Marriage Market Implications of the Demographic Transition

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Abstract

We present international evidence on the marriage market implications of cohort size growth, and set out a theoretical model of how marriage markets adjust to imbalances. Since men marry younger women, secular growth in cohort size in the second phase of the demographic transition worsens the position of women. This effect has been substantial in many Asian countries earlier, and continues to be true in sub-Saharan Africa. Secular decline in cohort sizes in phase III, as is happening in East Asia, improves the position of women. We show that the age gap at marriage will not adjust in order to equilibrate the marriage market in response to persistent imbalances, even though it accommodates transitory shocks. This is true under transferable utility even if age preferences are relatively minor, as well as non-transferable utility.

JEL Categories: J12, J13, J16

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1 Introduction

The demographic transition has major implications for the position of women within the family, and in society. Fertility declines as parents trade "quantity for quality" in their children, and investment in human capital becomes increasingly important. Doepke and Tertilt (2009) argue that increasing requirements for human capital investments is what made men want to relinquish their monopoly on property rights, and grant them to women. Fernandez (2010) sets out a model where the decline in fertility plays an important role in this, and uses data from the state-wise variation in the granting of property rights in the US to test this model. While the broad brush picture of the world in the last two centuries indicates a dramatic expansion in women's rights, within marriage and more generally in society, this trend has not been as universal or comprehensive as might be expected. The developing world offers a more nuanced picture. For example, in India, concerns have been raised about "missing women", and the trend of rising dowries, and the consequent financial "burden" that daughters impose on their parents. In sub-Saharan Africa, polygamy continues to be an important phenomenon despite modernization and economic development.

The paper focuses on one implication of the demographic transition, for the marriage market and for the consequent position of women. This appears not to have been previously been appreciated, although demographers have pointed out some aspects of this picture. Our focus is on the rate at which marriage cohorts grow, in the different phases of the demographic transition.\(^1\) Given that men marry younger women, systematic growth in cohort sizes implies that each cohort of men is matched with a larger cohort of women, giving rise to a marriage squeeze on women, i.e. their excess supply.\(^2\) Similarly, systematic decline in cohort sizes imply a reverse marriage squeeze, where men are in excess supply. The consequent marriage market effects are substantial. Deferring the detailed evidence to section II, the stylized facts are as follows (see Fig. 1). In phase I, cohort sizes are basically static or growing very slowly. In phase II, with the decline in mortality, especially infant mortality, cohorts grow rapidly, at 2-3% per annum in many developing countries. With an age gap at marriage of 4-5 years, this translates into a 8-15% increase in the effective supply of women, as compared to phase

\(^1\)While the demographic transition is normally phrased in terms of population growth, the relevant variable for the marriage market is the rate of growth in marriage cohort size, which differs somewhat from population growth. Empirically, the two measure can diverge quite substantially.

\(^2\)Demographers use the term marriage squeeze (see Akers, 1977; Schoen, 1984) to denote a marriage market imbalance, due to the effective excess of women or men. This may arise due to shocks to the marriage market sex ratios, e.g. due to wars or due to variations in the sex ratio at birth, or transitory shocks to cohort size, due to baby booms or famines, give rise to imbalances. Bergstrom and Lam (1989a) demonstrate that there are large variations in cohort size in 19th century Sweden. In the Chinese famine of 1959-61, cohort sizes fell by 75% (Brandt, Siow and Vogel, 2008) The focus of the present paper is on the effects of systematic growth or decline in cohort sizes.
I. In phase III, cohort size growth becomes either zero or significantly negative, at -1 to -2% per annum, implying an increase in the supply of men of 4-10% as compared to phase I. As compared to phase II, the change can be as much as 25%.

These changes in the effective excess supply of women have major implications for the balance of power between the sexes and for the allocation of resources within the household. Angrist (2002) and Chiappori, Fortin and Lacroix (2001) find that important effects on female labor supply and household allocation even for significantly smaller changes in marriage market balance. In the Indian context, demographers such as Bhatt and Halli (1999) have argued that the marriage squeeze is responsible for the deterioration of the position of women in India, and replacement of the institution of bride price in many regions and communities by dowries (payment from the bride’s family to the groom). 3 Rao (1993) analyzes data on dowries from a sample of Indian villages and attributes the increase in dowries in India to the marriage squeeze.

Our focus in the paper is on marriage market adjustment mechanisms. One obvious mechanism is polygyny – i.e. polygamy in phase II and polyandry in phase III. This can be an important factor, especially in societies where legal and social sanctions against polygyny are absent. For example, in sub Saharan Africa, cohort size growth continues to be substantial, implying a significant excess of women in the marriage market relative to the number of men. This could be an important factor explaining the surprising persistence of polygamy in the face of modernization. Similarly, in Punjab in India, the persistent excess supply of women has been a historical feature, due to male biased sex ratios. Polyandry has been historically prevalent in the Punjab, but has declined in the last century. Our argument suggests that cohort size growth may have more than offset male biased sex ratios, and that this may have played an important role. Polygyny may not, however, be very appealing for the more abundant sex, and in this case, it utility consequences for the abundant sex may not be very different from non-marriage.

If we abstract from polygyny, the marriage market is an assignment market, in the sense of Shapley and Shubik (1972) since a man can be matched to at most one woman. The key potential margin of adjustment is via the age gap at marriage. If the age gap at marriage were to adjust in response to marriage market imbalances, they could be reduced, if not eliminated. Indeed, a reduction in the age gap reduces both the excess of women in phase II and the excess of men in phase III. While the age gap at marriage tends to fall in the process of development, as women become more educated, our question is whether, the age

3Anderson (2007) examines the time path of dowries in response to an one-off increase in the number of women. She finds that that dowries rise and then fall. This analysis is less relevant to the question of sustained population growth.
gap falls *endogenously*, in response to marriage market imbalance. Bergstrom and Lam (1989a, 1989b) and Brandt, Siow and Vogel (2008) have examined the effects of temporary shocks to birth cohort size, using data from Sweden and from the Chinese famine of 1959-61 respectively. These papers use a transferable utility assignment model in the tradition of Becker (1981), and find that marriage markets display considerable flexibility – the age gap at marriage adjusts in order to accommodate large shocks to cohort size. The empirical findings of Brandt et al. are particularly noteworthy, given that the Chinese famine reduced cohort sizes by 75%.

In the light of these results, one might be sanguine about the how the marriage market adjusts to secular growth in cohorts, in the demographic transition. However, our analysis shows that there is a considerable difference between the marriage squeeze due to temporary shocks, and that arising from *systematic growth* in cohort size. We find that with transferable utility, the age gap at marriage is *completely* insensitive to systematic marriage market imbalances. Moreover, this is true no matter how small the relative weight of age preferences is relative to other considerations such as wealth or attractiveness. If age related preferences play a relatively minor role in men’s and women’s utility, a social planner could greatly reduce imbalances by mandating a reduced age gap, with only minor negative utility consequences. However, such an outcome would not arise in a marriage market equilibrium. Thus our analysis highlights that the demographic transition has important implications for the balance of power and resources between the sexes. It suggests that the cause of women’s equality has been hindered by high cohort size growth, but that this may now be reversed in many countries, as cohort growth becomes negative.

Why is it that the age gap adjusts to transitory shocks in cohort size, but not to secular growth? Suppose that a shock reduces the size of the cohort indexed by date $t$. If the age gap at marriage is $\tau$, this results in an excess supply of men at date $t - \tau$ (and an excess supply of women at $t + \tau$). Thus attractive men from date $t - \tau$ now become available. Women from cohorts that are adjacent to that affected by the shock (i.e. $t - 1$ and $t + 1$) now have more attractive options, and would prefer to match with these men rather than their "customary" match. Thus the age gap adjusts, and the magnitude of adjustment is greater the smaller the weight of age related preference relative to other considerations. We should therefore expect age gap adjustments, the more important considerations such as wealth, education, attractiveness or agreeability are to marrying individuals, as compared to considerations of age.

Consider now secular positive growth, at some rate $g$. The equilibrium age gap will be

\[^{4}\text{While our focus is on transferable utility, with non-transferable utility the equilibrium age gap may increase, aggravating the imbalance.}\]
the preferred age gap \( \tau^* \),\(^5\) no matter how large \( g \) is, and no matter how much of a marriage market imbalance that this causes. (Indeed, the equilibrium age gap will be \( \tau^* \) even if growth were to be negative.) The basic reason is that there is no margin for profitable adjustment available. In a steady state equilibrium, there will be an excess of women in every cohort. So the unmatched women from cohort \( t \) are as attractive as unmatched women from any other cohort \( \bar{t} \). This implies that there is no incentive to choose a woman other than from ideal age gap, \( \tau^* \). Indeed, this is true no matter how weak the preferences for a specific age gap are, relative to other characteristics.

Our focus on marriage market flows differs from that of the large literature on the number of "missing women" in the population stock (see Sen (1990), Coale (1991) and Anderson and Ray (2010), for a very partial list). This is not to deny the importance of missing women overall, but rather because they are unlikely to have similar behavioral consequences.\(^6\)

This paper is related to two main strands of literature. First, there is the literature on marriage markets, following Gale and Shapley (1962) who assume non-transferable utility and Shapley and Shubik (1972) and Becker (1981), who assume transferable utility. More specifically, there is the literature on the marriage squeeze, by demographers as well as economists, including Akers (1977), Schoen (1983), Bergstrom and Lam (1989a, 1989b), Bhatt and Halli (1999), Anderson (2007), Brandt, Siow and Vogel (2008). Second, our work is related to a large volume of work on empirical work on the sex ratio. Apart from the literature on missing women, Neelakantan and Tertilt (2008) is particularly relevant since they focus on the marriage market.

The organization of the rest of the paper is as follows. Section 2 sets out the empirical evidence on cohort growth and marriage market balance in number of developing countries in Asia and Africa. Section 3 sets out a transferable utility model of the marriage market, and examines whether the marriage market permits an adjustment mechanism, via the endogeneity of the age gap. Section 4 considers non-transferable utility. The final section concludes. The appendix provides details of the formal proofs that are not dealt with in the body of the paper.

## 2 Marriage market balance

The effective supply-demand situation in the marriage market depends upon the sex ratio at birth, and upon rate of growth of female cohorts relative to the males that they are matched

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\(^5\)More precisely, \( \tau^* \) is defined as the age gap that maximizes the sum of payoffs of the two sexes.

\(^6\)One could argue that if women are more abundant, their greater political voice may ensure that public policy is more female friendly; however, this mechanism reinforces imbalances rather than correcting them.
with. Consider a society where cohort sizes are growing at an annual rate \( g \). Let the age gap at marriage, between men and women, be given by \( \tau \) years. The required number of boys, \( R \), per 100 girls, is given by

\[
R = 100(1 + g)^\tau \frac{\Delta G}{\Delta B} \frac{\lambda_G}{\lambda_B},
\]

where \( \Delta B \) is the survival rate for boys, between infancy and the age of marriage, and \( \Delta G \) is that for girls. \( \lambda_G \) (resp. \( \lambda_B \)) is the proportion of girls (resp. boys) who would like to marry.\(^7\) Thus if the marriage market is to be balanced, the actual number of boys per hundred girls, \( A \), must be close to or equal to \( R \). The difference \( G = A - R \) measures the extent to which there is an excess of men (or missing women) on the marriage market.

We now present estimates from a range of countries in Asia and Africa on \( g \), \( R \), and \( G \). This is based on UN data on the population by sex in the age group 0-4, at five yearly intervals starting 1950. Since cohort data is noisy, \( g \) for any year \( t \) is based on a regression estimate of the growth rate using observations at \( t \), \( t-5 \) and \( t+5 \). To estimate survival rates, we use infant mortality data for each year, since this is the main component of mortality. Since it is problematic to use observed data on the proportions marrying in order to estimate \( \lambda_G \) and \( \lambda_B \), we use the singles rate from a fixed year, around 1990 in most cases. The age gap is that from ....

The charts in figures 1-9 present the evolution of \( g \), \( R \) and \( G \) in three groups of countries: East Asia, South Asia + North Africa, and sub-Saharan Africa. Within each group we present a selection of countries. These figures show that:

- Cohort size growth was a significant factor in phase 2, resulting in a large excess supply of women.

- Phase 2 ended around the 1970s (i.e. among the marriage cohorts of the 1990s) in East Asia. In other countries of Asia such as India, Indonesia, Pakistan and Egypt, it continued into the 1990s, and appears to be ending only now. Thus the marriage markets of today are still in a situation of excess supply of women in these South Asia and North Africa.

- Phase 2 continues to be operative in current birth cohorts in most of sub Saharan Africa, with a value of \( G \) between -20 and -10 in the 2000 birth cohort.

- Negative cohort growth, and a large reverse marriage squeeze is operative in East Asia, South India and Tunisia.

\(^7\)This is valid if \( \lambda_G \) and \( \lambda_B \) are the proportions who desire marriage. However, in empirical work, demographers use the observed proportions, and we have some reservations about this, since the actual proportions of women and men that marry will reflect marriage market conditions, i.e. will be endogenous. As we shall see, our substantive conclusions are not much affected by this adjustment.
3 A model of the equilibrium age gap

Our empirical findings raise several questions. The first relates to the endogeneity of the age gap. Our estimates in the previous section assume that the age gap at marriage, $\tau$, is exogenous. Clearly, $\tau$ may change, for exogenous reasons. As women become more educated, their marriage age increases, while that of men does not increase by the same amount. Does the age gap at marriage adjust in order to help "clear" the marriage market? Economic "intuition" suggests that it should do so, at least on reading much of the literature on the subject. If this is the case, this could restore balance in the marriage market. A reduction in the age gap would help reduce the excess supply of women that is predicted to prevail in most parts of India. Similarly, even in China, the large excess in the actual number of boys could be reduced if the age gap fell, and even more if men began marrying older women.

This point assumes greater relevance in view of the existing literature on how the age gap responds to the marriage squeeze. The pioneering work in this regard is by Bergstrom and Lam (1989a, 1989b), who study 19th century Sweden, where there were large fluctuations in the effective sex ratios across marriage cohorts, due to the age gap at marriage and the fluctuations in the size of birth cohorts. They set out an assignment model of the marriage market with transferable utility, in the style of Shapley-Shubik (1972) and Becker (1981), and conclude that marriage markets showed considerable flexibility, in the sense that age gaps at marriage adjusted relatively easily in order to clear marriage markets. More recently, Brandt, Siow and Vogel (2008) use a similar transferable utility framework to examine the effects of the large shocks to cohort size arising from the Chinese famine of 1959-61. They find that participants in the marriage market showed considerable flexibility, so that the marriage rates of the cohorts who were matched with the famine affected cohorts were damaged, but not to the extent that one might imagine.

A positive age gap seems a very pervasive phenomenon – data from the United Nations (1990), from over 90 countries, show that the difference between the ages at first marriage for men and women is positive in every single country, for every time period. This suggests that both men and women prefer a positive age gap, and such preferences may have an evolutionary foundation. Kaplan and Robson (2003) present evidence from hunter-gatherer societies on age-productivity profiles. The productivity of women in gathering is relatively constant over time, while the productivity of men in hunting rises sharply from a low initial base, so that between the ages of 25 and 50, men produce a large surplus relative to their

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8If men preferred a positive age gap while women preferred a negative age gap, then one would expect a negative age gap to emerge in marriage markets where women are in short supply – see section 3.2.
consumption requirements. In addition, women reach peak fertility relatively quickly, and their fertility declines more rapidly than that of men. In this paper, we shall assume that the pervasiveness of a positive age gap is grounded in preferences, of men as well as women. This may have evolutionary foundations. With development, as women become more educated and take on paid employment, they will prefer to delay marriage, while the preferred age of marriage for men may not rise as much. Thus the preferred age gap may fall, for exogenous reasons. An alternative approach is set out by Bergstrom and Bagnoli (1990) who attribute the age gap to the differential roles of women and men in marriage, with the suitability of women for their role (the production of offspring) being revealed earlier, while that of men in their role (bread-winning) being revealed only later. We have also briefly examined how the age gap in the Bergstrom-Bagnoli model adjusts to imbalances, and find results that are broadly consistent with those reported here.

3.1 The model

Let time be discrete, and index it by the integers, positive and negative.\(^9\) We assume a continuum population, that grows at a constant rate \(g\) per year. At each date, the relative measure of women to men and women equals \(\bar{r}\), i.e. \(\bar{r}\) is the sex ratio at birth or within each cohort. There are two dimensions to an individual’s marriage market characteristics and preferences. Age constitutes a "horizontal dimension of differentiation, and "quality" is the vertical dimension. Turning to age first, assume that men and women have preferences defined on the age gap, i.e. the difference between the man’s age and the woman’s age. Both sexes prefer a positive age gap. Specifically, assume that men have single-peaked utility function, \(U(\tau)\), defined on the age gap at marriage, \(\tau\), that reaches a unique maximum at \(\tau_B > 0\). Similarly, women also have single peaked preferences \(V(\tau)\) with the ideal point \(\tau_G > 0\). We allow for the possibility that \(\tau_B > \tau_G\) or the reverse. Define \(S(\tau) \equiv U(\tau) + V(\tau)\), and let \(\tau^*\) be the age gap that maximizes \(S(\tau)\). We assume that \(\tau^*\) is unique, as will generically be the case. Single peakedness implies that \(\tau^*\) lies between the ideal points of the two sexes. We normalize \(S(\tau^*)\) to zero, so that \(S(\tau) < 0\) for any \(\tau \neq \tau^*\).

The second dimension is match quality, that depends upon the characteristics of both individuals in the pair. Assume that the quality of a man in any cohort, \(\varepsilon\), is distributed with a continuous, strictly increasing cumulative distribution function \(F(\cdot)\) on \([\varepsilon_{\min}; \varepsilon_{\max}]\). Similarly, the quality of a woman, \(\eta\), is distributed with with a continuous, strictly increasing cumulative distribution function \(G(\cdot)\) on \([\eta_{\min}; \eta_{\max}]\). If a man of type \(\varepsilon\) is matched with woman of type \(\eta\), total match quality equals \(q(\varepsilon, \eta)\). The function \(q\) assumed to be increasing in

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\(^9\)Since we are focusing on steady states, we assume that time extends indefinitely, backwards and forwards, thus avoiding any initial date effects that would preclude existence of steady state.
both arguments, weakly supermodular.

The overall payoff from a match characterized by the triple \((\varepsilon, \eta, \tau)\) is given by \(Q(q(\varepsilon, \eta), S(\tau))\), where \(Q\) is strictly increasing in both the vertical dimension of quality \(q(.)\) and the horizontal dimension, \(S(\tau)\). Without loss of generality, we can normalize \(Q(q(.), 0) = q(.)\).

Since we assume transferable utility, the total match payoff \(Q\) that any matched pair achieve can be divided between the two parties in any way that they choose. The payoffs from being single are \(\bar{u}\) for any man and \(\bar{v}\) for any woman. For simplicity, we assume that these reservation values do not depend on type of the individual.

Let \(M\) be the set of men and let \(W\) be the set of women. Each element of \(M\) has a pair of characteristics, \((\varepsilon, t)\), i.e. has a quality type and a cohort date. A matching is a function \(\phi : M \rightarrow W \cup \{\emptyset\}\) that satisfies the following properties. First, if \(w \in W = \phi(m)\), then \(w\) is not the image of any other \(m' \in M\) under \(\phi\), i.e. any woman can be matched only to a single man. Second, if \(M'\) is any finite Lebesgue measure subset of \(M\), such that \(\phi(M') \subset W\), the Lebesgue measure of \(M'\) equals that of the set \(\phi(M')\). That is, any matching must be measure preserving. Associated with any matching is a payoff allocation, \((u : M \rightarrow \mathbb{R}, v : W \rightarrow \mathbb{R})\). The payoff allocation must be feasible, i.e. for any matched pair \((m, w)\), \(u(m) + v(w) = Q(\varepsilon(m), \eta(w), \tau(m, w))\), and for any unmatched individual, the payoff allocation equals his/her payoff from being single.

We now turn to equilibrium matchings. We focus on \textit{stable, steady-state} matchings. Stability implies that \(u(m) \geq \bar{u}\) and \(v(w) \geq \bar{v}\), i.e. any individual gets at least the payoff from being single. Secondly, for any \((m, w)\) who are not matched to each other, \(u(m) + v(w) \geq Q(\varepsilon(m), \eta(w), \tau(m, w))\). That is, the payoff allocations of this pair must be weakly greater than the total match value that they could generate by matching together, since otherwise, this pair would have a profitable pair-wise deviation from the matching. A \textit{stable payoff allocation} is the allocation associated with a stable match.

Our final requirement is that we shall restrict attention to steady states. The steady state requirement is that every couple that matches will have the same age gap, \(\tau\). This requirement, of course, applies only to equilibrium matches – deviating couples could have an arbitrary age gap. In a steady state with age gap \(\tau\), men born at date \(t\) will be matched with women born at \(t + \tau\), and the ratio of the latter to the former will equal \(r = \bar{r}(1 + g)^{\tau}\). Since the matching must be measure preserving, if \(r > 1\), a there will be more unmatched females than males in every cohort, while if \(r < 1\), the reverse is the case.

\textbf{Assumption 1} \(Q(\varepsilon, \eta, \tau)\) is a continuous and differentiable in \((\varepsilon, \eta)\). \(Q(\varepsilon_{\min}, \eta_{\max}, \tau) < \bar{u} + \bar{v}\) and \(Q(\varepsilon_{\max}, \eta_{\min}, \tau) < \bar{u} + \bar{v}\).

Assumption 1 implies that the worst quality man (or woman) can never be profitably matched.
We now define the following matching and payoff allocation – this will be the unique stable payoff allocation under assumption 1. Let the age gap of every matched pair equal \( \tau^* \), and let \( r^* = \bar{r}(1 + g)^{\tau^*} \) be the consequent sex ratio in the marriage market. Let \( \tilde{\varepsilon} \) and \( \tilde{\eta} \) denote the lowest matched types of men and women respectively, where \( \phi(\tilde{\varepsilon}) = \tilde{\eta} \). \( \tilde{\varepsilon} \) and \( \tilde{\eta} \) are the unique solutions to the pair of equations:

\[
1 - F(\tilde{\varepsilon}) = r^*[1 - G(\tilde{\eta})] 
\]

(1)

\[
Q(\tilde{\varepsilon}, \tilde{\eta}, \tau^*) = \bar{u} + \bar{v}.
\]

(2)

Let matching be assortative, with \( \phi(\varepsilon) \geq \tilde{\eta} \) being defined, for \( \varepsilon \geq \tilde{\varepsilon} \) by

\[
1 - F(\varepsilon) = r^*[1 - G(\phi(\varepsilon))].
\]

(3)

Having defined payoffs for the marginal types in the market, we now turn to the rest. Let \( v(\eta) \) denote the payoff of a woman of type \( \eta \). Let \( \phi(\varepsilon) \) denote the match of a man of type \( \varepsilon \). Stability implies that \( \phi(\varepsilon) \) is the optimal choice for a man of type \( \varepsilon \), i.e.

\[
\arg\max_{\eta} [q(\varepsilon, \eta) - v(\eta)] = \phi(\varepsilon).
\]

This gives us the differential equation

\[
q_{\eta}(\varepsilon, \phi(\varepsilon)) = v'(\eta).
\]

The solution to this differential equation, in conjunction with the boundary condition, that \( v(\tilde{\eta}) = \bar{v} \), gives us the payoffs all types of matched women. The payoff to men equals the residual, \( q(\varepsilon, \phi(\varepsilon)) - v(\phi(\varepsilon)) \). The payoffs to unmatched men, of type \( \varepsilon < \tilde{\varepsilon} \), equals \( \bar{u} \). The payoff to unmatched women, of type less than \( \tilde{\eta} \), equals \( \bar{v} \). This defines a payoff allocation, for every type of man and woman. Let us denote this payoff allocation by \( (u^*, v^*) \). Note that this payoff allocation depends upon the sex ratio \( r^* \), which in turn depends upon the age gap. We are now in a position to state the main result of this section.

**Theorem 1** In any steady state stable match, the age gap equals \( \tau^* \), independent of \( g \). There is an essentially unique steady state stable matching and a unique steady state payoff allocation, \( (u^*, v^*) \).

**Proof.** Consider a steady state with age gap \( \tau^* \), with the matching \( \phi \) specified by 3 and
payoffs \((u^*, v^*)\) specified by the differential equation and boundary condition. Consider an arbitrary woman of type \(\eta\). Since her choice in the allocation maximizes her payoff given \(p(\varepsilon)\), she cannot do better by choosing any other man with age gap \(\tau^*\). Furthermore, she can also not do better by choosing any other age gap man and ensuring him a payoff \(p(\varepsilon)\) – since the total payoff will be lower than in the case where she chooses type \(\varepsilon\) with age gap \(\tau^*\), her own payoff will be strictly lower. Thus the allocation is stable.

To show that there is no other stable steady state, suppose that there is a steady state with an age gap \(\tau \neq \tau^*\). Suppose that a man of quality \(\varepsilon\) is matched with a woman of quality \(\eta\) under this matching, and earn payoffs \(u(\varepsilon)\) and \(v(\eta)\) respectively, where \(u(\varepsilon) + v(\eta) = Q(\varepsilon, \eta, \tau) < Q(\varepsilon, \eta, \tau^*)\). Then the man of quality \(\varepsilon\) in an arbitrary cohort can propose a match to the woman of the same quality \(\eta\), but with age gap \(\tau^*\), such that both can have a higher payoff. Thus the candidate allocation is not stable.

In finite economies with transferable utility, Shapley and Shubik (1972) show that finding a stable matching is equivalent to finding an assignment of men to women that maximizes total surplus. That is, a stable matching is the solution to a linear-programming assignment problem. Gretsky, Ostroy and Zame (1992) extend this result to continuum economies with a finite measure of agents. Our society has a countable infinity of dates, so that the total measure is not finite. In consequence, total surplus is not well defined, since the infinite sum of payoffs associated with an allocation is not well defined, the series being non-convergent. This precludes using the existing results, and our proof of existence and uniqueness must be done directly. That is, we explicitly construct a stable allocation, and verify that no other steady state is stable.

It is noteworthy that this invariance result does not depend upon how important \(S(\tau)\) is relative to \(q(\varepsilon, \tau)\) in the overall match payoff function \(Q(\cdot)\). For instance, considerations of age could be arbitrarily unimportant, so that deviations from \(\tau^*\) reduce \(Q\) very little. However, a reduction in \(\tau\) below \(\tau^*\) could significantly reduce the number of matches.

While this model predicts complete uniformity in the age gap at marriage, more realistic results can be obtained by allowing heterogeneity in preferences. If we assume that age preferences are uncorrelated with quality, then there would be a distribution of age gaps in the steady state, and similar conclusions would follow. Choo and Siow (2006) set out a model where the underlying preferences are perturbed by shocks that have an extreme value distribution.

We have assumed that preferences are separable in the two dimensions, \(q\) and \(S(\tau)\), although the aggregator function \(Q(\cdot)\) could be arbitrary. This is the basis for the sharp result, that the equilibrium age gap does not depend upon the extent of market imbalance. However, without non-separability, it would still be true that the age gap does not generally
adjust in order to reduce imbalances – the equilibrium gap could possibly change, but without presumption that this would be in the right direction.

The main content of theorem 1 is that the age gap does not adjust to changes in $g$ or $\hat{r}$ that result in a marriage market imbalance. That is, the simple intuition of supply and demand does not operate in the context of the marriage market, due to the indivisibilities in the assignment problem, whereby one man can be assigned to at most one woman, and vice versa.\footnote{It is possible that if there is an excess supply of women (men), they may be more willing to accept polygamy (polyandry). This would be the case if the total payoff of a menage a trois is greater than the payoff a couple plus a single. We leave an exploration of this issue to future work.} Under transferable utility – i.e. precisely the assumptions made by Bergstrom-Lam and Brandt et al. – the age gap does not adjust in response to changes either in the sex ratio at birth or changes in the rate of growth of cohort size. Indeed, this result applies even if preferences regarding the age gap have relatively little weight in the utility functions of men and women – note that our assumptions allow for the possibility that the total match surplus declines very little as the age gap changes from $\tau^*$. Things are quite different if we consider a transitory shock to cohort size, as Bergstrom and Lam and Brandt et al. have shown. For example, take the case where $\hat{r} = 1$ and $g = 0$ and $\tau_B = \tau_G = \hat{\tau} > 0$, so that the steady state marriage market sex ratio is balanced. Suppose that there is positive shock to the birth cohort at date $t$. In this case, there is an excess supply of women born at date $t$ (whose ideal match, of men born at $t - \hat{\tau}$, is smaller) and an excess supply of women born at date $t$. This will reduce the price of men and women born at date $t$, and raise the price of those born in scarce cohorts. Since some types of women are scarce, and some types of men are scarce, these price changes induce adjustment in the age gap of the affected cohorts and those nearby, and magnitude of these adjustments will be sensitive to the weight that age sensitive preferences have. In contrast, a long run increase in the marriage market sex ratio, say due to a rise in $g$, has the effect of making all women more plentiful. Thus there are no relative price effects across cohorts that induce an adjustment in the age gap.

### 3.2 Welfare considerations

In marriage markets with finitely many agents and transferable utility, the Shapley-Shubik results imply that the first and second welfare theorems hold. With transferable utility, welfare equals the sum of payoffs across all agents, and a stable matching maximizes this, implying the first welfare theorem. Furthermore, if lump sum transfers across agents are feasible, they can be used to redistribute payoffs, without any consequences for the matching pattern. Thus the second welfare theorem also holds.

In the present context, the sum of payoffs across all agents is not well defined since
the infinite series is necessarily non-convergent. If \( g \geq 0 \), so that cohort sizes are growing or constant, then the infinite series is clearly non-convergent. However, this is true even if \( g < 0 \), since the infinite sum going backwards in time is non-convergent. So we must adopt alternative welfare criterion. One possibility is to consider average per-capita payoffs. However, even here there is some ambiguity. Consider a steady state where the age gap is \( \tau \), and where the associated marriage market sex ratio equals \( r = \bar{r}(1 + g)^\tau \). Since payoffs are weakly supermodular, efficiency requires that matching is assortative. Thus the lowest types on the long side of the market are unmatched, and for a man of type \( \varepsilon \) who is matched, \( \phi(\varepsilon) \), satisfies

\[
[1 - F(\varepsilon) = r[1 - G(\phi(\varepsilon))].
\]

The total surplus associated with matches between men at date \( t \) and women at date \( t + \tau \) equals

\[
y(\tau) \equiv \int_{\xi}^{\xi_{\text{max}}} Q(q(\varepsilon, \phi(\varepsilon)), S(\tau))dF(\varepsilon).
\]

Now consider the welfare per man at date \( t \). The surplus generated per man equals \( y(\tau) \) if all men are matched, i.e. if \( r \geq 1 \). However, if \( r < 1 \), then the surplus per man equals \( ry(\tau) \). That is,

\[
W_m(\tau) = \min \{y(\tau), ry(\tau)\}.
\]

Consider instead the surplus the surplus generated per woman at date \( t \). This is given by

\[
W_f(\tau) = \min \left\{y(\tau), \frac{1}{r}y(\tau)\right\}.
\]

A third possible welfare criterion is the surplus generated per person in each cohort. Since the proportion of women to men within cohort is \( \frac{1}{1+r} \), this is equal to \( \frac{1}{1+r}W_m(\tau) + \frac{r}{1+r}W_f(\tau) \), i.e. it is a convex combination of the previous two welfare measures.

The steady state stable age gap \( \tau^* \) does not, in general, maximize any of these welfare criteria. To examine this issue further, let us consider the expression \( Q(q(\varepsilon, \phi(\varepsilon)), S(\tau)) \) at any realization of \( \varepsilon \). Since \( \phi(\varepsilon) \) is increasing in \( r \) (i.e. a man of a given quality gets a better match if the sex ratio increases), this implies that \( q(\varepsilon, \phi(\varepsilon)) \) is increasing in \( r \). The second argument of \( Q \) is \( S(\tau) \) and this is single peaked around \( \tau^* \). This establishes that at \( \tau^* \), a change in \( \tau \) reduces \( W_m(\tau) \) if this change in \( \tau \) reduces \( \bar{r} \); however, this change in \( \tau \) may increase \( W_m(\tau) \) if the consequence is to increase \( r \), and thereby \( q(\varepsilon, \phi(\varepsilon)) \) sufficiently. This argument is summarized in the following theorem.

**Theorem 2** The stable age gap \( \tau^* \) does not, in general, maximize \( W_m(\tau) \) or \( W_f(\tau) \). If
\[ r(\tau^*) = \bar{r}(1 + g)^{\tau^*} < 1 \] and age preferences are sufficiently weak, then \( W_m(\tau) \) is maximized at a value of \( \tau \) that increases \( r \) relative to \( r(\tau^*) \). If \( r(\tau^*) = \bar{r}(1 + g)^{\tau^*} > 1 \) and age preferences are sufficiently weak, then \( W_f(\tau) \) is maximized at a value of \( \tau \) that reduces \( r \) relative to \( r(\tau^*) \).

### 3.3 Distributional effects

What are distributional effects of changes in the effective sex ratio in the marriage market, whether due to the marriage squeeze or due to sex selective abortions? Clearly, if \( r \) increases, men benefit and women lose out, due to a worsened competitive position. This occurs regardless of whether there is transferable utility or non-transferable utility, since more men will be able to marry, while less women are able to. However, if utility is transferable, there will be distributional effects, even on women who are able to find partners. We now show that the magnitude of distributional effects depends upon the nature of the marriage market. Specifically, the distributional effects of sex ratio imbalances will be large in Asian societies, while they will be more muted in North European societies.

To elucidate the reasons for this difference, one must consider the difference between marriage institutions across cultures. In his seminal work, Hajnal (1965) pointed out the difference between marriage patterns in Northern Europe and Southern or Eastern European society. As Hajnal (1982) observes, these differences are accentuated all the more when one compares Northern Europe with Asia – i.e. Southern or Eastern Europe lie somewhere in between the Asian and Northern European marriage pattern. The salient features are as follows:

- A high age at marriage for both men and women (NE), as compared to earlier marriage.
- A small age gap at marriage (NE).
- A large fraction of the population who never married (NE), with a never married rate of about % (NE), as compared to a 98 or 99% marriage rate in Asia.

Clearly, a large age gap at marriage magnifies the marriage squeeze, both due to temporary shocks and due to secular growth. For example, a 3.5% rate of growth translates into a 19% excess supply of women if the age gap is five years, but only a 7% excess supply if the age gap is 2 years – such are the dramatic effects of compound growth.

More subtle is the effect of the last factor – the distributional effects of the marriage squeeze will be large in societies where marriage is near universal, but much smaller in North European type societies, where marriage rates are lower.
Let us now consider the effects of a change in $r^*$, that may arise either due to a worsening of the sex ratio at birth, or due to a change in the age gap. An increase in $r^*$ raises the relative supply of women. From the conditions for the marginal type (1) and (2), one may verify that this increases $\bar{\eta}$ and reduces $\bar{\varepsilon}$. That is, the the singles rate for men declines and that of women increases. Turning to payoffs, consider the payoffs of a man of a given type, $\varepsilon$. This can be written as

$$u^*(\varepsilon, r) = \tilde{u} + \int_{\bar{\varepsilon}(r)}^{\varepsilon} u'(x) dx.$$  

Recall that the derivative, $u'(\varepsilon)$, is given by

$$u'(\varepsilon) = q_x(\varepsilon, \phi(\varepsilon, r)).$$

Thus the payoff can be written as

$$u^*(\varepsilon, r) = \tilde{u} + \int_{\bar{\varepsilon}(r)}^{\varepsilon} q_x(x, \phi(x, r)) dx.$$  

The derivative of this payoff with respect to a change in the equilibrium sex ratio, $r$, is given by

$$\frac{\partial u(\varepsilon)}{\partial r} = \int_{\bar{\varepsilon}(r)}^{\varepsilon} q_{xy}(x, \phi(x, r)) \frac{\partial \phi(x, r)}{\partial r} dx - q_x(x, \phi(x)) \bigg|_{x=\bar{\varepsilon}(r)} \frac{\partial \bar{\varepsilon}}{\partial r}.$$  

Suppose now that the payoff function is strictly supermodular. Then $q_{xy} > 0$, and $\frac{\partial \phi(x, r)}{\partial r} > 0$, i.e. an increase in the supply of women improves the match quality of any type of man. Thus the first term is strictly positive. Furthermore, $\frac{\partial \bar{\varepsilon}}{\partial r} < 0$, i.e. the marginal type of man worsens. Since $q_x$ is positive, the second term is also positive, and the payoff of any type of man increases. The same argument implies that the payoff of any type of woman worsens.

If the payoff if weakly supermodular, so that $q_{xy} = 0$, then the first term is zero, but the second effect persists, so that an increase in the sex ratio worsens the position of women and increases that of men. In this case, we may write the payoff $q(x, y) = x + y$, so that

$$u^*(\varepsilon, r) = \tilde{u} + [\varepsilon - \bar{\varepsilon}(r)].$$

This verifies that the payoff of men increases at a rate that depends upon how quickly $\bar{\varepsilon}$ declines as $r$ increases. Thus in the case where the payoff function is additive, distributional effects will be larger, the greater the change in the marginal type induced by the sex ratio.
From the conditions (1) and (2), the derivative of the marginal type of woman, as a function of the sex ratio, is given by

\[
\frac{d\tilde{\eta}}{dr} = -\frac{1 - G(\tilde{\eta})}{f(\tilde{\xi}) \frac{q}{q} + rg(\tilde{\eta})} = -\frac{1 - G(\tilde{\eta})}{f(\tilde{\eta}) + rg(\tilde{\eta})}.
\]

Thus the distributional effects are larger if the densities associated with the marginal types, \(f(\tilde{\xi})\) and \(g(\tilde{\eta})\), are small. Suppose that \(\varepsilon\) and \(\eta\) are given by single peaked distributions, where \(f\) and \(g\) are strictly increasing up to a point and then strictly decreasing. Consider two different societies. First, an Asian one where \(\bar{u}\) and \(\bar{v}\) are low, so that the equilibrium marriage rate is high, over 95%. Second, a North European one where \(\bar{u}\) and \(\bar{v}\) are high, so that the equilibrium marriage rate is low, say below 90%. The numerator will be smaller in the latter, and the denominator will be larger. We summarize our results in the following proposition.

**Proposition 3** An increase in \(r^*\), induced by a change in the sex ratio or in cohort size growth, increases the singles rate for women and reduces that for women. The payoff for a non-marginal type of man strictly increases, and that for a non-marginal woman falls. If payoffs are additive, the magnitude of distributional effects is larger in societies where the density of marginal types is sparse.

### 3.4 The effect on dowries

Our analysis has focused on the payoffs to men and women, as a function of the sex ratio. This is separate from the effects on dowries (or bride-prices), which are monetary transfers at the time of the marriage. Becker has argued that dowries arise due to the inflexibility in the division of the marital surplus within the marriage. Under this interpretation, the total surplus \(Q(\varepsilon, \eta, \tau)\) is divided into *premuneration values* \(U(\varepsilon, \eta, \tau)\) and \(V(\varepsilon, \eta, \tau)\), for the man and woman respectively. These will, in general, differ from the payoffs in the stable allocations \(u^*(\varepsilon)\) and \(v^*(\eta)\), and a dowry or brideprice is paid to reconcile the two. Our analysis speaks to the equilibrium payoffs, not dowries per se. If the equilibrium premuneration value of given type of woman, \(V(\eta, \phi^{-1}(\varepsilon, r^*), \tau^*)\), does not respond as quickly to an increase in \(r^*\) as the equilibrium payoff \(v^*(\eta, r^*)\), then the dowry paid must rise. However, it is also possible that premuneration values for some types rise rapidly enough that dowries may even fall.\(^{11}\) The reason is, as \(r^*\) increases, each man is matched with a

\(^{11}\)There is a controversy on whether an increase in dowries can be attributable to the marriage squeeze – see Anderson (2003, 2007a, 2007b).
woman of higher quality, since \( \phi(\varepsilon) \) increases with \( r \). This may induce a large increase is premuneration value for the man.

Let us assume that payoffs are additive so that \( q(\varepsilon, \eta) = \varepsilon + \eta \). More precisely, let us assume that \( q(\varepsilon, \eta) = \varepsilon + \eta + \rho \varepsilon \eta \), where \( \rho > 0 \) and consider the limit case where \( \rho \to 0 \). This ensures strict supermodulatity and ensures that the matching is unique and assortative. There are two possible interpretations one may give to "types" in this context. In the first case, an individual’s type measures his or her attractiveness to the partner.

Let \( \varepsilon \) and \( \eta \) be each uniformly distributed on \([0,1]\) and let \( \bar{u} = \bar{v} = \frac{1}{4} \). When the sex ratio is balanced, the marginal types are \( \bar{\varepsilon} = \frac{1}{4} \) and \( \bar{\eta} = \frac{1}{4} \). The matching satisfies \( \phi(\varepsilon) = \varepsilon \). The equilibrium payoff of any type must satisfy

\[
u^*(\varepsilon, r = 1) = \frac{1}{4} + (\varepsilon - \frac{1}{4}) = \varepsilon.
\]

The premuneration value is given by

\[
U(\varepsilon, \phi(\varepsilon), 1) = \phi(\varepsilon) = \varepsilon.
\]

Thus, for any type of man, his premuneration value equals his equilibrium payoff, and the dowry \( t(\varepsilon) = 0 \) for any \( \varepsilon \).

Now let us consider an unbalanced sex ratio, where \( r^* = 2 \). Now the marginal types are \( \bar{\varepsilon} = 0 \) and \( \bar{\eta} = \frac{1}{2} \). The equilibrium payoff for a type \( \varepsilon \) of man satisfies

\[
u^*(\varepsilon, r = 2) = \frac{1}{4} + \varepsilon.
\]

His premuneration value satisfies

\[
U(\varepsilon, \phi(\varepsilon), r = 2) = \phi(\varepsilon) = \frac{1 + \varepsilon}{2}.
\]

Thus the dowry received equals

\[
t(\varepsilon) = \frac{2\varepsilon - 1}{4}.
\]

We see that the increase in the ratio of women to \( r = 2 \) causes a bride price to be paid by lower quality men, but dowry to be paid for higher quality men.

Let us now consider a different interpretation, where an individual’s type denotes his or her desire for marriage. This does not affect the equilibrium payoffs as compared to the previous case, but does affect the premuneration value, which is now equal to the person’s own type as long as the individual is matched. That is, \( U(\varepsilon, \phi(\varepsilon), r) = \varepsilon \) if type \( \varepsilon \) is matched, and \( V(\eta, \phi^{-1}(\eta), r) = \eta \) if \( \eta \) is matched. Now if the sex ratio is 2, \( u^*(\varepsilon, r = 2) = \frac{1}{4} + \varepsilon \). Thus
the dowry $t(\varepsilon) = \frac{1}{4}$, independent of the man’s type.

One may combine these two elements, by allowing each individual man’s type to consist of two dimensions, attractiveness ($\varepsilon_1$) and the desire for marriage, $\varepsilon_2$, and similarly for a woman. Thus is a type $\varepsilon$ man is matched with a type $\eta$ woman, then the prenumeration values for the man would be given by

$$U(\varepsilon, \eta, \tau) = (1 - \lambda)\varepsilon_2 + \lambda\eta_1 - \frac{1}{2}S(\tau).$$

$$V(\varepsilon, \eta, \tau) = (1 - \lambda)\eta_2 + \lambda\varepsilon_1 - \frac{1}{2}S(\tau).$$

Assuming that there is a slight supermodularity between $\varepsilon_1$ and $\eta_1$, this model would predict an effect of sex ratios on dowries that is intermediate between the two polar cases. That is, an increase in $r$ would raise dowries for most men, except possibly those at the very bottom end of the attractiveness distribution. However, the increases for higher quality men would be greater than for those lower down.

### 4 Non-transferable utility

Now let us consider the other polar extreme, of non-transferable utility. Since our model has two dimensions of preferences – quality and age – the non-transferable utility is hard to analyze with any degree of generality. If men and women have different ideal age gaps, then the vertical dimension, then the possibility of trading "quality" for the age gap dimension becomes complicated. To avoid this, we can consider two cases. First, when the quality dimension is absent, so that only the age gap matters for preferences. Second, when both women and men have the same ideal point, so that $\tau_B = \tau_G = \tau^*$.  

Non transferable utility implies that the total payoff corresponding to a match, $Q(q(\varepsilon, \eta), S(\tau))$ can only be divided in a fixed way. Let $Q^m(q(\varepsilon, \eta), S(\tau))$ and $Q^f(q(\varepsilon, \eta), S(\tau))$ be functions such that

$$Q(q(\varepsilon, \eta), S(\tau)) = Q^m(q(\varepsilon, \eta), S(\tau)) + Q^f(q(\varepsilon, \eta), S(\tau)),$$

for every value of $(\varepsilon, \eta, \tau)$. Assume further that each sexes share in the share increases (weakly) as the total payoff increases.

**Theorem 4** Suppose that men and women have identical ideal age gaps, i.e. $\tau_B = \tau_G = \tau^*$. Then the unique steady stage age gap under stable matching is $\tau^*$, and matching is assortative in the quality dimension, so that $\phi(\varepsilon)$ is increasing in $\eta$.  

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Proof. Suppose that the age gap is \( \tau^* \) and matching is assortative. If a man of type \( \varepsilon \) prefers a woman of type \( \eta \) and age gap \( \tau' \), then \( \eta > \phi(\varepsilon) \), and thus \( \varepsilon < \phi^{-1}(\eta) \) and this women strictly prefers her current match to him. A similar argument applies to any woman. Now suppose that we have a steady state where \( \tau \neq \tau^* \). Matching must be assortative – otherwise, if \( \varepsilon > \varepsilon' \) and \( \phi(\varepsilon) < \phi(\varepsilon') \), the pair \( (\varepsilon, \phi(\varepsilon')) \) have a profitable deviation. Thus in every cohort, any man of type \( \varepsilon \) is matched to the same type of woman, \( \phi(\varepsilon) \). Consider a pair of individuals of types \( (\varepsilon, \phi(\varepsilon)) \) and age gap \( \tau^* \). This pair can deviate from the proposed equilibrium matching, and increase both their payoffs.

Consider next the case where there is no vertical element of quality. Let \( \bar{r} \) be given, and suppose that the equilibrium age gap is \( \tau \). Let \( r \) be the marriage market sex ratio corresponding to the pair \( \bar{r}, \tau \); i.e. \( r = (1 + g)^\tau \). We claim that \( \tau \) must equal the ideal point of the short side of the market. That is, if \( r > 1 \), then \( \tau \) must equal \( \tau_B \), and if \( r < 1 \), \( \tau = \tau_G \). To see this, let us suppose that \( r > 1 \). Then in every cohort, there are unmatched women. Thus a man can always propose to his ideal woman, i.e. one with an age gap of \( \tau_B \), and this proposal will be accepted since the woman is unmatched. This implies that any age gap \( \tau \) different from \( \tau_B \) cannot be stable. To see that \( \tau_B \) is stable, note that no man can do better, and furthermore, any woman who is matched can also not do any better, since no age gap other than \( \tau_B \) will be acceptable to a man. A similar argument establishes that the unique steady state stable match must equal \( \tau_G \) when \( r < 1 \).

Consider finally the case where \( r = 1 \) so that the marriage market is balanced, e.g. because \( \bar{r} = 1 \) and \( g = 0 \) – this situation is a good approximation to marriage market conditions for much of human history. In this case, any age gap \( \hat{\tau} \) that lies between \( \tau_G \) and \( \tau_B \) is an equilibrium steady state age gap. Consider a steady state where every man is matched to a woman with age gap \( \hat{\tau} \). Since \( r = 1 \), every woman is also matched with age gap \( \hat{\tau} \). Suppose that a man who proposes to woman he prefers – such a woman must have an age gap relative to him of \( 0 \) that is closer to \( \tau_B \) than \( \hat{\tau} \) is. However, since \( \hat{\tau} \) lies between \( \tau_B \) and \( \tau_G \), this implies that \( \tau \) is further away from this woman’s ideal point \( \tau_G \) than \( \hat{\tau} \) is, and so this proposal is not acceptable to the woman.

Theorem 5 There exists a stable steady state matching, for all parameter values. i) If \( \bar{r}(1+g)^\tau_G > 1 \) and \( \bar{r}(1+g)^\tau_B > 1 \), the unique equilibrium age gap equals \( \tau_B \). ii) If \( \bar{r}(1+g)^\tau_G < 1 \) and \( \bar{r}(1+g)^\tau_B < 1 \), the unique equilibrium age gap equals \( \tau_G \). iii) If neither the two above conditions hold, so that \( (\bar{r}(1+g)^\tau_G - 1)(\bar{r}(1+g)^\tau_B - 1) \leq 0 \), there exists a steady state where all women and all men are matched. In particular, if \( \bar{r} = 1 \) and \( g = 0 \), then every age gap \( \hat{\tau} \) between \( \tau_G \) and \( \tau_B \) is a stable age gap.

Proof. Parts (i) and (ii) have been proven in the text, so we turn to (iii). Let \( \hat{\tau} \) be
any value of \( \tau \) that solves the equation \( \bar{r}(1 + g)\hat{\tau} = 1 \), so that if the age gap is \( \hat{\tau} \), then the marriage market sex ratio \( r \) equals one. We show now that if \( \hat{\tau} \) is an integer between \( \tau_G \) and \( \tau_B \), then \( \hat{\tau} \) will be a stable steady state age gap. Consider a steady state where every man is matched to a woman with age gap \( \hat{\tau} \), and where every individual is matched – from the definition of \( \hat{\tau} \) such a matching is measure preserving and thus feasible. Suppose that a man who proposes to woman he prefers – such a woman must have an age gap relative to him of \( \tau' \) that is closer to \( \tau_B \) than \( \hat{\tau} \) is. However, since \( \hat{\tau} \) lies between \( \tau_B \) and \( \tau_G \), this implies that \( \tau' \) is further away from this woman’s ideal point \( \tau_G \) than \( \hat{\tau} \) is, and so this proposal is not acceptable to the woman. Suppose that no integer value of \( \hat{\tau} \) that solves the equation \( \bar{r}(1 + g)\hat{\tau} = 1 \). Consider the expression

\[
h(\lambda) \equiv \lambda \bar{r}(1 + g)^{\tau_G} + (1 - \lambda)\bar{r}(1 + g)^{\tau_B} - 1.
\]

Since \( h \) is continuous, \( h(0) > 0 \) and \( h(1) < 0 \), there exists a value \( \hat{\lambda} \) such that \( h(\hat{\lambda}) = 0 \). Let every individual be matched, with a fraction \( \hat{\lambda} \) in each cohort having an age gap \( \tau_G \) with the remainder having an age gap \( \tau_B \). This matching is constructed to be measure preserving, and also, no individual is left unmatched. It is also stable, since no woman with matched age gap \( \tau_G \) will accept a different match, and no man with age gap \( \tau_B \) will also accept a different match.

This theorem illustrates why the age gap does not necessarily adjust endogenously in order to equilibrate demand and supply in the marriage market, even with non-transferable utility. Suppose that the initial situation is one where the sex ratio at birth is the normal one, i.e. \( \bar{r} = 1 \) and where \( g = 0 \). In this case, the equilibrium age gap will be some \( \hat{\tau} \) between \( \tau_B \) and \( \tau_G \). Now if \( g \) becomes positive, there will be an excess supply of girls, and so the equilibrium age gap will shift to \( \tau_B \). So the age gap increases, aggravating the marriage market imbalance if \( \tau_B > \tau_G \), if men prefer a larger age gap than women. The age gap will decrease if \( \tau_G > \tau_B \). Similarly, the effect of sex selection for males, that reduce \( r \), is to make the age gap that preferred by women – this may reduce marriage market imbalance, or aggravate it.

To summarize, we have examined two leading models of the marriage market, transferable and non-transferable utility, and found that the age gap at marriage does not necessarily adjust in order to reduce the systematic marriage market imbalance arising from the marriage squeeze or biases in the sex ratio at birth. The underlying reasons for the age gap in these models are individual preferences – this seems a plausible foundation given the overwhelming evidence from over 90 countries, that the age gap is always positive. An alternative explanation for the age gap is provided by Bergstrom and Bagnoli (1990), who set out a
signalling model. Men are privately informed about their ability, and the more able ones delay marriage, in order to secure a better quality match. While a complete discussion of their model is beyond the scope of this paper, we have analyzed the implications of changing marriage market sex ratios upon the age gap in their model, and also find that gap does not necessarily adjust in order to reduce marriage market imbalance.

References


Gap Between R and Sex Ratio Africa