Advances in Polarimetric Radar Scattering

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Overview

- Scattering and Polarimetry
- Revisiting the Fredholm Integral Method
- Geometric Polarimetry
- Conclusions

Part I Scattering and Polarimetry

Spatial anisotropy of hydrometeors implies polarization sensitivity.

Weather radars exploit flattening of raindrops as a function of size to help with identification and estimation of drop size distribution.

Ice crystals come in many habits, many of which are plates or columns, which can exhibit alignment relative to gravity or electric fields.

For weather radar, the melting layer in which snowflake aggregates contain air, ice and meltwater and is complex to model.

What to measure?

The first polarimetric weather radars measured the horizontal and vertical reflectivities,

Zh ,v

A new observable Zdr was introduced:

This has the advantage (because Shh and Svv are generally highly correlated) of being much more stable and measurable to ~0.1 dB

Circular Polarization Radars

In Alberta, Canada, McCormick and Hendry developed techniques in observing storms with circular polarization radar.

CDR appeared to have a great disadvantage in being extremely sensitive to propagation effects – at S-band this was mainly due to differential phase.

Differential Phase - KDP

Targets are generally observed after the radar pulse has passed through (mainly)rain and the H/V anisotropy of the raindrops induces a differential phase in the forward (and return) paths.

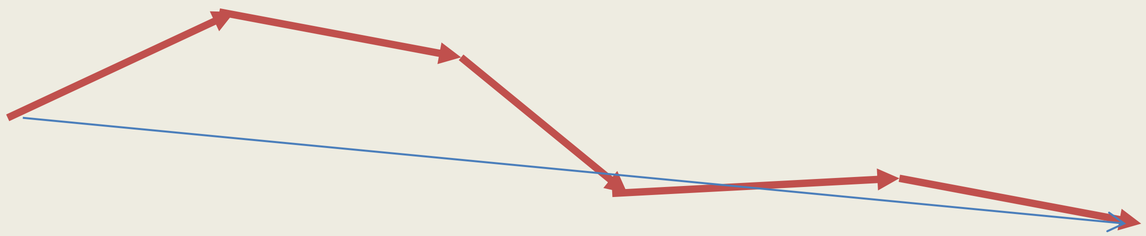
But this is a unitary transformation, and invertible if full polarmetric data is recorded.

At S-band this was easy because the Shh and Svv amplitudes were in an almost real ratio.

Degree of Polarization

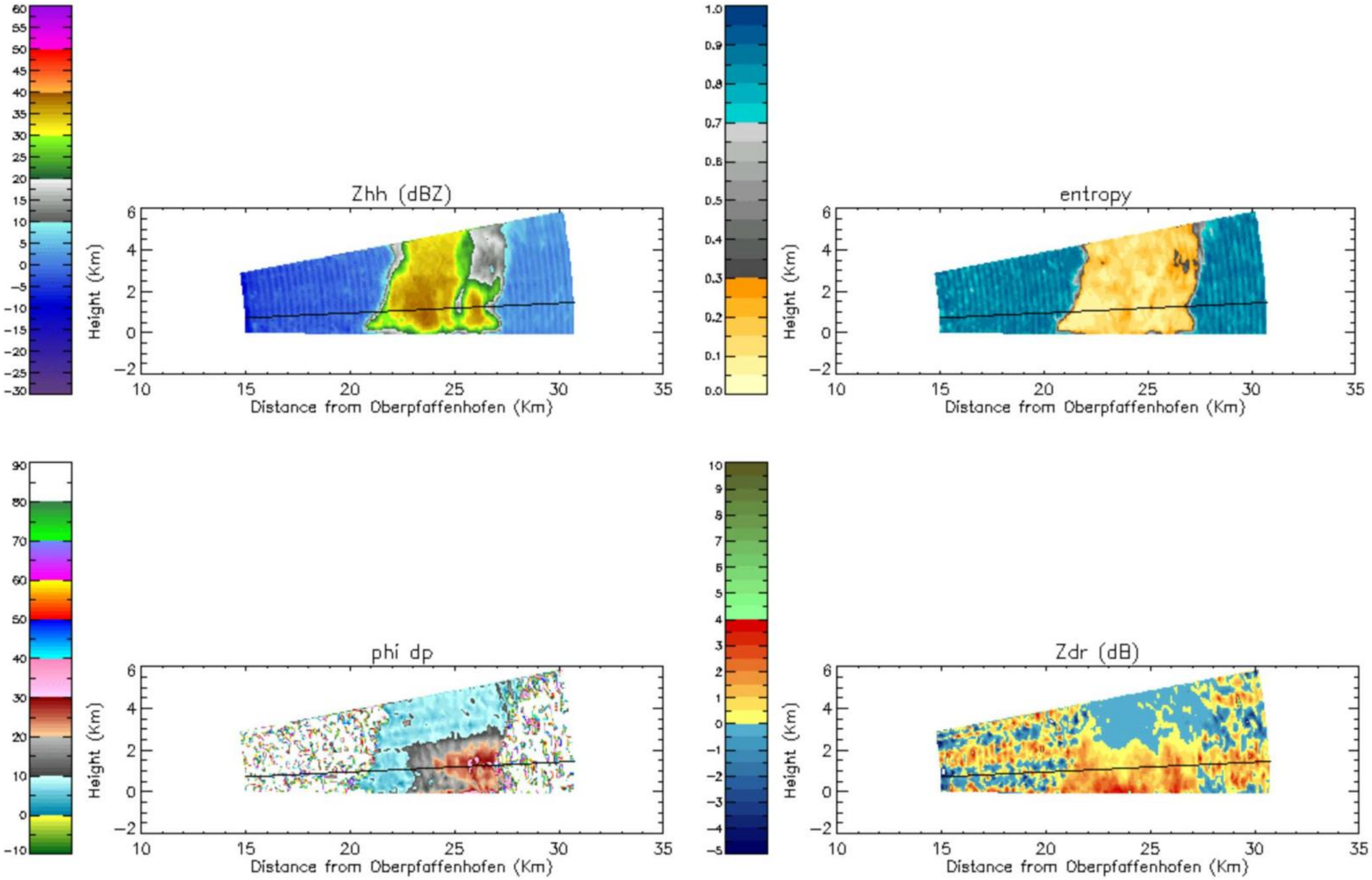
While a volume target returns a random signal, the responses in rain for different polarizations are highly correlated. This implies that the degree of polarisation is almost unity.

Loss of degree of polarization requires directional randomness in the Stokes vectors...

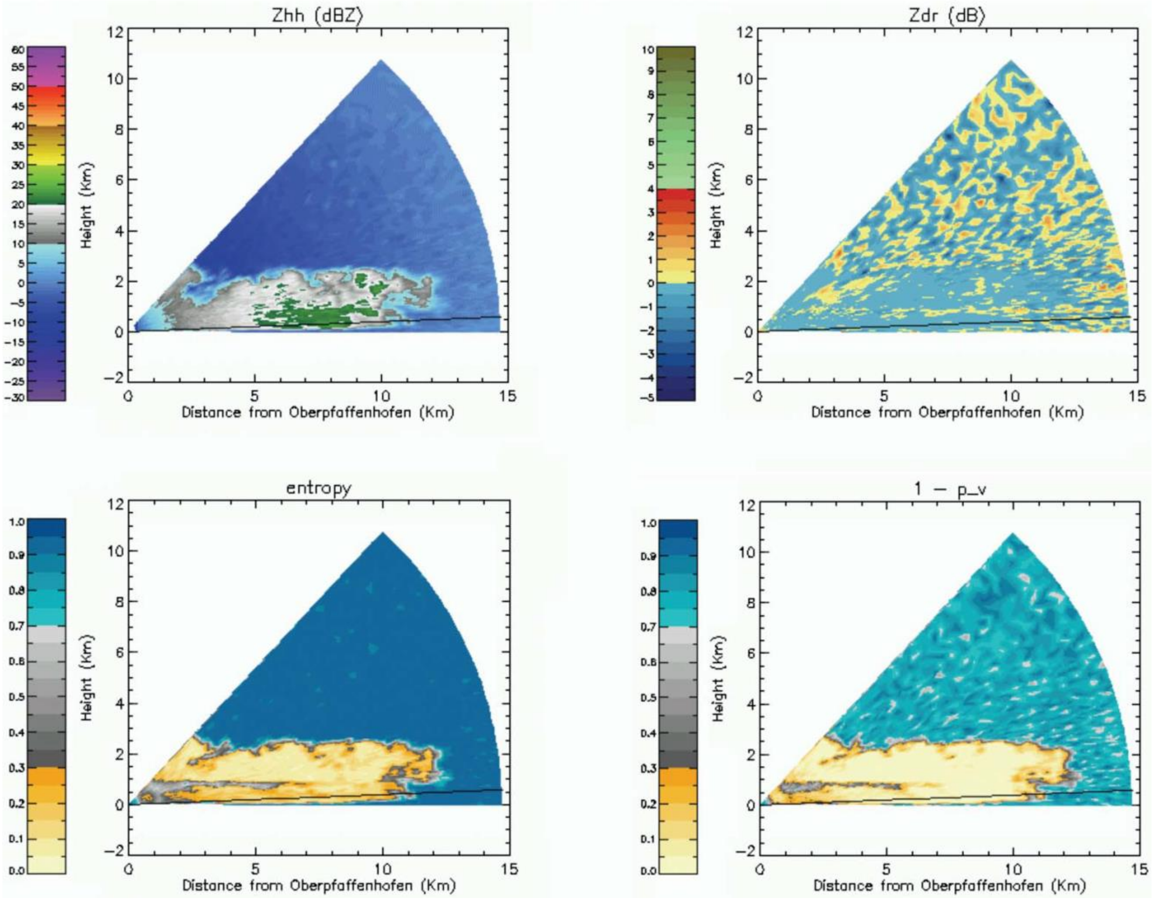


This relates to pairwise summation over scatterers, and to the heterogeneity of the scatterer distribution.

Convective precipitation event
(Galletti et al 2009)



Stratiform precipitation event



Fredholm Integral Method

In 1970's satellite telecommunications were in their heyday – it was not fully anticipated how soon and how much optical fibre communications would take over.

National PTT's funded research on attenuation by rain and its management in microwave up- and down links.

A number of approaches to calculating the scattering amplitudes were taken.

Methods for calculating Hydrometeor scattering amplitudes

- Point matching (Morrison & Cross)
- T-Matrix Extended Boundary Condition (Waterman)
- Fredholm Integral Method (Holt, Uzunoglu and Evans)

In the weather radar and microwave propagation fields, the T-Matrix approach supported by freely available codes is now ubiquitous.

Advantages of the FIM

- Does not require explicit solution of boundary conditions
- Inherently very stable
- Non-iterative
- Solution satisfies a Schwinger variational principle
- Is extensible to inhomogeneous dielectrics

Outline of the method

$$E(\mathbf{r}) = J_i \exp(i\mathbf{k.r}) + \int_V G(\mathbf{r}, \mathbf{r'})\gamma(\mathbf{r'}).E(\mathbf{r'})d^3\mathbf{r'}$$

$$\gamma(\mathbf{r}) = \varepsilon_r(\mathbf{r}) - 1$$

G is the dyadic free space Green’s function and Ji is the incident field unit dyadic.

The Green’s function is singular and the aim is to transform to the Fourier domain and deal with the singularity analytically.

Fourier domain

$$E(\mathbf{r}) = \int C(\mathbf{k}_2) \exp(i\mathbf{k}_2 . \mathbf{r}) d^3\mathbf{k}_2$$

$$U(\mathbf{k}_1, \mathbf{k}_2) = \int_V \gamma(\mathbf{r}) \exp(-i(\mathbf{k}_1 - \mathbf{k}_2).\mathbf{r}) d^3\mathbf{r}$$

$$W(\mathbf{k}_1, \mathbf{k}_2) = \mathbf{1}\int_V \varepsilon(\mathbf{r})\gamma(\mathbf{r}) \exp(-i(\mathbf{k}_1 - \mathbf{k}_2).\mathbf{r}) d^3\mathbf{r}$$

$$K(\mathbf{k}_1, \mathbf{k}_2) = W(\mathbf{k}_1, \mathbf{k}_2) - Z(\mathbf{k}_1, \mathbf{k}_2)$$

$$Z(\mathbf{k}_1, \mathbf{k}_2) = \frac{1}{8\pi^3 k_0^2} \lim_{\varepsilon \to 0} \int \frac{p^2 d^3\mathbf{p}}{p^2 - k_0^2 - i\varepsilon} \quad .$$

$$(\mathbf{1} - \hat{\mathbf{p}}\hat{\mathbf{p}}) U(\mathbf{k}_1, \mathbf{p}) U(\mathbf{p}, \mathbf{k}_2)$$

Solution

The exact solution is that of the coupled Fredholm integral equations,

$$\int K\left(\mathbf{k}_1,\mathbf{k}_2\right)C\left(\mathbf{k}_2\right) = J_i U\left(\mathbf{k}_1,\mathbf{k}_i\right)$$

$$f\left(\mathbf{k}_s,\mathbf{k}_i\right) = \frac{1}{4\pi} J_s . \int U\left(\mathbf{k}_s,\mathbf{k}_2\right)C\left(\mathbf{k}_2\right)d^3\mathbf{k}_2$$

In practice, the integrals are replaced by weighted summations and the problem reduces to an approximate solution by solving matrix equations.

The kernel is non-singular and the method known to be remarkably stable

Extension to Bodies of Revolution

The original code of Holt was restricted to ellipsoids

This arose because it was desired to expand the spatial integrals in a Neumann series with Gegenbauer polynomial terms

If, however, we perform the integral for K using the spherical analogue of trapezoidal integration we re free to use arbitrary shapes.

In particular, for bodies of revolution, we can calculate the U integrals in cylindrical coordinates. The z-integral becomes a 1-D Fourier transform.

Extension to Prisms

The extension to homogeneous prisms is straightforward

The 2-D Fourier transform of a basal slice can be computed analytically by transforming to the line integral of a fictitious vector field with constant curl.

Hollow columns and prisms can also be handled as contributions to the U-integral only occur in the dielectric.

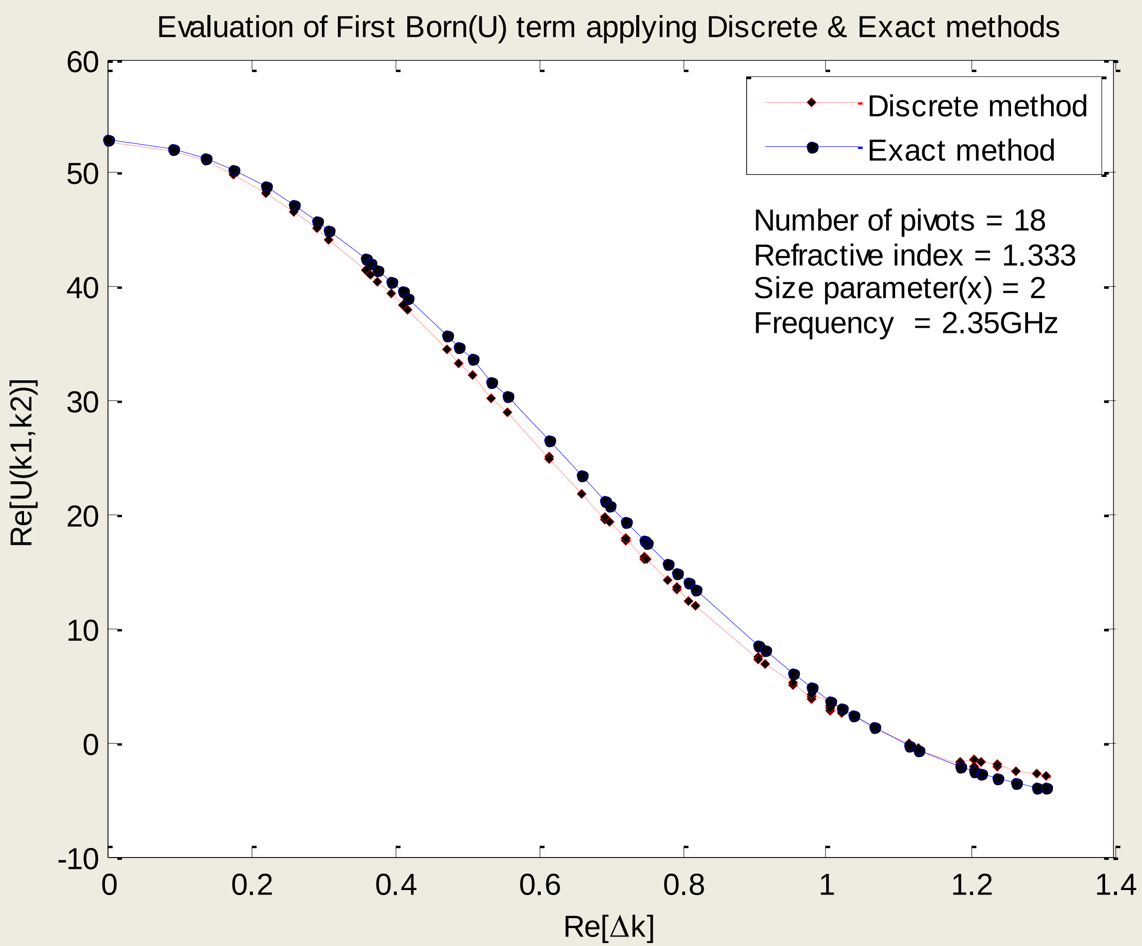
Extension to Inhomogeneous Dielectrics (with Felix Nghobiga)

The case of snowflakes is complex computationally because there are two dielectric constants besides air.

We have effective medium theories for two-part dielectrics where it can be determined one is an inclusion in the other.

There is as yet no known unique effective medium mixing formula when there are three components.

Example calculation of the U-integral Mie scatterer



Approaches to calculating the 2nd Born term

$$Z\left(\mathbf{k}_1,\mathbf{k}_2\right)=\frac{1}{8\pi^3 k_0^2}\lim_{\varepsilon\to 0}\int\frac{p^2 d^3\mathbf{p}}{p^2-k_0^2-i\varepsilon}\left(\mathbf{1}-\hat{\mathbf{p}}\hat{\mathbf{p}}\right)U\left(\mathbf{k}_1,\mathbf{p}\right)U\left(\mathbf{p},\mathbf{k}_2\right) .$$

This looks innocuous enough, but the magnitude p integrates from zero – we need to be able to cast the integral as a closed contour integral. U does not normally exhibit the required symmetry.

Ellipsoids

For ellipsoids, Holt, Uzunoglu and Evans (1978) exploited the Neumann series,

$$\frac{J_\nu(\varpi)}{\varpi^\nu} = 2^\nu \Gamma(\nu) \sum_{m=0}^{\infty} J_{\nu+m}(k) J_{\nu+m}(p) C_m^\nu(\cos\varphi)$$

$$\varpi^2 = |\mathbf{k} - \mathbf{p}|^2 = k^2 + p^2 - 2kp\cos\varphi$$

Products of the Bessel functions in p integrate the singularity analytically

For Discretized volume

In the case of a discrtized volume, the integral for each cell is related to a definite integral

$$\int_0^{\infty} \frac{\exp(ipz)}{p^2 - 1 - i\varepsilon} dp \to \sim \int_0^{\exp(i\omega)\infty} \frac{\exp(ipz)}{p^2 - 1} dp$$

$$\omega = \frac{1}{2} \ln(1 + i\varepsilon)$$

This can be expressed in terms of the Struve function of order 1/2

Problems arising

Whereas with a uniform dielectric, the interior field expansion has wave vectors lying on a 'shell' the interior field in a complex dielectric is more complex.

In principle, we need a full 3-D Fourier expansion of the polarizatibility tensor – making the Matrix problem of large N and dense.

There is no reason why this cannot be done, and this case is no worse than DDA in complexity, while involving no more approximation than discretization.

Can the expansion be truncated?

Looking at the solution to the scattering amplitude,

For moderately sized scatterers the scattered fields are the ones with k < inverse scale of dielectric.

The integral is a convolution – to get output terms of k, even if there are large amplitudes in U(k’) for large k’ do we get any significant contributions for C(k’-k) U(k’). That is, is there any significant Bragg-like diffraction between the internal field and the Polarizability?

Part 2 – Overview of Geometric Polarimetry (with Laura Carrea)

Why are coherent polarization states represented as carriers of the unitary group SU(2)?

In quantum terms such a representation appears to be more appriate to spin ½ fermions, no spin 1 bosons.

This can be resolved by spinor representation when it is recognized that a second half-spinor is implicit but suppressed.

$$\psi_A = \Phi_{AA'}\vartheta^{A'} = \begin{pmatrix} 0 & \Phi_{01'} \\ \Phi_{10'} & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Polarimetric scattering as geometrical construction

We can exploit analogy between space-time algebra and projective geometry in homogenous coordinates to provide a (complex) geometric representation in coordinate (or Fourier) space of scattering representations.

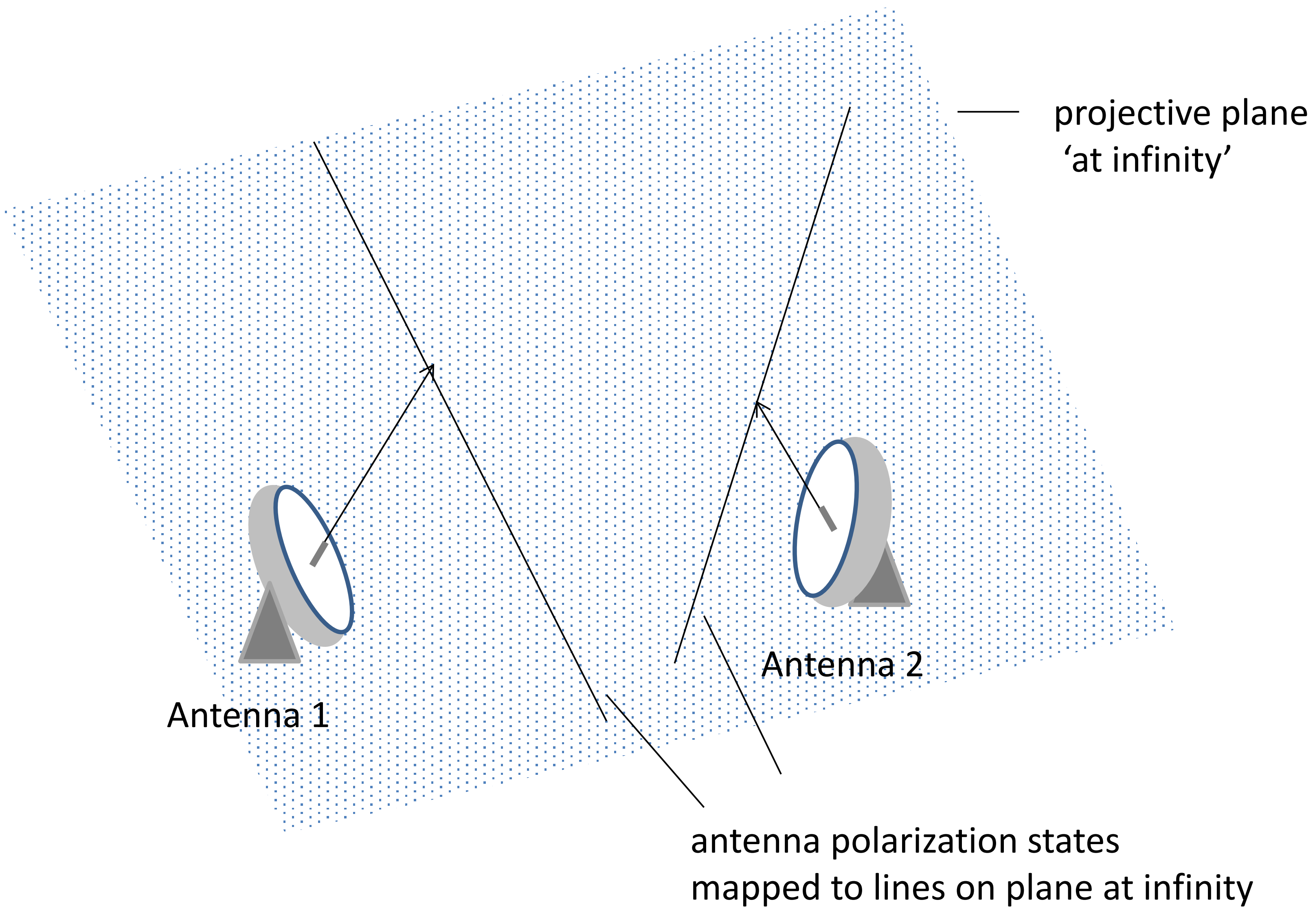
This fully reconciles the analytic signal representation and the configuration geometry of monostatic and bistatic scattering

Complex Bistatic Invariant

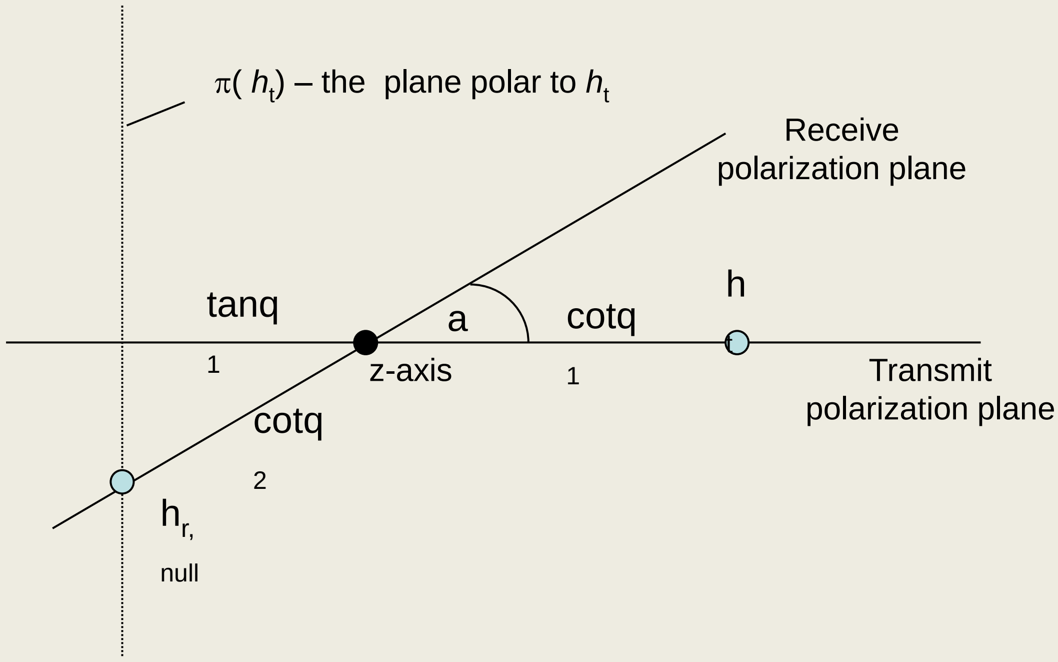
A new result emerging from this is that one can find a complex invariant associated with a bistatic scattering matrix.

For any transmit polarization there is always one unique receive polarization state which nulls the voltage. By projecting the two dipoles to a complex projective plane at infinity, this sets up a 1:1 correspondence which is related to a unique conic, which has a complex projective invariant.

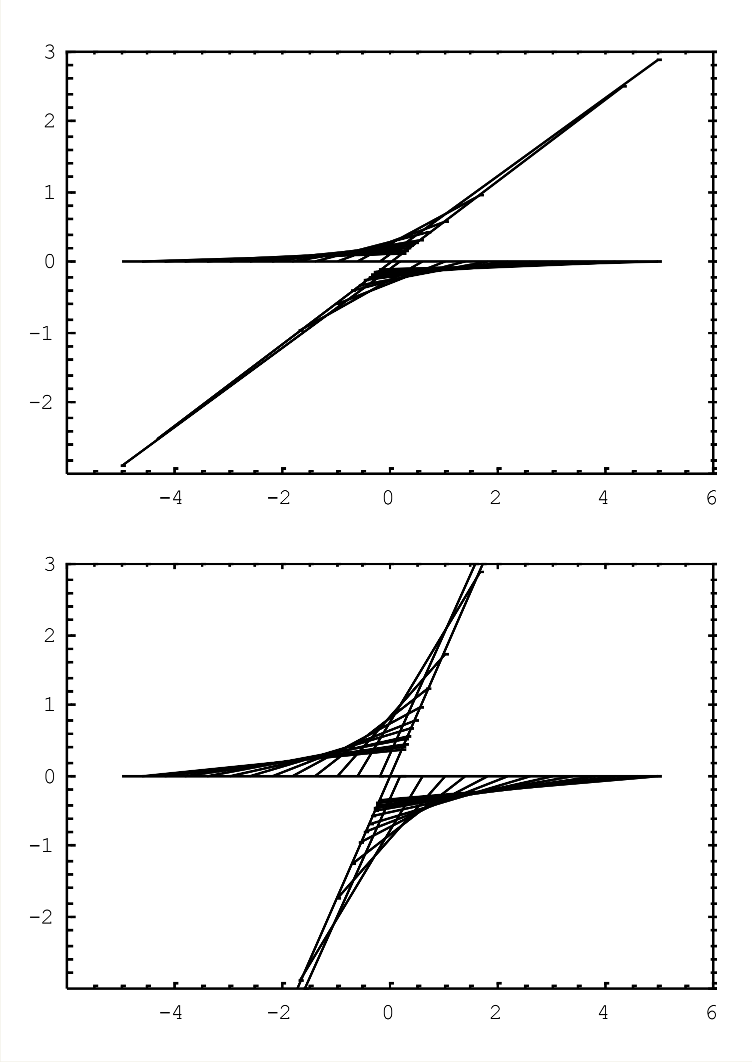
Projective construction



Corresponding states



Constructions for Spherical scatterer

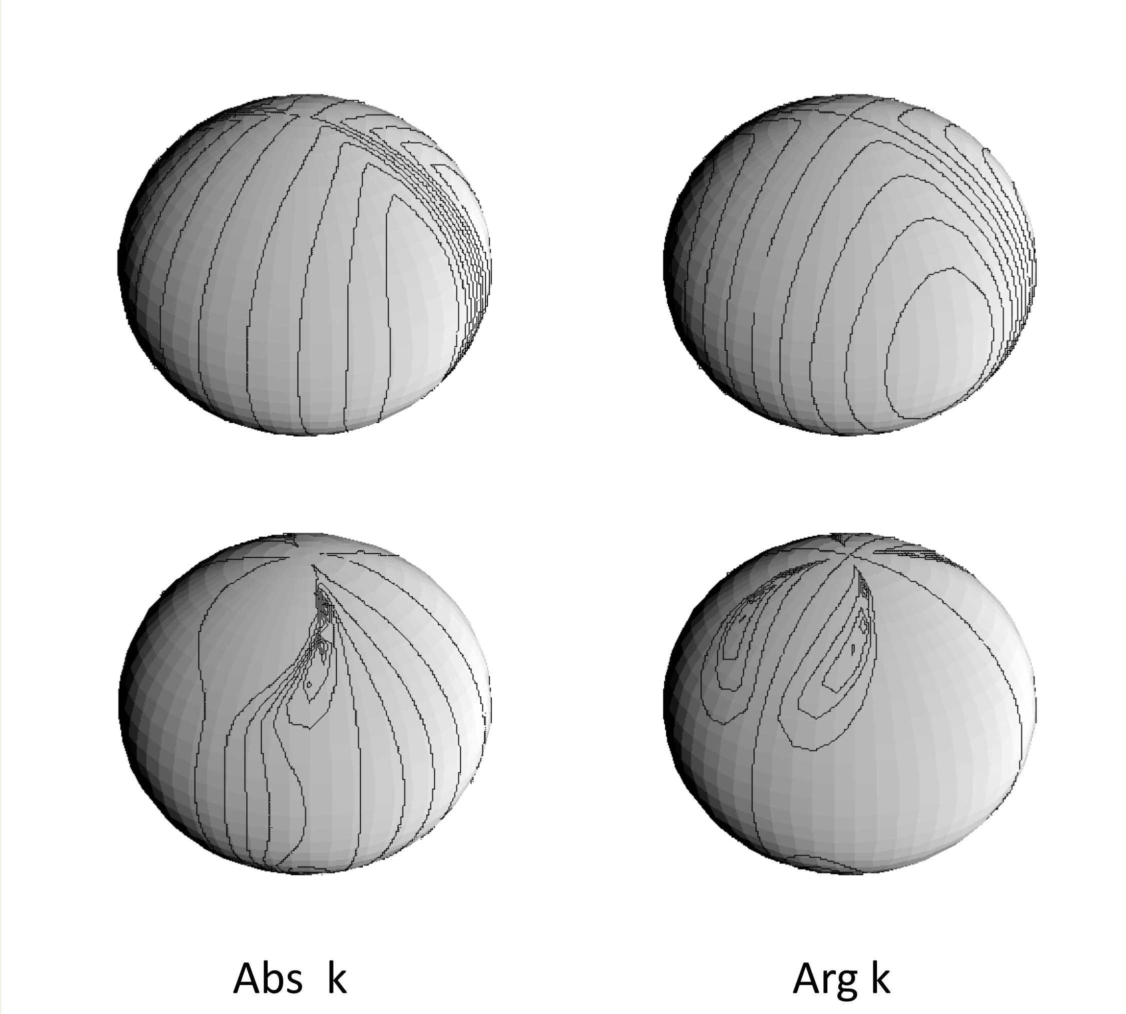


For spherical scatterers the conic is a real hyperbola.

(a) Bistatic angle = 30 deg

(b) Bistatic angle = 60 deg

Mapping the complex k-invariant

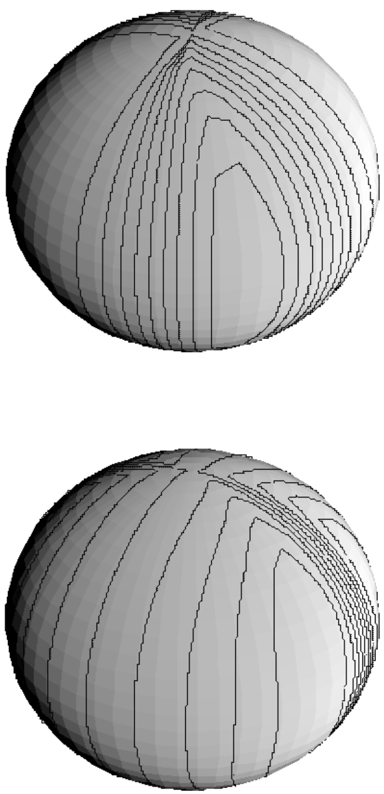


Oblate Rayleigh scatterers
Bistatic angle 60 deg

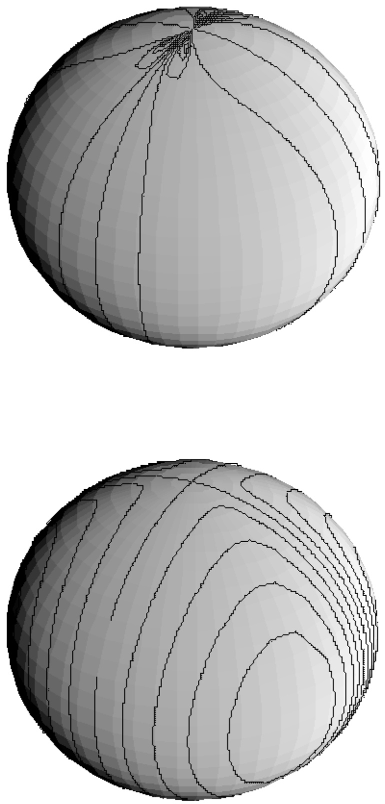
(a) Axial ratio 0.4

(b) Axial ratio 0.1

Oblate v prolate (0.1 axial ratio)



Abs k



Arg k

Bistatic angle 120 deg

(a) Oblate

(b) Prolate

Conclusions

- Polarimetry and anisotropy/shape
  - Alignment probed by linear polarization
  - Shape independently of alignment with Cpol
- Polarimetry and propagation effects
  - Alignment in volume gives rise to exploitable propagation effects
- Polarimetry and distributions
  - Heterogeneity in scatterers revealed in degree of pol
- Polarimetry and scattering geometry
  - Bistatic scattering may in future provide further degrees of freedom to explore particle morphology