Problem Sheet 7: Differential Equations

Assessed questions (1b, 1c, 2 and 3) are marked with a star. You are welcome to check your answers by differentiating.

1. Solve the following initial value problems. You must express your answers in the form $y = f(x)$.

   (a) $\dfrac{dy}{dx} = \dfrac{\sin x}{\cos y}, \quad y(0) = \pi,$

   (c*) $y\dfrac{dy}{dx} = e^{2x-y^2}, \quad y(0) = 0,$

   (b*) $x\dfrac{dy}{dx} + 3y = \dfrac{\sin x}{x^2}, \quad y(\pi) = 0,$

   (d) $(1+x)\dfrac{dy}{dx} + y = \sqrt{x}, \quad y(0) = 1.$

2.* The slope of a curve is $\dfrac{-x}{y}$, and it passes through the point with $y(0) = 1$. By setting up a differential equation and solving it, find the equation of that curve. Now describe the shape of the curve.

3.* **Maths applied:** Newton's Law of Cooling states that the surface temperature of an object changes at a rate that is proportional to the difference between the temperature of said object and the ambient temperature of the surroundings:

$$\frac{d\theta}{dt} = -k(\theta - T),$$

where $\theta(t)$ is the body temperature, T is the environment temperature and k is a positive constant.

   (a) Don't solve the ODE yet! What is the sign of $\dfrac{d\theta}{dt}$ when $\theta > T$ (i.e. the object is hotter than its surroundings)? Does the object warm up or cool down?

   (b) Now suppose $\theta < T$ (i.e. the object is colder than its surroundings). What is the sign of $\dfrac{d\theta}{dt}$? Again, does the object warm up or cool down?

   (c) Now find the general solution of the ODE.

   (d) I make a cup of tea. At first the temperature of the tea is 100°C, which is far too hot for me! So I decide to let it cool down before drinking. The tea will be just right to drink at 80°C. The room temperature is a chilly 16°C, $k = 0.03$ and the units for the time t is in minutes. How long will I have to wait before I can finally have my cuppa? Give your answer to the nearest minute.

Due in by the start of the lecture on **Friday 2nd December, 11am**. On the front page, please clearly write your name with your surname underlined and your student number. All pages must be **stapled together**, otherwise you will lose a mark!