused to find that RTTOV is a doubly-nested acronym:

RTTOV = Radiative Transfer model for TOVS

TOVS = TIROS Operational Vertical Sounder

TIROS = Television and Infrared Observation Satellite)

Anthony Baran remarked that a **more general package (applicable to visible wavelengths) is DISORT**, details of which can be found at <http://arm.mrcsb.com/sbdart/html/>. He also mentioned that the Met Office will be releasing a generalised scattering radiative transfer model around September 2014, which will be fast, will cover the full spectrum at any wavelength resolution, is sensor independent and can be applied in any atmosphere (Earthly!) and over any surface. This new fast model does not as yet include polarization but this will be included by March 2015.

Anthony Baran pointed out that there is a **webpage documenting all the currently available light scattering codes** that users can simply download (including BEM++), located at <http://www.scattport.org/>.

Anthony Baran remarked that the study of scattering by ice crystals forms part of a wider effort in the meteorological community to understand **scattering by more general atmospheric particles, including aerosols** (dust, smoke, volcanic ash etc.). Future meetings could consider this more general setting.

Regarding **birefringence**, Anthony Baran noted that Takano and Liou (1989) did study the effect, and found it not to be particularly important. This finding has influenced others, which might be why it is not generally included in models. In principle it should be included,

and the importance of its contribution carefully considered; in the case of some aerosols it may have a much greater significance than in ice.

Regarding the idea of a **review paper on methods for EM scattering**, Anthony Baran pointed out that such reviews have been published quite recently in the literature, so any such publication would need to go beyond the existing review, e.g. by considering new methods (e.g. FEM, BEM) not previously considered by the meteorological community.

Picking up on this theme, Dave Hewett noted that BEM++ is more competitive with the T-matrix method than one might have thought after listening to Anthony Baran and Timo Betcke's talks. Anthony reported results from Yang's group where the T-matrix method was applied to a hexagonal ice crystal column at size parameter 150, whereas Timo showed results for size parameter 8. But this "headline" comparison is not fair because Yang's group used  $kH$  as a size parameter, where  $H$  is the height of the column and  $k$  is the wavenumber, whereas Timo used  $H/\lambda$ , where  $\lambda=2\pi/k$  is the wavelength. So in Yang's terminology Timo was actually reporting results for size parameter approximately 50. Moreover, the BEM++ computation time was significantly lower than those reported in Yang's paper - but it should be noted that the computation times are not directly comparable because Yang's group's calculations were for randomly orientated crystals (the random orientation being dealt with "analytically" using the averaging technique of Mishchenko) whereas the BEM++ results were for single orientations. Notwithstanding this caveat about random orientations, it would appear that BEM++ could well be competitive, especially once further efficiencies have been implemented. Dave noted that Samuel Groth (Reading) will be visiting Anthony Baran at the Met Office for an extended period this summer and one of his goals during his time there will be to explore the potential for using BEM++ in meteorological applications.

Simon Chandler-Wilde remarked that there is a very large community of mathematicians working on inverse scattering problems, who may be interested in meteorological applications such as interpreting the scattering patterns from SID-3 probes. Expertise from this community could help to design new measurement methodologies.

John Ockendon (Oxford) suggested that it might be interesting to consider whether radiative transfer models can be approximated by even simpler models in particular regimes, e.g. when looking only at forward/backscatter. (This seems to have some overlap with Robin Hogan's talk.)