Workshop:
Wave propagation in complex domains
Programme and Abstracts

University College London

Thursday 30th March 2017

This workshop is supported by EPSRC grant EP/P511262/1

https://www.ucl.ac.uk/~ucahdhe/WaveScatteringWorkshop.htm

1100	Welcome and introduction
1105	**David Hewett (UCL)**
	Scattering by fractal screens
1130	**Andrea Moiola (Reading)**
	Sobolev spaces on non-Lipschitz domains
1200	**Mikael Slevinsky (Manitoba)**
	A numerical method for the solution of wave scattering by fractal screens
1230	**Timo Betcke (UCL)**
	A computational framework for Calderon projectors for Maxwell problems
1300	Lunch
1345	**Ralf Hiptmair (ETH)**
	Boundary integral equations on complex screens
1415	**Xavier Claeys (UPMC)**
	Second kind boundary integral equation for multi-subdomain diffusion problems
1445	**Annalisa Buffa (EPFL)**
	Electromagnetic scattering with splines
1515	Tea/coffee
1545	**Euan Spence (Bath)**
	Acoustic transmission problems: wavenumber-explicit bounds and resonance-free regions
1615	**James Christian (Salford)**
	On Fresnel optics problems with fractal boundaries: apertures, screens, unstable resonators
1645	**Chris Westbrook (Reading)**
	Microwave scattering by atmospheric ice particles
1715	Drinks reception
1800	Free time for networking and hotel check-in
1900	Dinner

All talks will take place in lecture room **505** on the fifth floor of the Department of Mathematics at University College London on 25 Gordon Street (UCL Union Building), London WC1H 0AY.
Lunch and refreshment breaks will be served in the fifth floor common room (**502**).

Abstracts of the talks

Scattering by fractal screens

David Hewett, UCL

Joint work with Simon Chandler-Wilde and Andrea Moiola (Reading)

The analysis of acoustic and electromagnetic scattering by planar screens is a classical topic in wave propagation. But previous works assume that the screen is a smooth (e.g. Lipschitz or smoother) open subset of the plane. In this talk I will present recent work on developing well-posed boundary value problem and boundary integral equation formulations for the case where the screen is an arbitrary subset of the plane. In particular, the screen could be fractal, or have fractal boundary. Such problems are of interest in the study of fractal antennas in electrical engineering, and of fractal models of snowflakes/ice crystals in atmospheric physics. The roughness of the screen presents interesting questions concerning how boundary conditions should be enforced, and the appropriate function space setting. One observes some surprising solution behaviours - for example, waves can penetrate a sound-hard screen from which a set of measure zero has been removed, provided the fractal dimension of the removed set is large enough.

Sobolev spaces on non-Lipschitz domains

Andrea Moiola, Reading

Joint work with Simon Chandler-Wilde (Reading) and David Hewett (UCL)

Recent studies into scattering by fractal screens has motivated investigations into the properties of fractional Sobolev spaces (the classical Bessel potential spaces) on non-Lipschitz domains. We wanted to investigate the extent to which the properties of these spaces, and the relations between them, that hold in the well-studied case of a Lipschitz open set, generalise to non-Lipschitz cases. Our motivation is to develop the functional analytic framework in which to formulate and analyse integral equations on non-Lipschitz sets.

A numerical method for the solution of wave scattering by fractal screens

Mikael Slevinsky, Manitoba

Fractals have surprising analytical properties when considered as screens or apertures in wave scattering problems. An optimal numerical method abstracts the self-similarity of a pre-fractal screen and assembles boundary integral operators hierarchically. In this setting, a recursive block operator factorization exploits the bounded off-diagonal numerical rank of the integral operators. This hierarchical solver involves a pre-computation independent of the incident wave. Once an integral operator is factored, the solution for multiple incident waves exhibits even lower complexity, and may even be simulated live.

A computational framework for Calderon projectors for Maxwell problems

Timo Betcke, UCL

Calderon projectors are a fundamental building block for the design and analysis of integral equations in a wide range of settings and directly lead to preconditioners with excellent numerical properties. Within the Galerkin boundary element library BEM++ we have created an operator algebra to directly work with Calderon projectors and efficiently represent typical operations, including squaring, as a fundamental building block for complex formulations such as the implementation of multi-trace problems. We will give various interesting examples and show how this framework can be used to obtain stable software implementations of electric field, magnetic field and combined field integral equations.

Boundary integral equations on complex screens

Ralf Hiptmair, ETH Zürich

Joint work with Xavier Claeys (UPMC)

A complex screen is an arrangement of panels that may not be even locally orientable because of junction points (2D) or edges (3D). Whereas the situation of simple screens that are locally orientable Lipschitz manifolds with boundary is well understood, the presence of junctions compounds difficulties encountered in the definition of appropriate trace spaces and boundary integral operators associated with second-order elliptic PDEs outside the screen. Our approach to overcome these difficulties is guided by the intuition that a screen is the limiting case of a massive object, heavily relies on the understanding of trace spaces as quotient spaces of functions defined on the complement of the screen, and employs Green's formula to define duality pairings in trace spaces. Using these ideas and tools, we generalize the notions of trace spaces with boundary conditions and jump traces to complex screens. In the process we introduce layer potentials and boundary integral operators for scalar second-order elliptic PDEs and derive their properties like jump relations. Extensions to electromagnetic field equations will be discussed briefly.

Second kind boundary integral equation for multi-subdomain diffusion problems

XAVIER CLAEYS, UPMC

We study elliptic boundary value problems where the coefficients are piecewise constant with respect to a partition of space into Lipschitz subdomains, focusing on the case of jumping coefficients arising in the principal part of the partial differential operator. We propose a boundary integral equation of the second kind posed on the interfaces of the partition, and involving only one unknown trace function at each interface. We provide a detailed analysis of the corresponding integral operator, proving well-posedness. We also present numerical results that exhibit a systematically stable condition number for the associated Galerkin matrices, so that GMRES seems to enjoy fast convergence independent of the mesh resolution.

Electromagnetic scattering with splines

ANNALISA BUFFA, EPFL

I will discuss spline discretization techniques for the integral equation representing the scattering at a perfect conductor. I will discuss both wellposedness and preconditioning issues.

Acoustic transmission problems: wavenumber-explicit bounds and resonance-free regions

EUAN SPENCE, Bath

Joint work with Andrea Moiola (Reading)

We consider the Helmholtz transmission problem with one penetrable star-shaped Lipschitz obstacle. Under a natural assumption about the ratio of the wavenumbers, we prove bounds on the solution in terms of the data, with these bounds explicit in all parameters. These bounds then imply the existence of a resonance-free strip beneath the real axis. The main novelty of these results is that the only comparable results currently in the literature are for smooth, convex obstacles with strictly positive curvature.

On Fresnel optics problems with fractal boundaries: apertures, screens, and unstable resonators

JAMES CHRISTIAN, Salford

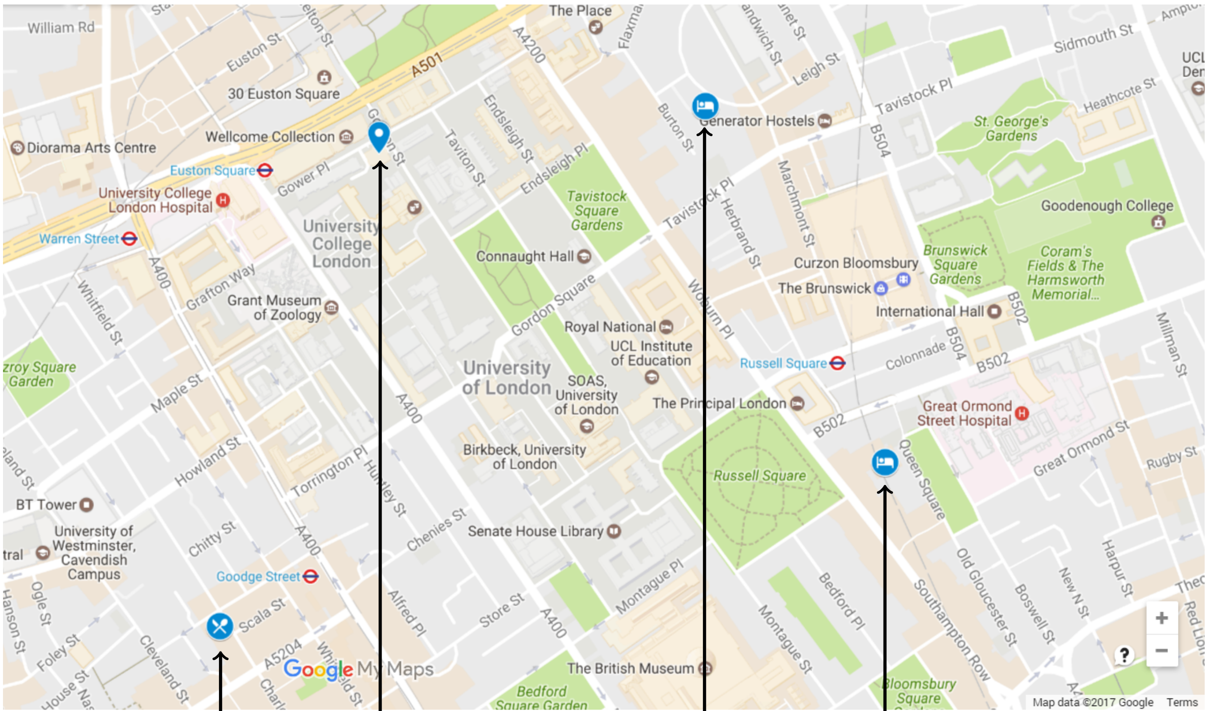
The diffraction of scalar plane waves by complex apertures is a well-known configuration in optics, with the overwhelming majority of experiments and analyses performed in the Fraunhofer (far field) limit. Here, we will show how the two-dimensional Fresnel integral can be deployed to solve a wide class of propagation problem involving irregular (or 'rough') hard-edged boundaries comprising hierarchies of straight-line segments. By transforming the area integral into a circulation, exact expressions can be obtained for near-field diffraction patterns from fractal-type apertures and, by exploiting Babinet's principle, for their complementary screens. These patterns are formulated mathematically using a generalization of Young's edge waves. The single-aperture Fresnel patterns provide essential basis functions for modelling geometrically-unstable optical resonators (linear systems with inherent magnification and periodic aperturing that naturally possess scale-free eigenmodes). We will conclude with a survey of recent results from virtual-source computations for cavities whose small feedback mirror has a boundary corresponding to increasing iterations of some classic self-similar curves (e.g. snowflakes, pentaflakes, Gosper islands, and Cesaro fractals).

Microwave scattering by atmospheric ice particles

CHRIS WESTBROOK, Reading

I will discuss how remote sensing by radar and microwave radiometers are used to measure the properties of clouds and precipitation in the ice phase, and explain how one of the key limitations to our capabilities for doing this results from poor knowledge of their scattering properties. I will present observational evidence that large snowflakes, which are aggregates of many individual crystals, have a fractal geometry, and what this means for their scattering properties and retrievals, and mention some new work on the accuracy of the Rayleigh-Gans-Debye method for computing scattering vs more sophisticated techniques.

Map



Navarro's restaurant,
67 Charlotte Street, W1T 4PH

Crescent hotel,
49–50 Cartwright Gardens,
WC1H 9EL

Department of Mathematics,
25 Gordon Street, WC1H 0AY

Imperial hotel,
61-66 Russell Square WC1B 5BB