Problem Sheet 8: Vectors

Assessed questions (1, 2, 3c) are marked with a star.

1.* (a) As mentioned in the lectures, **i**, **j** and **k** are the unit vectors in the direction of the x, y and z-axes respectively. Moreover, they are mutually perpendicular, i.e. if we choose any two of these three vectors, then the two vectors are perpendicular to each other. Hence, using the definition of the dot product, prove the following nine results:

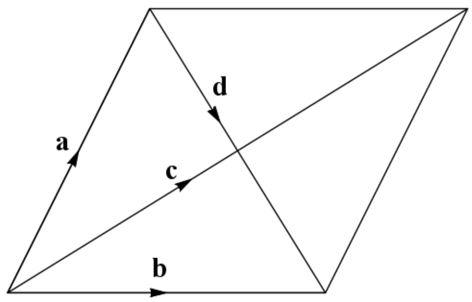
$$\begin{array}{llllll} \mathbf{i}.\mathbf{i} & =1, & \mathbf{i}.\mathbf{j} & =0, & \mathbf{i}.\mathbf{k} & =0, \\ \mathbf{j}.\mathbf{i} & =0, & \mathbf{j}.\mathbf{j} & =1, & \mathbf{j}.\mathbf{k} & =0, \\ \mathbf{k}.\mathbf{i} & =0, & \mathbf{k}.\mathbf{j} & =0, & \mathbf{k}.\mathbf{k} & =1. \end{array}$$

(b) Let

$$\begin{aligned} \mathbf{a} = & \quad (a_1, a_2, a_3) = & a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}, \\ \mathbf{b} = & \quad (b_1, b_2, b_3) = & b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}. \end{aligned}$$

be two vectors.
Using the result from part (a), multiply out their dot product $\mathbf{a} \cdot \mathbf{b}$ and show that the dot product equals $a_1b_1 + a_2b_2 + a_3b_3$.
Note: This problem is based on part of a 2011 exam question.

2.* A rhombus is a paralellogram with sides of equal length, just like this one:



Using vector methods, show that the diagonals **c** and **d** are perpendicular.

3. You are given three points $P = (1, -1, 2)$, $Q = (2, 0, -1)$ and $R = (0, 2, 1)$. Using vector methods...

   (a) Find the area of the triangle whose vertices are at these three points.

   (b) Find the angle between the sides PQ and PR.

   (c*) Find a unit vector perpendicular to $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

Due in by the start of the lecture on **Friday 9th December, 11am**. On the front page, please clearly write your name with <u>your surname underlined</u> and your student number. All pages must be **stapled together**, otherwise you will lose a mark!