

## Problem Sheet 5: Integration

1. Evaluate the following integrals by substitution:

(a)  $\int x^3 \sqrt{x^2 + 2} \, dx$

(d)  $\int \cos(3x - 7) \, dx$

(b\*)  $\int x^2 \cos(x^3) \, dx$

(e)  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$

(c)  $\int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) \, dx$

(f\*)  $\int e^{(x+e^x)} \, dx$  (use  $u = e^x$ )

2. Evaluate the following integrals. You are advised (but not required) to use a trigonometric/hyperbolic substitution:

(a)  $\int \frac{2}{x^2 - 6x + 10} \, dx$

(b\*)  $\int \frac{1}{\sqrt{-x^2 + 4x - 3}} \, dx$

(c)  $\int \frac{2}{\sqrt{4x^2 - 9}} \, dx$

3. Evaluate the following integrals using any method you find appropriate. Hints are given in square brackets.

(a\*)  $\int \tan x \, dx$   $\left[ \tan x \equiv \frac{\sin x}{\cos x} \right]$

(b)  $\int \sin^5 x \, dx$   $[\sin^2 x \equiv 1 - \cos^2 x]$

(c)  $\int \frac{1}{x \ln x} \, dx$   $\left[ = \int \frac{\frac{1}{x}}{\ln x} \, dx \right]$

4.\* **Maths applied:** Suppose that a company's marginal revenue from the manufacture and sale of whisks is

$$\frac{dr}{dx} = 2 - \frac{2}{(x+1)^2}, \quad (1)$$

where  $r$  is measured in thousands of pounds and  $x$  in thousands of units. How much money should the company expect from a production run of  $x = 3$  thousand pans? To find out, integrate the marginal revenue from  $x = 0$  to  $x = 3$ .

Due in by the start of the lecture on **Friday 18th November, 11am**. On the front page, please clearly write your name with your surname underlined and your student number. All pages must be **stapled together**, otherwise you will lose a mark!