Problem Sheet 2: More Differentiation

Assessed questions are marked with a star.

1.* Using Leibnitz's rule, find $\frac{d^6}{dx^6}\left((x^2-1)\cos 2x\right)$.

2. A cycloid is a curve given by $x = t - \sin t$, $y = 1 - \cos t$. It appears in problems relating to mechanics, cosmology and architecture.

   (a*) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ using parametric differentiation.

   (b*) A curve is said to be convex if $\frac{d^2y}{dx^2}$ is always positive, while it is concave if $\frac{d^2y}{dx^2}$ is always negative. Using your result from part (a), is the cycloid convex, concave or neither?

   (c) Find the Cartesian coordinates for the stationary point on the cycloid. Is it a maximum, minimum or point of inflection? Justify your answer.

3.* A function is given by $y = \frac{x+2}{x^2+4x+5}$. Find its stationary points, and classify each of them as a maximum, minimum or point of inflection. Does this function have any vertical or horizontal asymptotes? Now sketch the curve.

4. (a*) The curve described by the equation $y^2(2-x) = x^3$ is called the cissoid of Diocles. Find the equations for the tangent and normal to this curve at the point where $x = 1$ and y is positive.

   (b) The normal to the curve $x^2 + 2xy - 3y^2 = 0$ at $(1,1)$ intersects the curve at one other point. Find the coordinates of this point.

5. **Maths applied:** A piston moves straight up and down so that its position at time t seconds is

$$s(t) = A\cos\left(2\pi bt\right), \tag{1}$$

with positive constants A and b. The value of A is the amplitude of the motion, while b is the frequency (number of times the piston moves up and down each second).

(a) Compute the first, second and third derivatives of s with respect to t (these are the velocity, acceleration and jerk of the piston respectively).

(b) What effect does doubling the frequency have on the piston's velocity, acceleration and jerk respectively?

Due by the start of the lecture on **Friday 21st October, 11am.** On the front page, please clearly write your name with your surname underlined and your student number. All pages must be **stapled together**, otherwise you will lose a mark!