Coherent backscattering effects for single particles and distributions of particles

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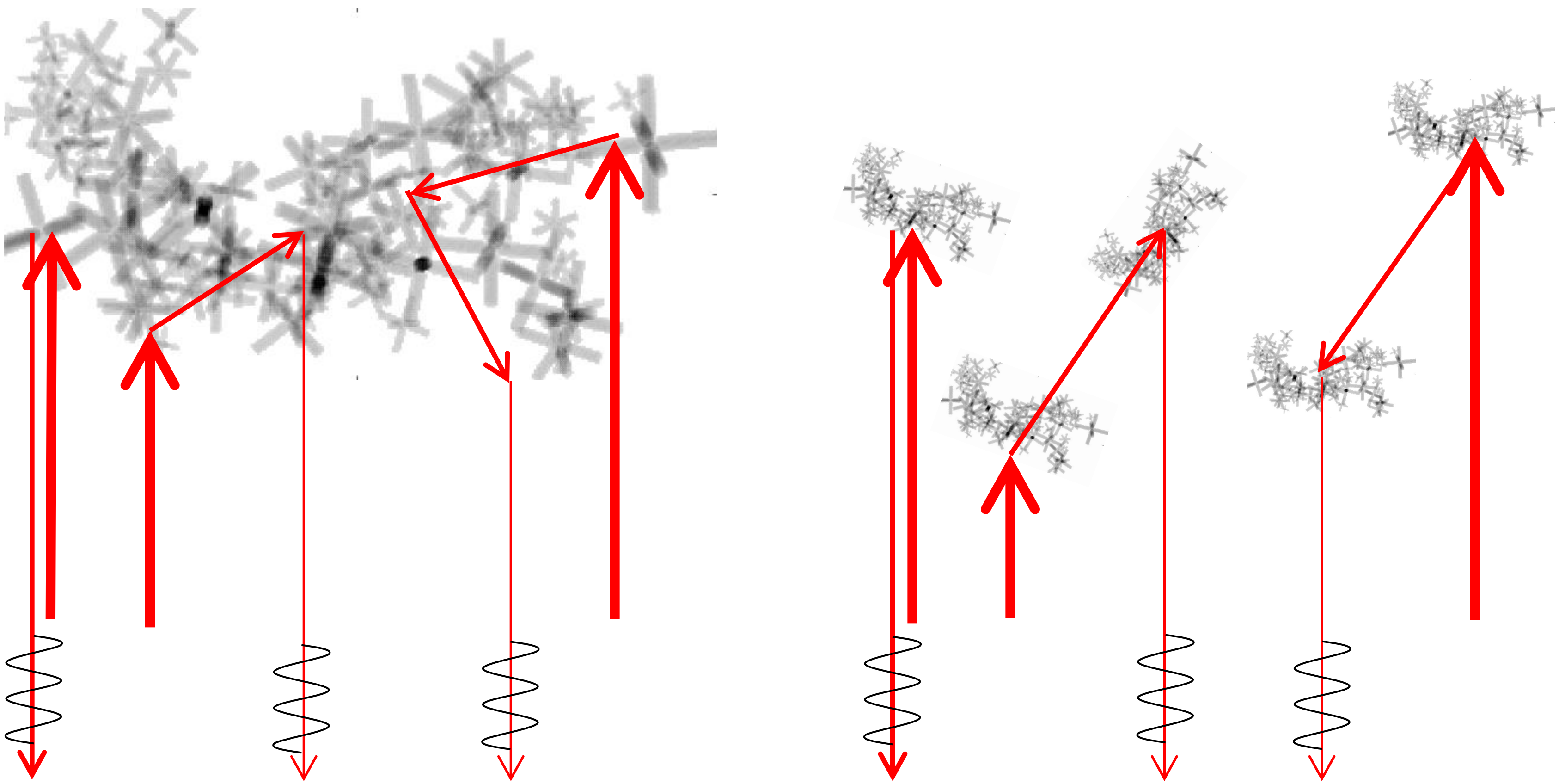
Overview

- Motivation
  - We need to be able to model the backscattered signal from clouds in order to interpret radar and lidar observations (particularly from space) in terms of cloud properties
- Coherent backscattering effects for single particles
  - Radar scattering by ice aggregates and snowflakes
  - The Rayleigh-Gans approximation
  - A new equation for the backscatter of an ensemble of ice aggregates: the *Self-Similar Rayleigh-Gans approximation*
- Coherent backscattering effects for distributions of particles
  - Coherent backscatter enhancement (CBE) for solar illumination
  - The multiple scattering problem for radar and lidar
  - How important is CBE for radar and lidar?
- A prediction
  - Coherent backscatter enhancement occurs for individual particles so ray tracing could underestimate backscattering by a factor of two

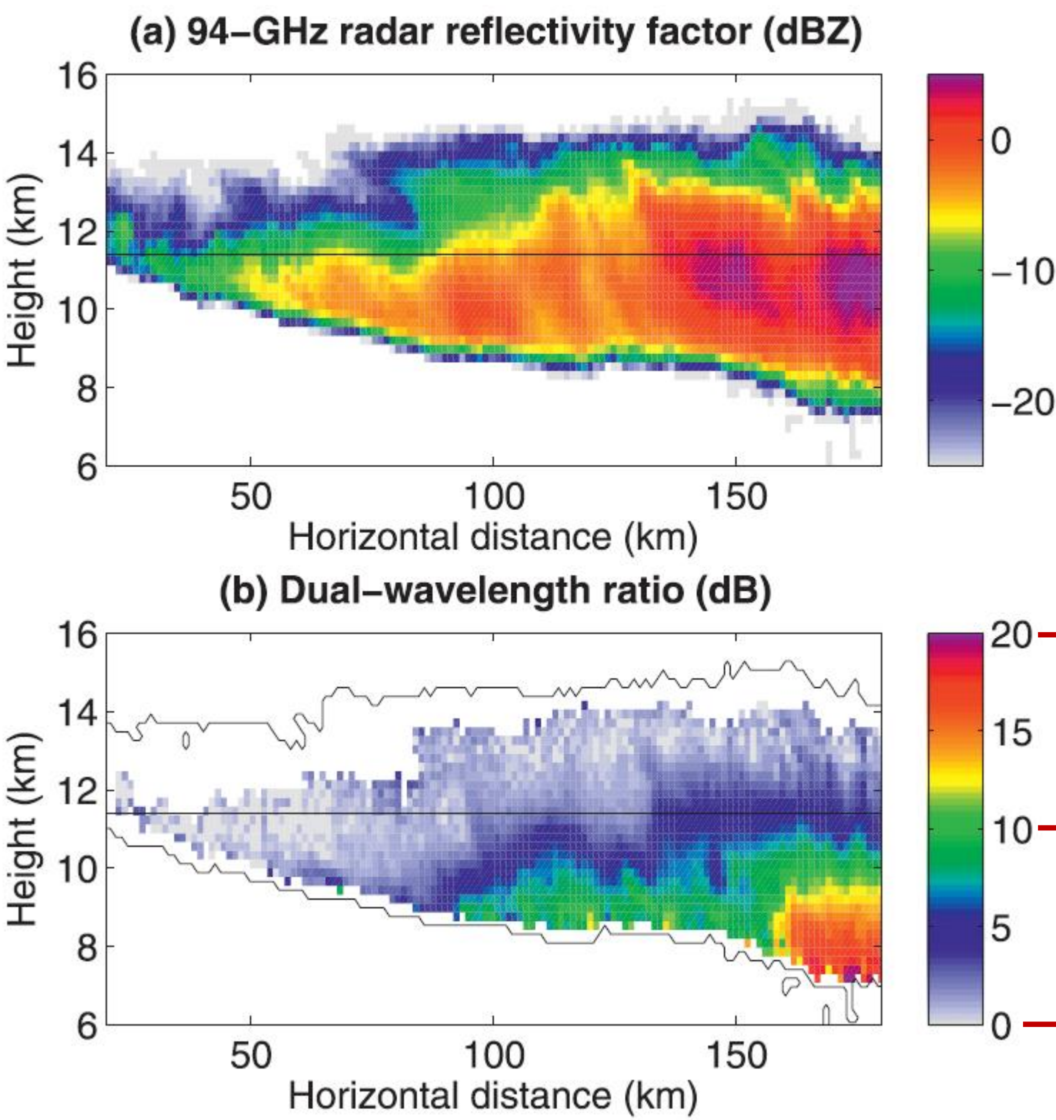
The principle unifying this talk

Single particle **Distribution of particles**



Backscattered amplitude is found by summing the returned rays *coherently*

Radar observations of tropical cirrus



- Two airborne radars: 3 mm (94 GHz) and 3 cm (10 GHz)
  - Most ice particles scatter in Rayleigh regime only for 3-cm radar
- How can we interpret deviations from Rayleigh scattering in terms of particle size?

20 → 3-mm wavelength scattered 100 times less

10 → 3-mm wavelength scattered 10 times less

0 → 3-mm wavelength in Rayleigh regime

Hogan et al. (2012)

Two problems

1. Snowflakes have complicated shapes
2. Methods to compute their scattering properties are slow

Is the best we can do to (somehow)generate a large ensemble of 3D snowflake shapes and compute their scattering by brute force?

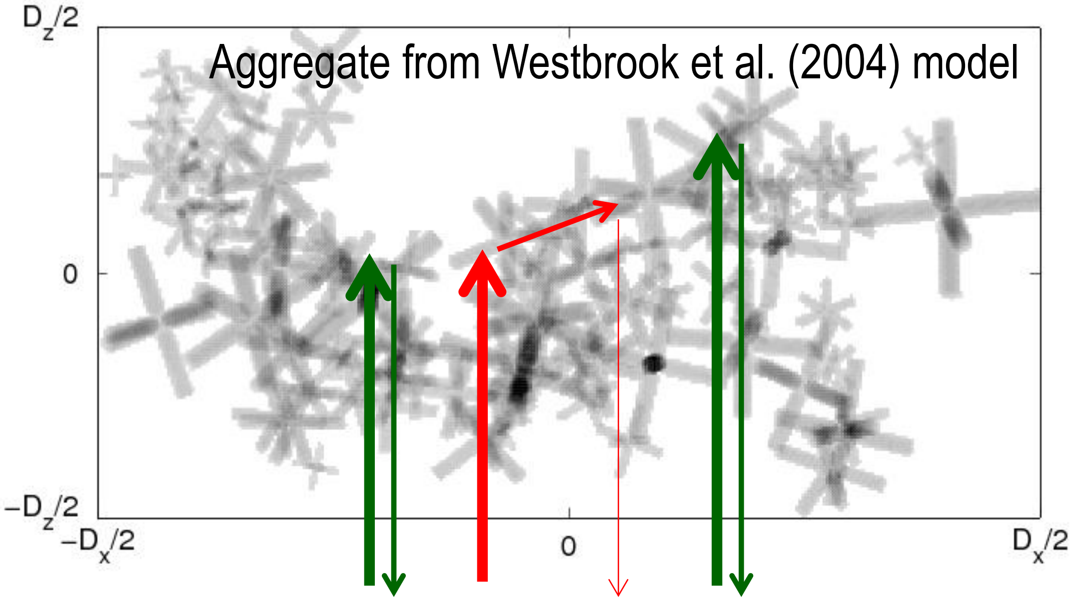
Potentially not:

1. Snowflakes have fractal structure that can be described statistically
2. The Rayleigh-Gans approximation is applicable

Images from Tim Garrett, University of Utah

The Rayleigh-Gans approximation

- Approximate the field at any point by the incident field
- Sum backscattered returns from each volume element coherently



Backscatter depends only on $A(s)$ function and dielectric constant ε (or refractive index $m = \varepsilon^{1/2}$)

- Rayleigh-Gans applicable if $|m - 1| \ll 1$ and $|\rho| \ll 1$
  - where $\rho = kD(m - 1)$ is the phase shift across the particle and $k = 2\pi/\lambda$
- Solid ice in the microwave has $m = 1.77$, but on the scale of the wavelength the particle is mostly air so effective m close to 1
  - Tyynela et al. (2012) found that Rayleigh-Gans is a good approximation compared to other uncertainties, e.g. in ice structure

The Rayleigh-Gans approximation

$$\sigma_b = \frac{9k^4|K|^2}{4\pi} \left| \int_{-D/2}^{D/2} A(s) \exp(\mathrm{i}2ks) ds \right|^2$$

- Backscatter cross-section is proportional to the power in the Fourier component of A(s) at the scale of half the wavelength

- Can we parameterize A(s) and its variation?

$$A(s) = a_0 \left[\left(1 + \frac{\kappa}{3}\right) \cos\left(\frac{\pi s}{D}\right) + \kappa \cos\left(\frac{3\pi s}{D}\right) \right]$$

← Mean structure, κ = *kurtosis parameter*

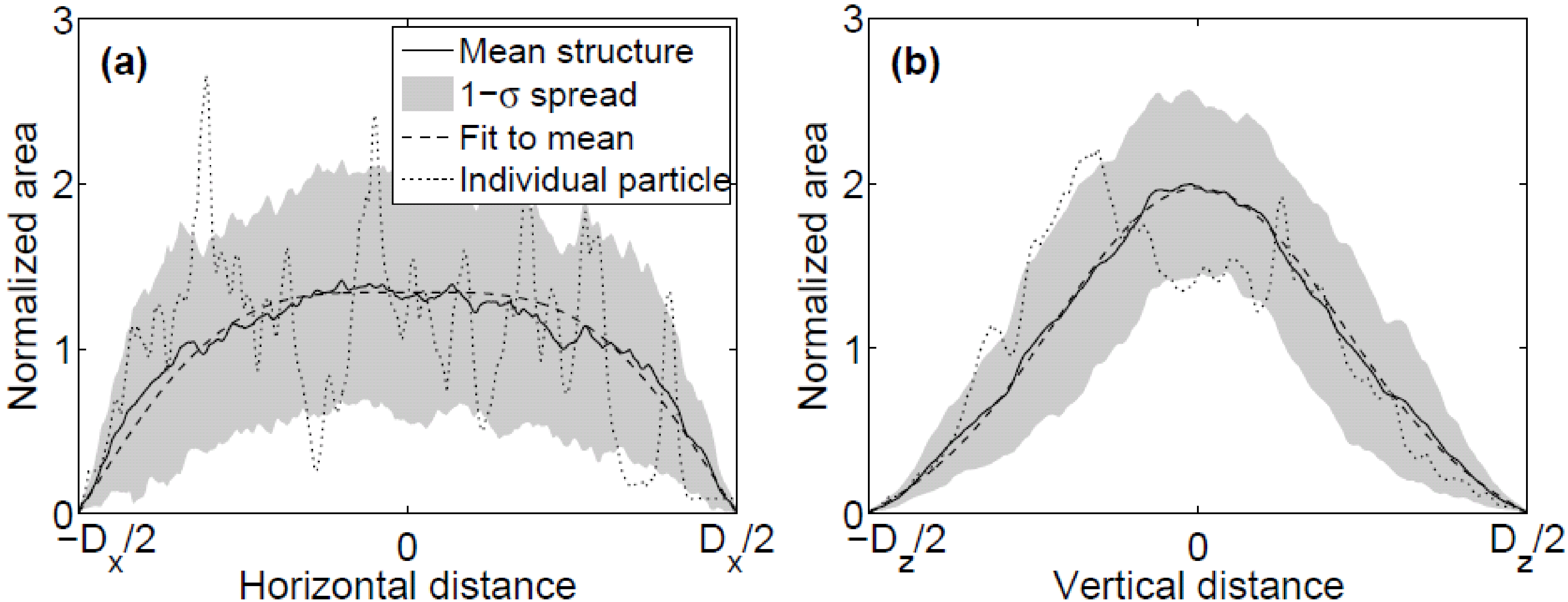
$$+ \sum_{j=1}^{n} a'_j \cos\left(\frac{2\pi j s}{D}\right) + a''_j \sin\left(\frac{2\pi j s}{D}\right),$$

← Fluctuations from the mean

- where $a_0 = \frac{\pi}{2D} V$, and V is the volume of ice in the particle

Aggregate mean structure

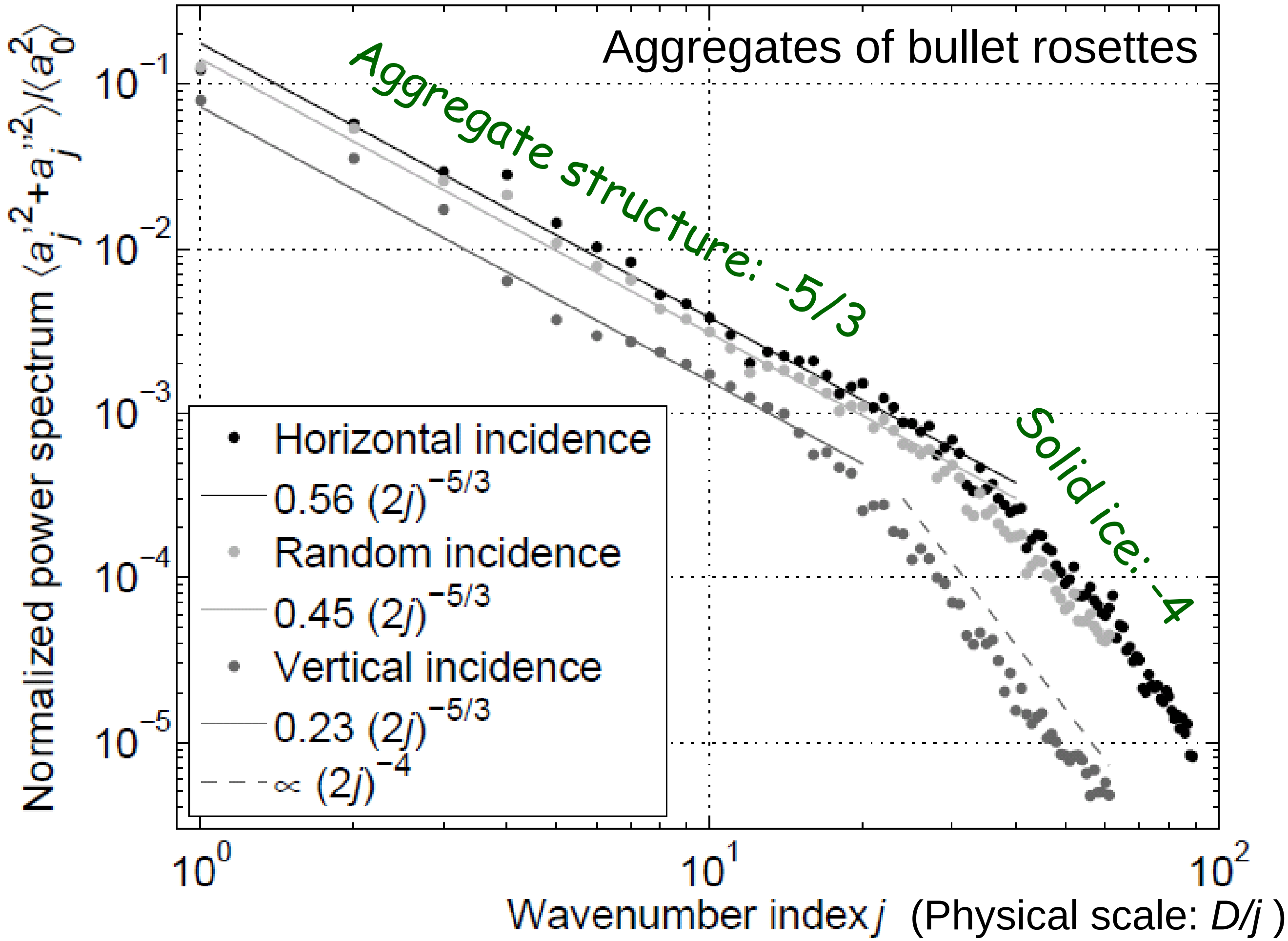
- Hydrodynamic forces cause ice particles to fall horizontally, so we need separate analysis for horizontally and vertically viewing radar



- Mean structure of 50 simulated aggregates is very well captured by the two-cosine model with kurtosis parameters of
  - $\kappa = -0.11$ for horizontal structure
  - $\kappa = 0.19$ for vertical structure

Aggregate self-similar structure

- Power spectrum of fluctuations obeys a -5/3 power law
  - Why the Kolmogorov value when no turbulence involved? Coincidence?
  - Aggregates of columns and plates show the same slope



New equation

- Assumptions:
  - Power-law: $\langle a_j'^2 + a_j''^2 \rangle / \langle a_0^2 \rangle = \beta(2j)^{-\gamma}$
  - Fluctuations at different scales are uncorrelated: $\langle a_j' a_k' \rangle = \langle a_j'' a_k'' \rangle = 0$
  - Sins and cosine terms at the same scale are uncorrelated: $\langle a_j' a_j'' \rangle = 0$

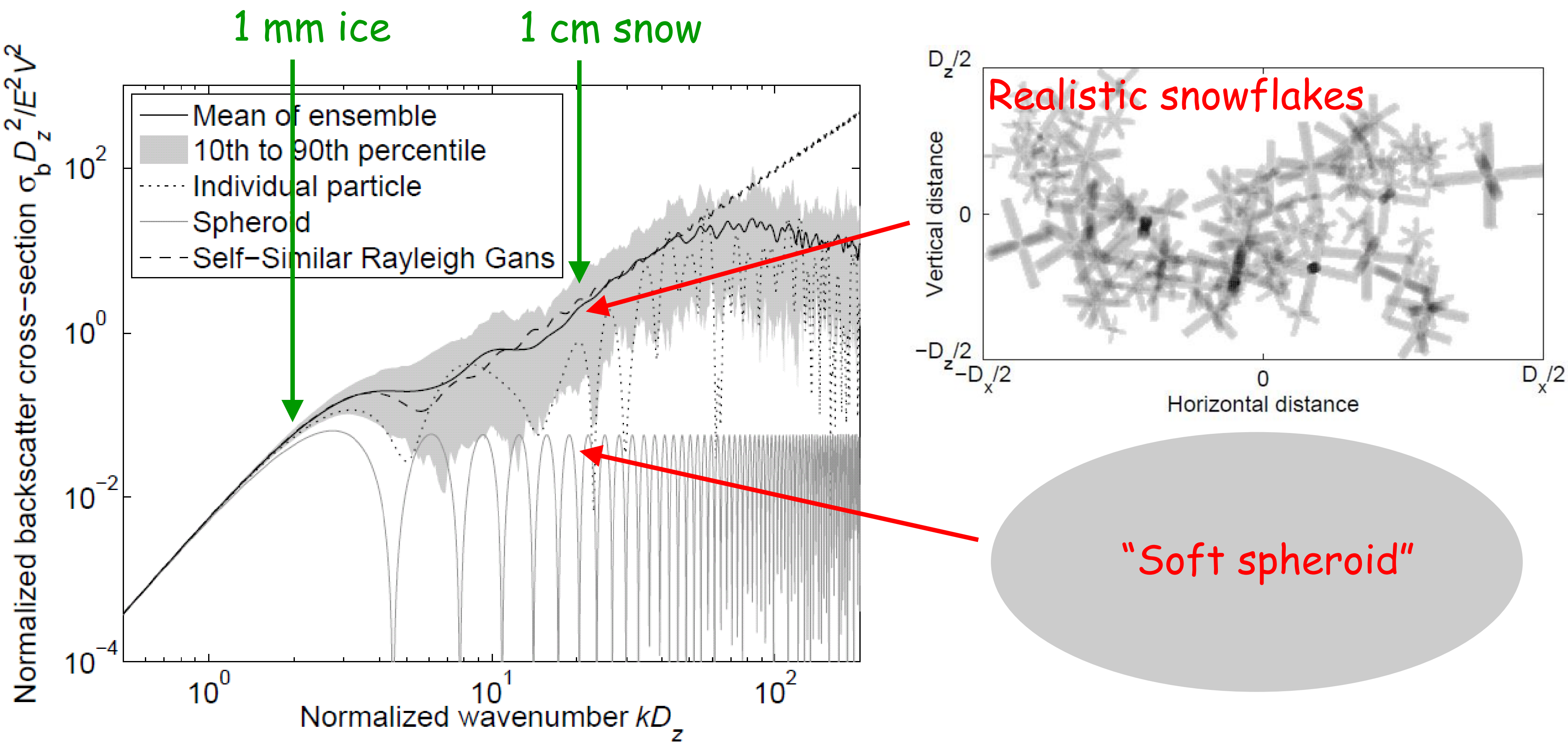
- Leads to the *Self-Similar Rayleigh-Gans approximation* for backscatter coefficient:

$$\langle \sigma_b \rangle = \frac{9k^4\pi|K|^2V^2}{16} \left\{ \cos^2(x) \left[\left(1 + \frac{\kappa}{3}\right) \left(\frac{1}{2x+\pi} - \frac{1}{2x-\pi} \right) - \kappa \left(\frac{1}{2x+3\pi} - \frac{1}{2x-3\pi} \right) \right]^2 + \beta \sum_{j=1}^{n} (2j)^{-\gamma} \sin^2(x) \left[\frac{1}{(2x+2\pi j)^2} + \frac{1}{(2x-2\pi j)^2} \right] \right\},$$

  - where $x = kD$

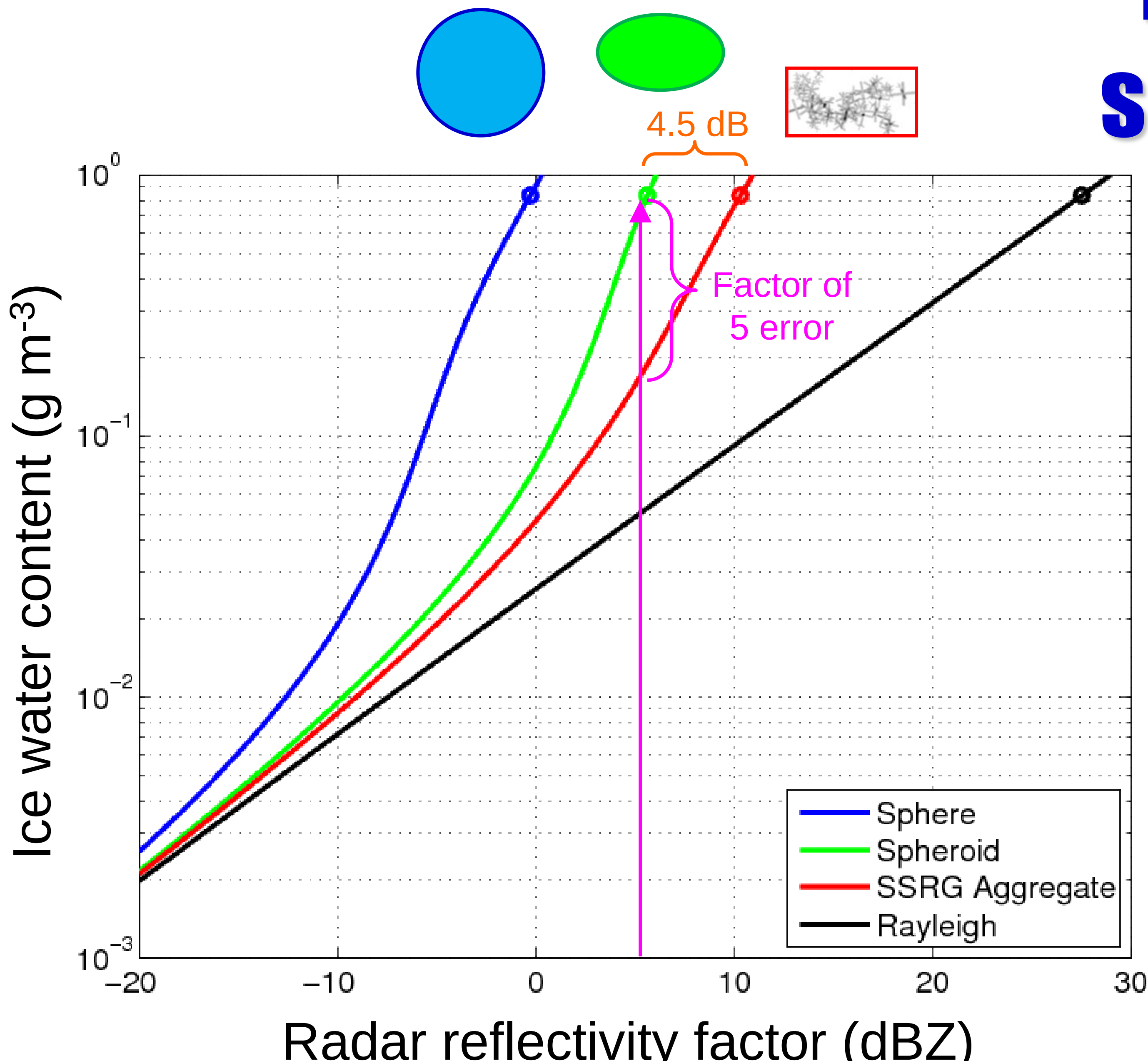
Hogan and Westbrook (2014, in revision)

Radar scattering by ice



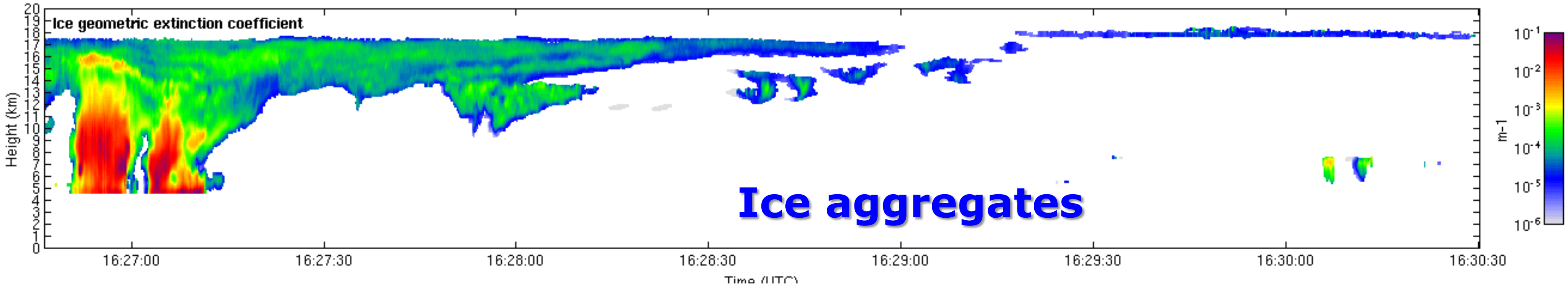
- Internal structures on scale of wavelength lead to significantly higher higher backscatter than "soft spheroids" (proposed by Hogan et al. 2012 and others)

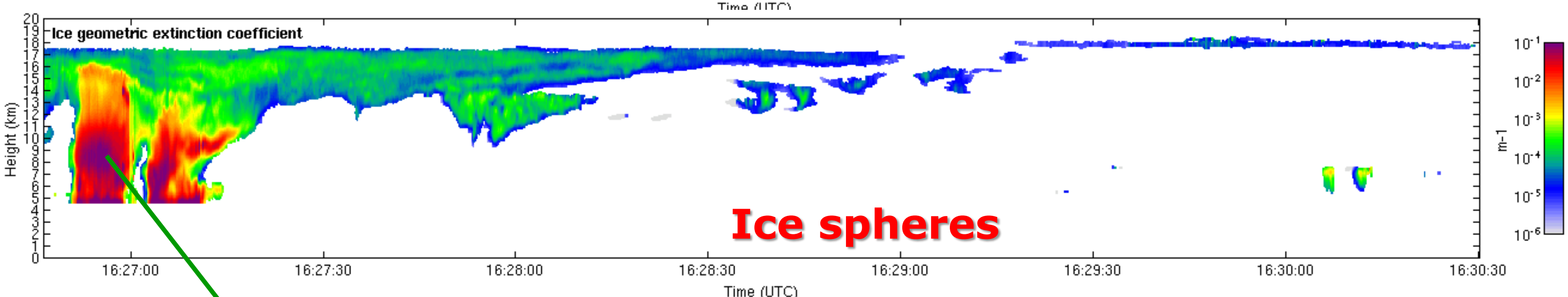
Impact of scattering model

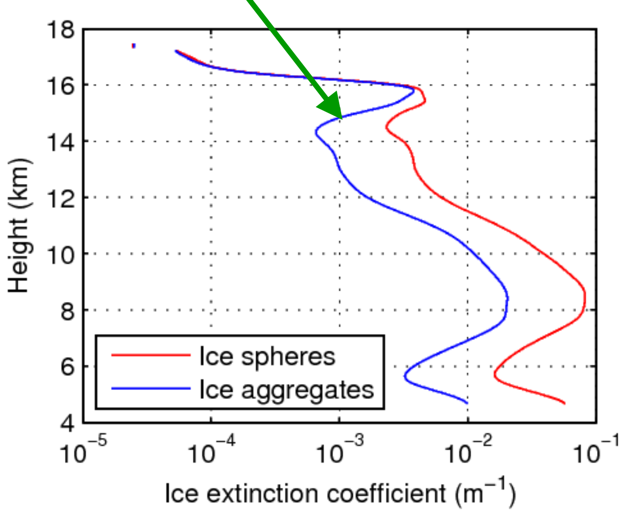


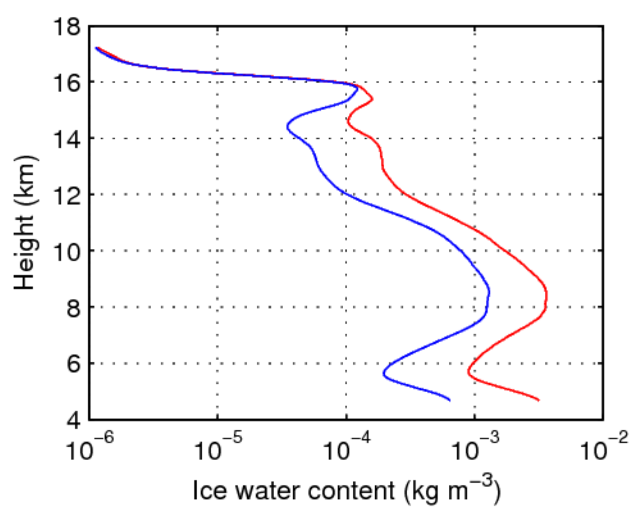
- Field et al. (2005) size distributions at 0°C
- Circles indicate D_0 of 7 mm reported from aircraft (Heymsfield et al. 2008)
- Lawson et al. (1998) reported D_0=37 mm: 17 dB difference

Impact of ice shape on retrievals



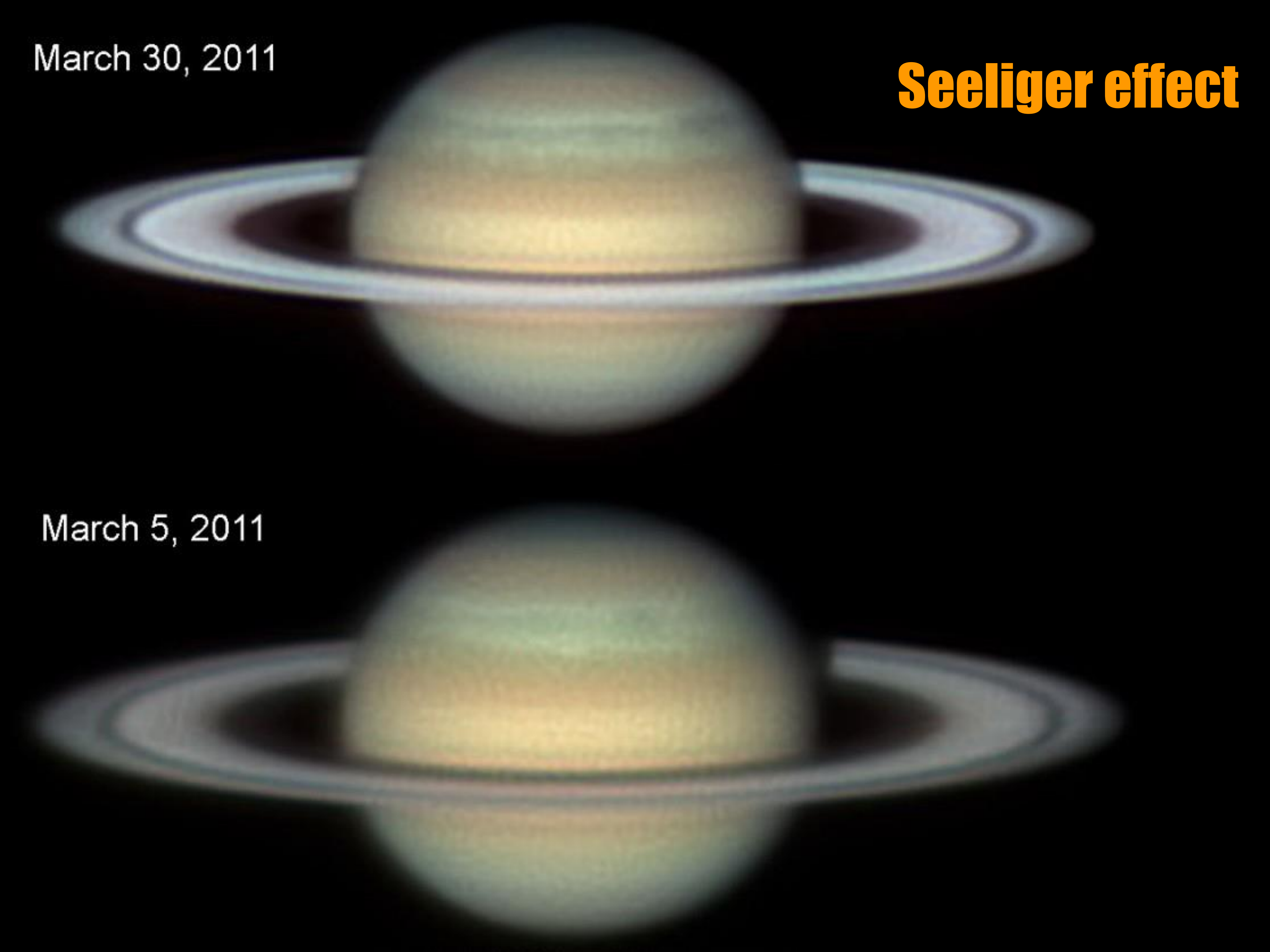






- Spheres can lead to overestimate of water content and extinction of factor of 3
- All 94-GHz radar retrievals affected in same way

Seeliger effect

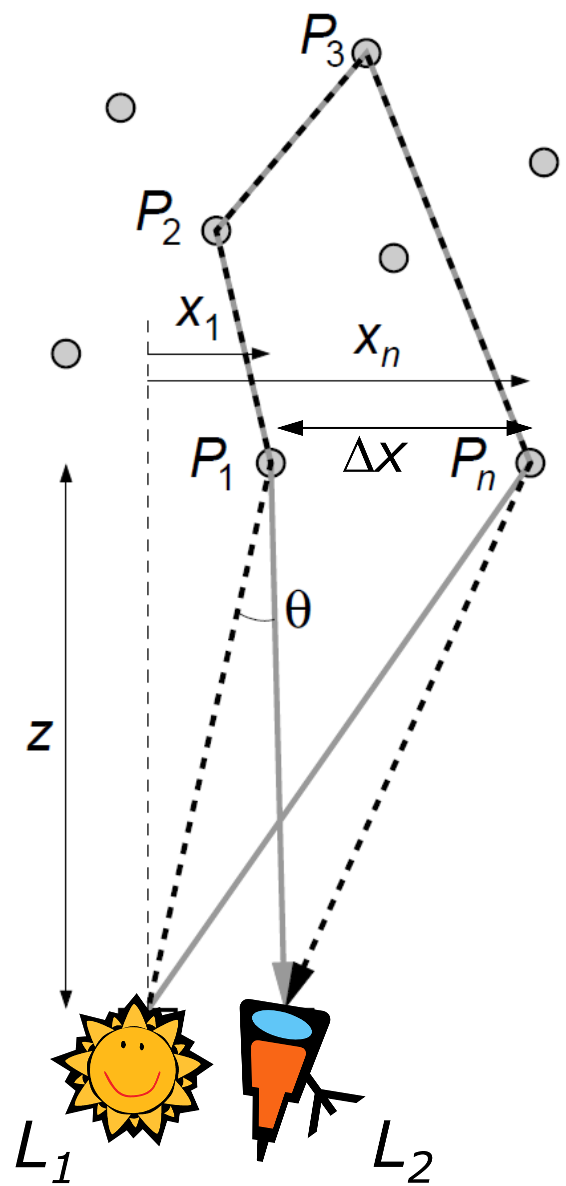


Why are Saturn's rings brighter when the sun is in opposition?

- *Shadow hiding* in the icy rocks that compose the rings (r >> λ)?
- *Coherent backscatter enhancement* (r<= λ)?
  - Multiply scattered light paths normally add incoherently
  - But for every path $L_1P_1P_2 \ldots P_nL_2$ there is an equivalent reverse path $L_1P_nP_{n-1} \ldots P_1L_2$ whose length differs by only

$$\Delta p \simeq \theta \Delta x$$

  - Where Δx is the lateral distance between the first and last particles in the scattering chain (P_1P_n in this example)
  - These paths will add coherently if $\Delta p \ll \lambda$
  - Reflected power *twice* what it would be for incoherent averaging



Observed enhancement

- Define coherent backscatter enhancement (0 = none, 1 = doubled reflection) for single pair of multiply scattered paths as

$$\widetilde{\mathrm{CBE}} = \cos\left(\frac{2\pi\Delta p}{\lambda}\right)$$

- Observed enhancement found by integrating over distribution of Δx:

$$\mathrm{CBE} = \int_{-\infty}^{\infty} P(\Delta x)\widetilde{\mathrm{CBE}}(\Delta x)\,d\Delta x.$$

- If this distribution is Gaussian with width σ, then integral evaluates as:

$$\mathrm{CBE} \simeq \exp\left(-\frac{1}{2}\frac{\theta^2}{\theta_0^2}\right)$$ where $$\theta_0 = \frac{\lambda}{2\pi\sigma}$$

- But remember that there is no enhancement for single scattering, so this effect is only observed if multiple scattering is significant

Laboratory measurements

- Measurements by Wolf et al. (1985)

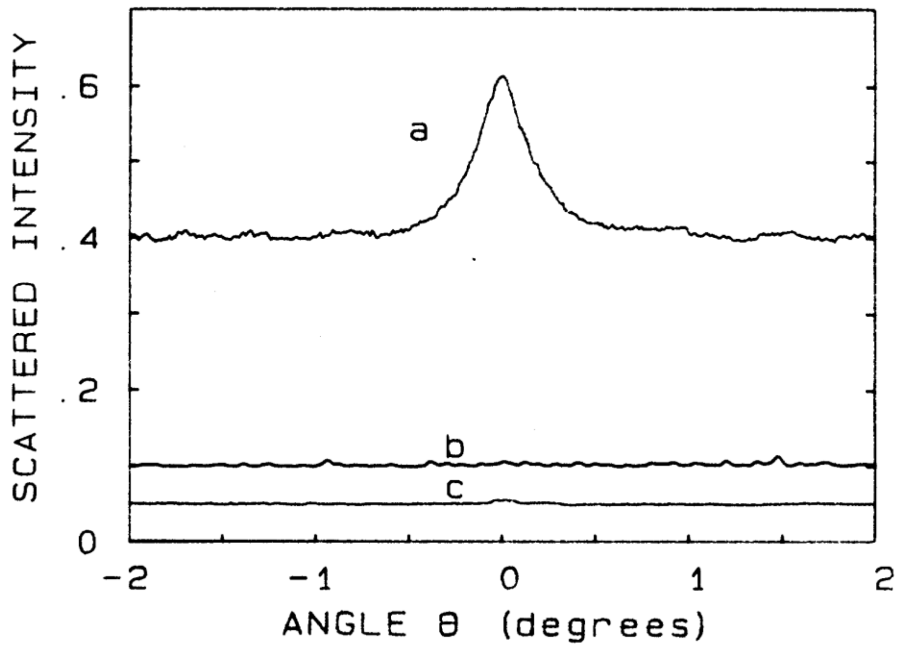
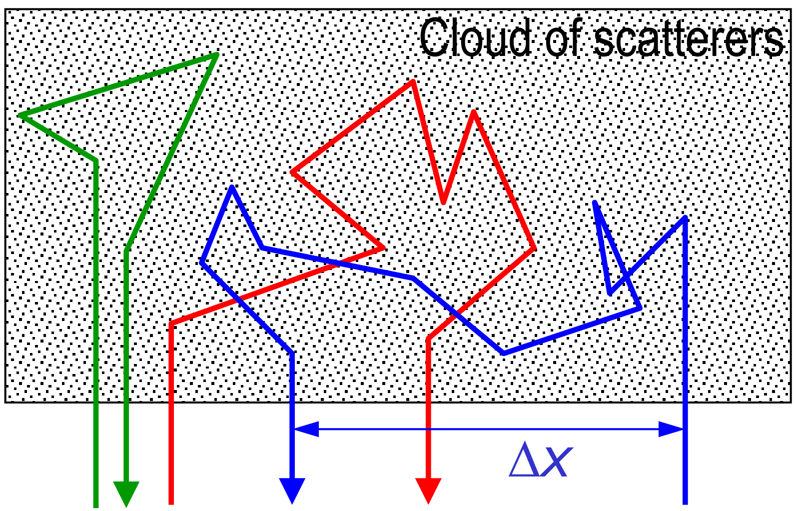
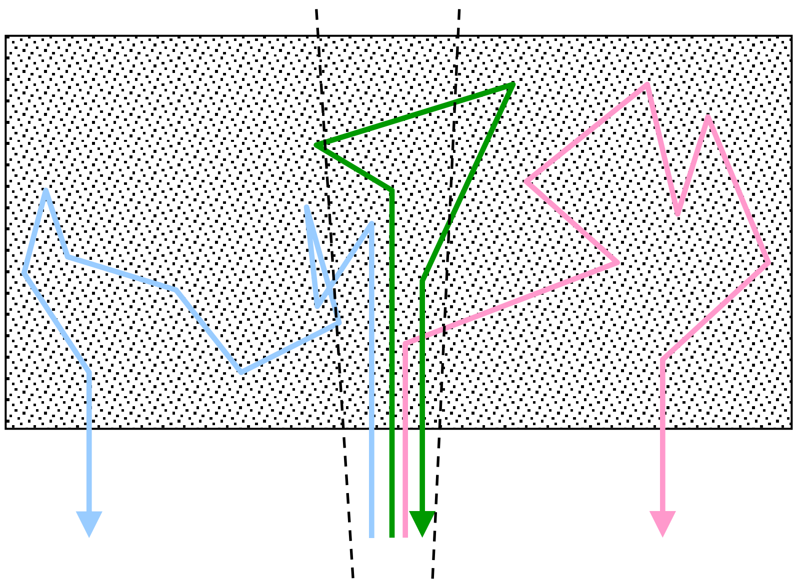


FIG. 2. Angular dependence of the scattered light intensity (curve *a*) by an aqueous suspension of 0.46-μm-diam polystyrene beads (solid fraction 10%), (curve *b*) by the same cell filled with water, and (curve *c*) in the absence of any cell. For these curves, no analyzer was used; scales are identical, but curves *b* and *c* are shifted by 0.1 and 0.05 vertical units, respectively.

Dependence on source

- Extended source (e.g. sun)
- Confined source (radar or lidar)

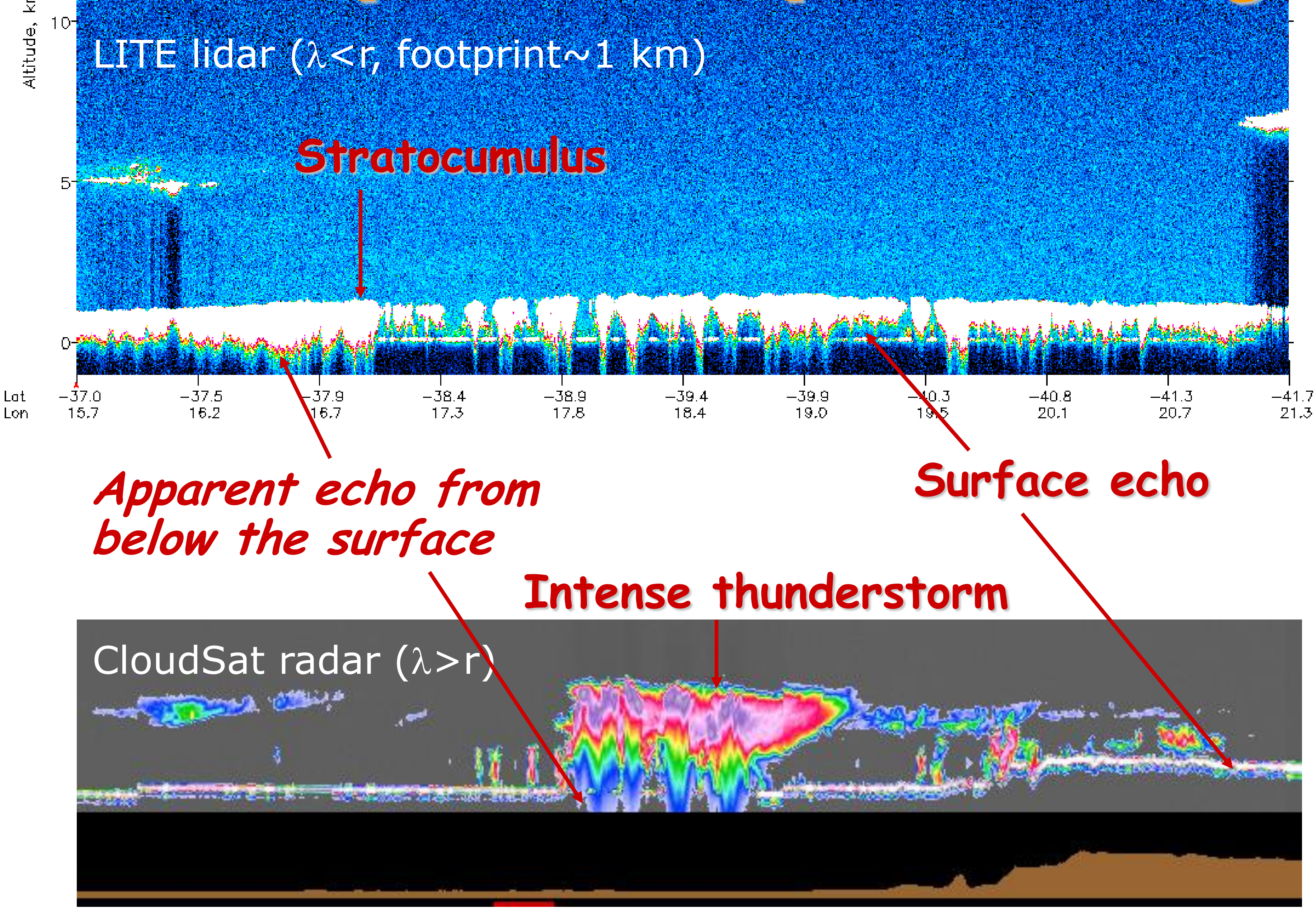




- Distance Δx determined by mean free path of light in the cloud of particles
- Most of the literature concerns this case

- Distance Δx determined by field-of-view of transmitter and receiver: transmitted light returning outside the FOV is not detected
- Lower Δx implies higher enhancement, but overall multiple scattering return is lower
- Very little literature

Examples of multiple scattering

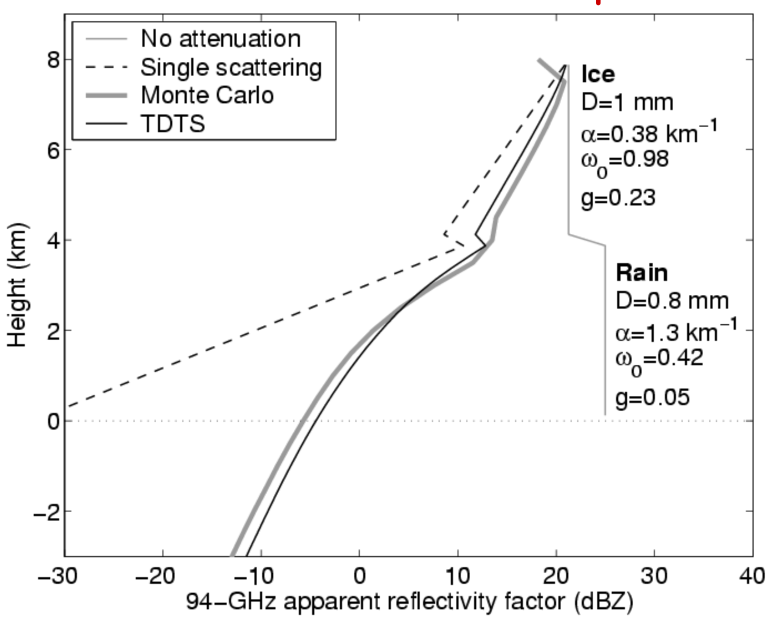


Fast multiple scattering model

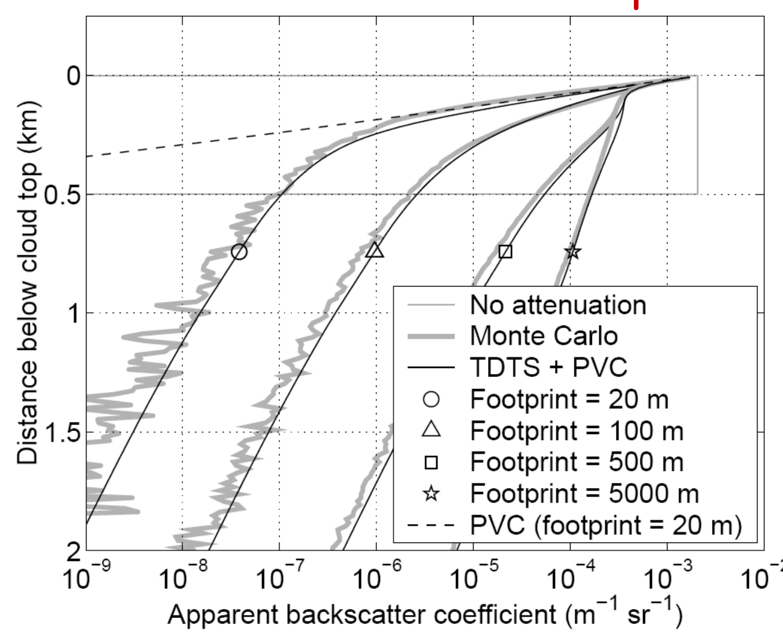
Hogan and Battaglia (JAS 2008)

- Uses the *time-dependent two-stream approximation*
- Agrees with Monte Carlo but ~10^7 times faster (~3 ms)
- Used in CloudSat operational retrieval algorithms

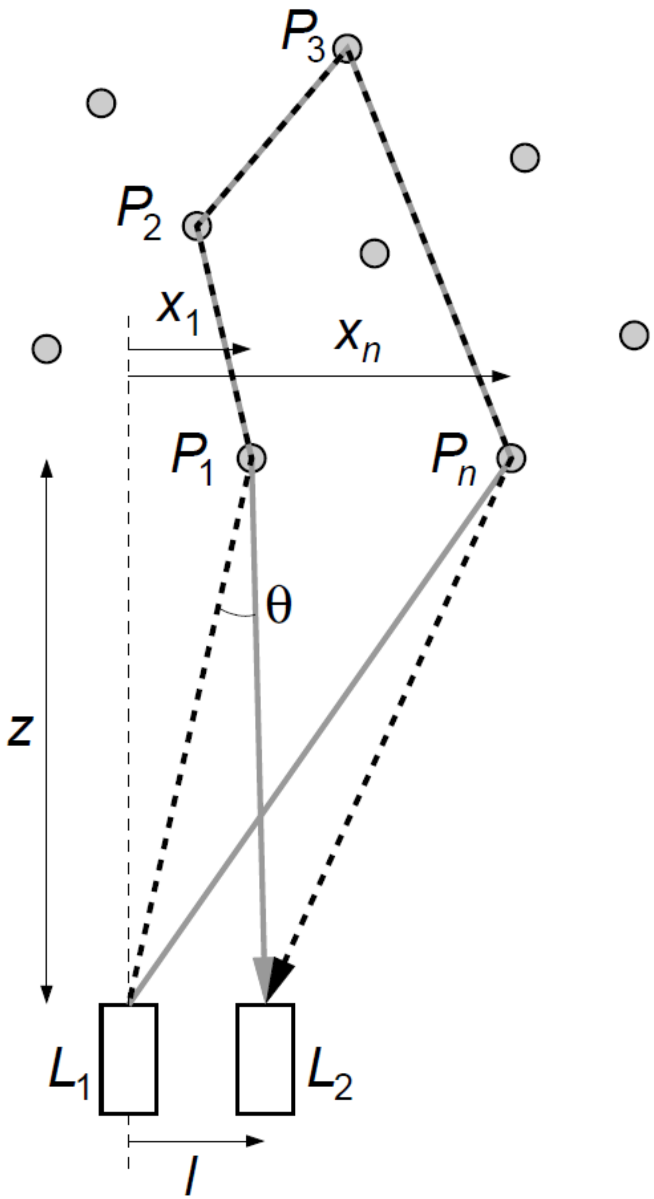
CloudSat-like example



CALIPSO-like example



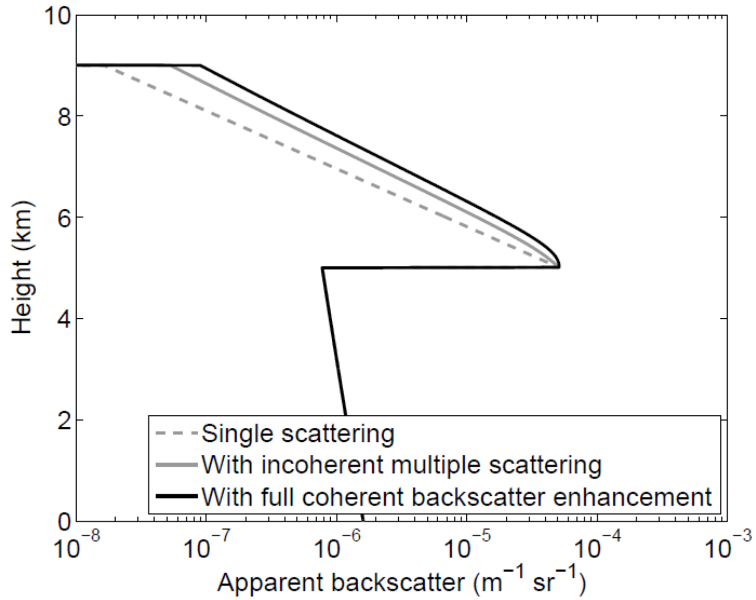
Moving platform: satellite radar or lidar



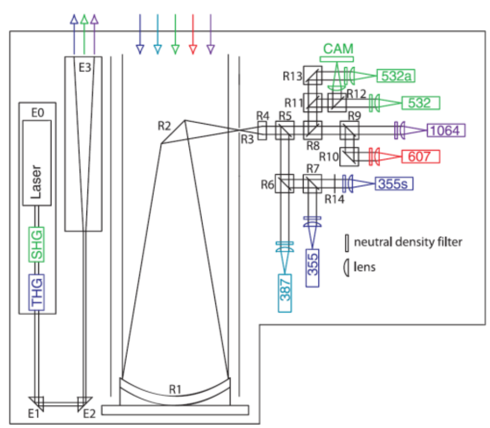
- Consider CloudSat & Calipso satellites at altitude of 700 km and speed of 7 km s^{-1}:
  - Distance travelled between time of reception and transmission is l = 33 m
  - So θ = 47 μrad
- Assuming most multiply scattered light escapes field-of-view, σ determined by receiver footprint on the cloud
- CloudSat: σ = 450 m, λ = 3 mm so
  - CBE = 10^{-9}
- Calipso: σ = 100 m, λ = 0.5 μm so
  - CBE = 0
- *Effect can be safely ignored for satellites*

Stationary platform: ground-based

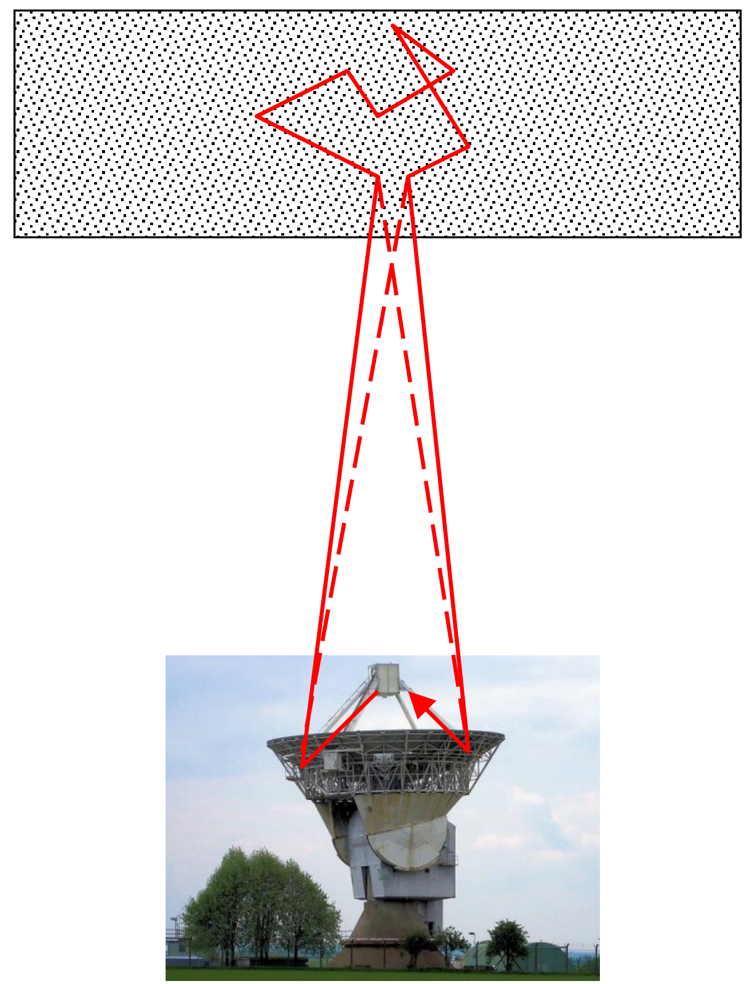
- θ = 0 so automatically we have CBE = 1 and the multiply scattered return is doubled?



- But most lidars are bistatic!



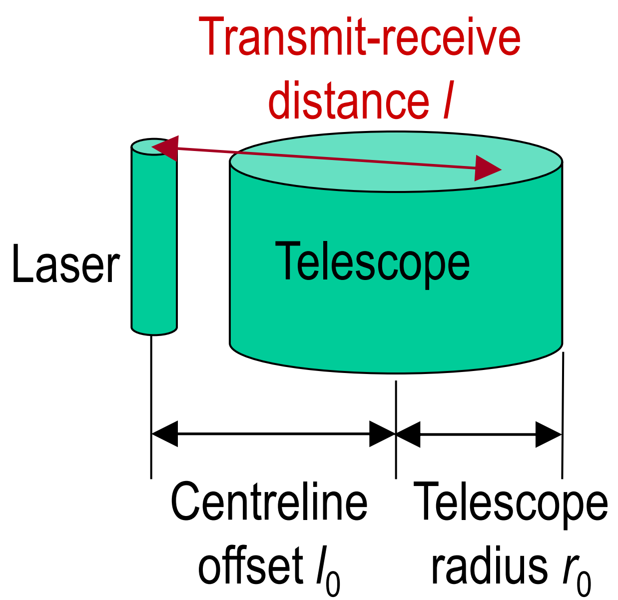
- And even for a monostatic radar, can't radiation be received from a different part of the antenna to where it was transmitted?

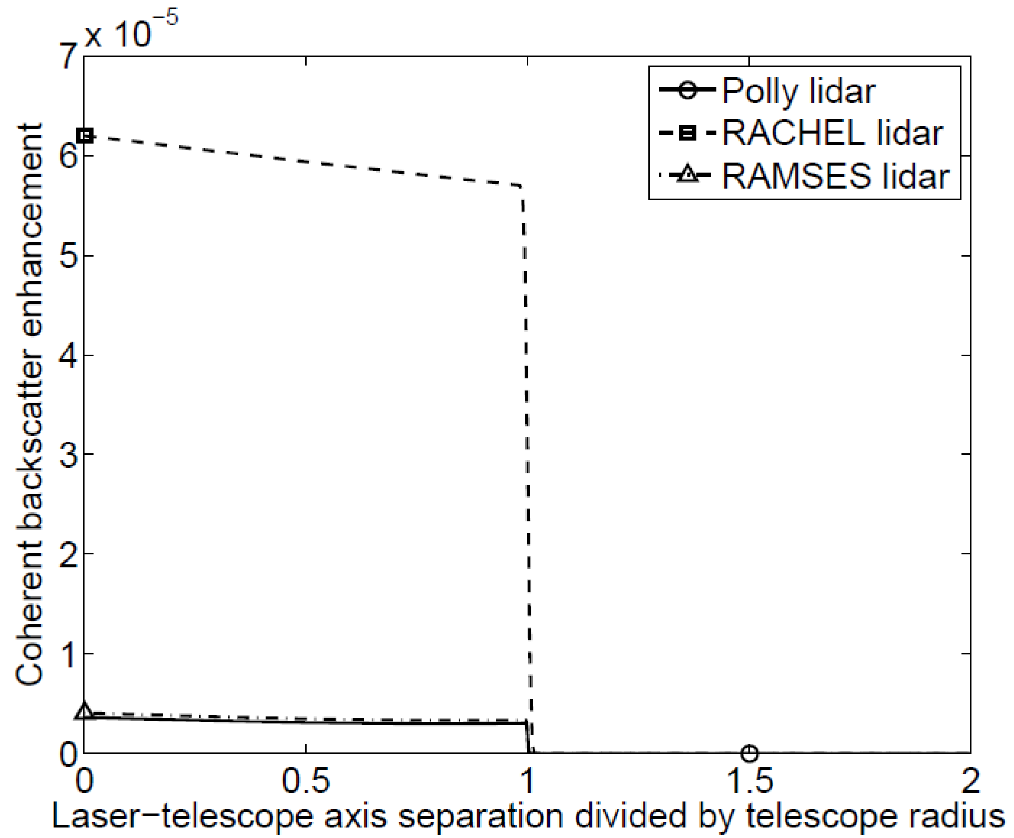


Stationary lidar

- Treat laser as infinitesimal point and integrate over all possible transmit-receive distances *l:*

$$\mathrm{CBE} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(l)P(\Delta x)\widetilde{\mathrm{CBE}}(\Delta x, l)\, d\Delta x\, dl$$





- *CBE is close to zero!*

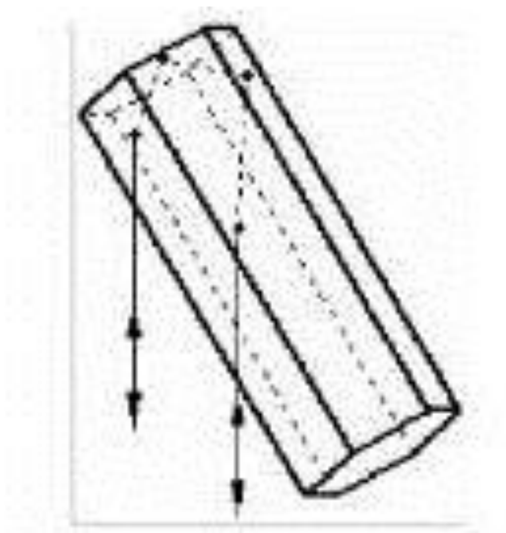
Stationary radar

- Again need to integrate over all possible transmit-receive distances

Transmit-receive
distance l

- Complication is that beam pattern is diffraction limited: field-of-view (and hence σ) is dependent on transmit-receive distance…
- Stationary radar should have fixed value of CBE, probably around 0.5, but theory needs to be developed

Coherent backscatter enhancement for particles?



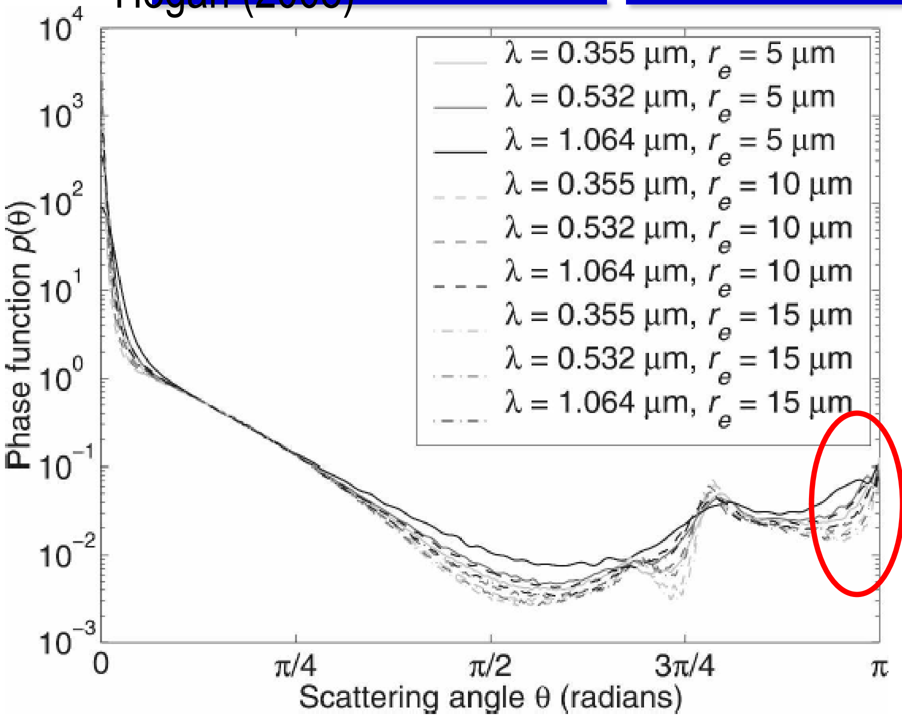
- Predictions for light scattering by particles r>>λ:
  - Coherent effects should double the backscatter due to light rays involving more than 1 reflection
  - The angular width of the enhancement is of order

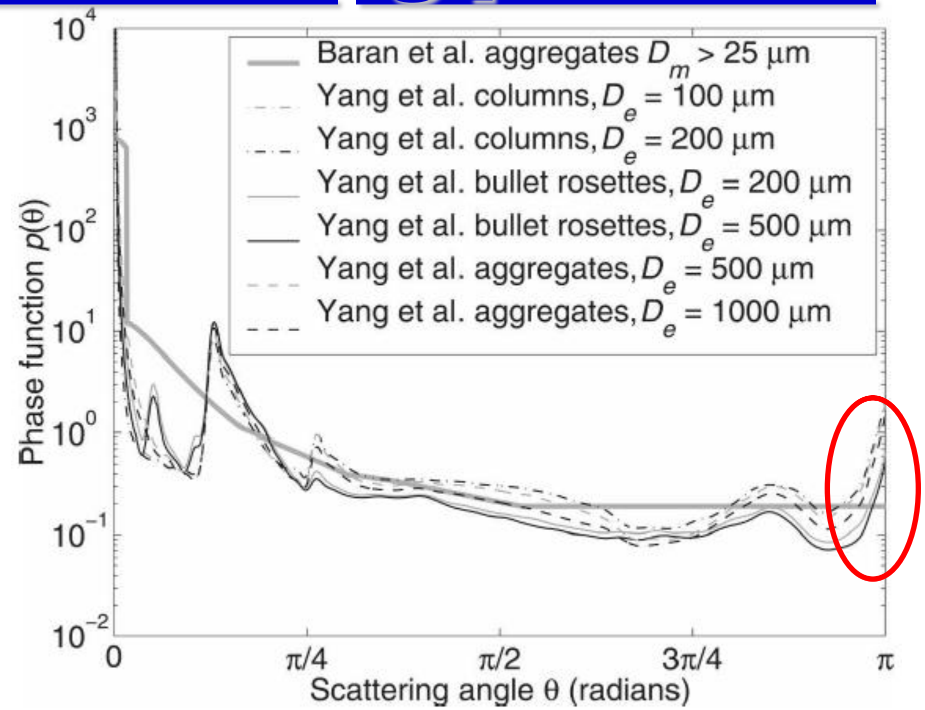
$$\theta_0 = \frac{\lambda}{2\pi\sigma}$$

    where σ is the RMS distance between entering and exiting light rays.
  - For 100 μm particles and λ=0.5 μm, θ_0 is 0.05 degrees
  - Ray tracing codes are unlikely to capture this effect, but explicit solutions of Maxwell's equations will (Mie, DDA)

Standard scattering patterns

Hogan (2008)





Liquid spheres (Mie theory)

- Width of backscatter peak is dependent on particle size
- *Is this peak underestimated by ray tracing?*

Ice particle phase functions

- Ping Yang's functions show size-independent enhancement of a factor of ~8
- Anthony Baran's functions are flat at backscatter
- *Neither seems right; do we need to model CBE?*

Summary

- A new equation has been proposed for backscatter cross-section of ice aggregates observed by radar
  - Much higher 94-GHz backscatter for snow than "soft spheroids"
  - Aggregate structure exhibits a power law with a slope of -5/3: why?
- Coherent backscatter enhancement (CBE) has been estimated for spaceborne and ground-based radar and lidar:
  - From space it can be neglected because of the distance travelled between transmission and receiption
  - From the ground, the finite size of a lidar laser/telescope assembly also makes CBE negligible
  - CBE can be significant for a ground based radar, and the exact value should be instrument/wavelength independent for monostatic radars, but value has not yet been rigorously calculated
- Coherent backscatter enhancement should apply to individual particles
  - Do current ray tracing algorithms underestimate backscattering by a factor of two because of this?