Review of Economic Studies (1990) 57, 255-277
0034-6527/90/00150255$02.00

Nonstationarity in Job Search Theory

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First version received January 1987; *final version accepted August* 1989 (*Eds.*)

Generally, structural job search models are taken to be stationary. In this paper models are examined in which every exogenous variable can cause nonstationarity, for instance because its value is dependent on unemployment duration. A general differential equation that describes the evolution of the reservation wage over time is derived. As an empirical illustration a nonstationary structural model is estimated that focuses on the consequences of a downward shift in the level of benefits. It appears that the elasticity of duration with respect to the level of benefits after the shift is much larger than the elasticity with respect to the level before the shift.

1. INTRODUCTION

This paper examines the movement of a job-seeking individual's reservation wage over time in a general nonstationary job search model. Also, results concerning comparative dynamics of the reservation wage and the distribution of the duration of unemployment are derived. As an empirical illustration a nonstationary structural model is estimated. The nonstationarity originates from the decrease in the level of benefits when unemployment duration equals two years. From the results some detailed policy recommendations can be deduced, as one is able to distinguish the effect of a change in the level of benefits in the first two years of unemployment from the effect of a change in the level after that period.

Recently the use of job search models for the analysis of unemployment duration has become widespread. The reduced-form approach in empirical studies (see e.g. Lancaster (1979)), in which only the hazard of the duration distribution is estimated, is gradually being replaced by a more structural approach. The latter way of modelling is characterized by the explicit use in empirical analysis of the reservation wage equation as stated by the theory. E.g. Yoon (1981), Lancaster and Chesher (1983), Lynch (1983), Narendranathan and Nickell (1985) and van den Berg (1988) use the complete theoretical framework of job search theory to make inferences about search behaviour.

However, the structural models used in these studies are stationary. This implies that variables like unemployment benefits or the rate of arrival of job offers are assumed to be constant over the spell of unemployment, which is often at variance with reality. What is more, various reduced-form empirical studies indicate a significant duration-dependence of the re-employment probability, which is generally interpreted as evidence in favour of the presence of nonstationarity (see e.g. Blau and Robins (1986), Kooreman and Ridder (1983), Lancaster (1979) and Narendranathan, Nickell and Stern (1985)). Consequently there is a need to model reservation wage movements over time based on a nonstationary theoretical framework.

In the last two decades a few papers have been published that pay some attention to nonstationarity in job search theory (see e.g. Burdett (1979), Gronau (1971), Heckman and Singer (1982), Lippman and McCall (1976*b*) and Mortensen (1986)). Though these articles draw important qualitative conclusions concerning the movement of the reservation wage over time, generally no attention is paid to a rigorous derivation of formulae for the time-dependence of the reservation wage. Furthermore, only very specific departures from stationarity are examined, like finite lifetimes or shifting wage offer distributions. Most models are specified in discrete time which means that an empirical implementation would require an arbitrary choice concerning the length of the unit time interval. These remarks also apply to Wolpin (1987) who estimates a structural model that allows for duration-dependence of the job offer arrival rate. Kiefer and Neumann (1979) estimate a discrete-time search model in which exactly one job offer per period is assumed to arrive and in which the reservation wage is a linear function of some explanatory variables including unemployment duration. This linear specification is not derived from theory so the model might be called semi-structural. It appears that duration has a significant negative influence on the reservation wage though it is not clear which economic causes should be held responsible for this effect.

In this paper we examine the consequences of nonstationarity in continuous-time job search models in a rather general setting. Section 2 gives a brief overview of job search theory. Various causes of nonstationarity that may arise are discussed, like macro-economic events and changes in the personal situation of individuals during the spell of unemployment. In Section 3 and 4 we present the main theorems concerning the movement of the reservation wage over time in nonstationary models. The exogenous variables like unemployment benefits and the wage offer distribution are allowed to vary over time in a very general way. The more specific the assumptions about the time paths of the exogenous variables, the more detailed are our inferences about the time path of the reservation wage. In Section 3 we also give some comparative dynamics results. These results concern the shift in the optimal reservation wage path if we replace some particular time path of an exogenous variable by another. We also examine the unemployment duration density in case of nonstationarity.

In Section 5 we illustrate by means of an empirical example the importance of allowing for nonstationarity. In The Netherlands at the beginning of the eighties the benefit level during the first years of unemployment was related to the pre-unemployment wage while the level after that was determined by the public assistance system. As a consequence benefits generally decrease substantially when duration equals about 2 years. In a nonstationary structural model one can analyze in detail the effects of these changes, not only on the expected duration but also on the optimal reservation wage path. Using survey data on unemployed individuals from 1983, a nonstationary continuous-time structural job search model that allows for such changes is estimated. Given the parameter estimates we calculate the elasticities of the expected duration with respect to the level of benefits before and after 2 years of unemployment. It appears that for most individuals the elasticity of duration with respect to the level of benefits after 2 years is much larger than the elasticity of duration with respect to the level before 2 years. Section 6 concludes.

2. JOB SEARCH THEORY AND THE INTRODUCTION OF NONSTATIONARITY

Job search theory tries to describe the bahaviour of unemployed individuals in a dynamic and uncertain world. Job offers arrive at random intervals following a (non-homogeneous)Poisson process with arrival rate λ. Such job offers are random drawings (without recall) from a wage offer distribution with distribution function $F(w)$. Every time an offer arrives the decision has to be made whether to accept the offer or reject it and search further. Once a job is accepted it will be kept forever at the same wage. It is assumed that individuals know λ and $F(w)$ but that they do not know in advance when job offers arrive and what wages are associated with them. During the spell of unemployment, unemployment benefits b are received. Unemployed individuals aim at maximization of their own expected present value of income (over an infinite horizon).

The job search model described here contains three exogenous variables (λ, b and $F(w)$) and one constant parameter ρ which is the subjective rate of discount. For expositional purposes the theoretical results in this paper are stated in terms of this basic model. At the end of Section 4 there is an outline of how the results can be generalized to a setting that is more realistic with regard to the function that is to be maximized and also with regard to the process of search.

We now discuss the concepts of stationarity and nonstationarity in the basic model. Let time T_0 denote the point of time at which an individual becomes unemployed. We call the job search model that describes the search behaviour of this individual stationary if the exogenous variables λ, b and $F(w)$ are constant on the time interval $[T_0, \infty\rangle$ and do not depend on realizations of offer times or wage offers. In combination with the infinite-horizon assumption this means that in case of stationarity the unemployed individual's perception of the future is independent of time or unemployment duration. Consequently, the optimal strategy is constant during the spell of unemployment. Let us assume that $F(w)$ is continuous in w, that this distribution has a finite first moment and that $0 \leqq \lambda < \infty$, $0 < \rho < \infty$ and $0 \leqq b < \infty$. For a stationary job search model satisfying these conditions it has been shown that the optimal strategy can be characterized by the reservation wage property (see e.g. Lancaster and Chesher (1983)). A job offer is acceptable if its wage exceeds the reservation wage ϕ while a wage below ϕ induces one to reject the offer and search for a better one. The reservation wage is the unique finite solution of

$$\phi = b + \frac{\lambda}{\rho} \int_{\phi}^{\infty} (w - \phi)\, dF(w). \tag{1}$$

Nonstationarity arises if one or more of the exogenous variables change after T_0. Such a change may be due to business cycle effects. For instance, an increase in the aggregate unemployment level may induce a fall in λ. Changes may also occur because of policy changes like a reduction of all unemployment benefits. Finally, for a job searcher the exogenous variables may change because of changes in his personal situation. Unemployment benefits and λ may be dependent on the elapsed unemployment duration. Sooner or later these features of the labour market and personal characteristics of job searchers are recognized and used in determining the optimal strategy. So, generally, the optimal strategy is not constant in the case of a nonstationary model.

In this paper we consider nonstationarity as a result of the time-dependence and duration-dependence of exogenous variables. Dependencies of exogenous variables on the number of rejected offers or the levels of wages associated with rejected offers are ruled out. Further, throughout the paper we will be concerned with job searchers with perfect foresight in the sense that they are assumed to correctly anticipate changes in the values of the exogenous variables. In other words, we expect people to know how the exogenous variables are related to unemployment duration. In Section 6 we turn to the issue of relaxing this assumption.

3. THE RESERVATION WAGE IN NONSTATIONARY JOB SEARCH MODELS

3.1. *Assumptions*

For ease of exposition we let calendar time start at the moment that one becomes unemployed, so that calendar time and unemployment duration coincide. In this way duration-dependence and other forms of nonstationarity can be considered simultaneously. In order to obtain properly defined present values and in order to restrict attention to economically meaningful cases, the following weak conditions concerning the exogenous variables and ρ are assumed.

Assumption 1. Wage offers at time t are drawn randomly from a distribution with a distribution function $F(w; t)$, which is a continuous function of w and strictly monotonically increasing in w on some interval $\langle \alpha(t), \beta(t) \rangle$ with $0 \leqq \alpha(t) < \beta(t) \leqq \infty$, $F(\alpha(t); t) = 0$ and $\lim_{\omega \to \beta(t)} F(w; t) = 1$ for every $t \geqq 0$. The mean of the distribution is a uniformly bounded function of t.

Assumption 2. For every $t \geqq 0$, $0 \leqq \lambda(t) \leqq K < \infty$ and $0 \leqq b(t) \leqq K < \infty$; K being a fixed number.

Assumption 3. $F(w; t)$, $\lambda(t)$ and $b(t)$ are continuous functions of t on $[0, \infty\rangle$ except possibly for a finite number of points. If an exogenous variable is discontinuous in t at some point, say t^*, then it is right-continuous, and the left-hand limit of this variable at t^* does exist (e.g. in the case of b: $\lim_{t \downarrow t^*} b(t) = b(t^*)$ and $\lim_{t \uparrow t^*} b(t)$ exists).

Assumption 4. There exists some number T such that all exogenous variables are constant on $[T, \infty\rangle$.

Assumption 5. $0 < \rho < \infty$.

Note that a model which satisfies Assumptions 1-5 allows for quite general patterns of movement of the exogenous variables over time, comprising virtually every nonstationary situation that may arise in practice.

3.2. *The optimal path of the reservation wage*

We now present a characterization of the time path of the optimal strategy.

Theorem 1. *Let Assumptions* 1-5 *be satisfied. Then the optimal strategy of a job searcher can be characterized by a reservation wage function* $\phi(t)$ *giving the reservation wage at time t.* $\phi(t)$ *is a unique, bounded and continuous function of t and it satisfies the following differential equation for every point in time at which* $b(t)$, $\lambda(t)$ *and* $F(w; t)$ *are continuous in t.*

$$\phi'(t) = \rho \cdot \phi(t) - \rho \cdot b(t) - \lambda(t) \cdot Q(\phi(t); t) \tag{2}$$

where $Q(\phi(t); t)$ *is defined as*

$$Q(\phi(t); t) = \int_{\phi(t)}^{\infty} (w - \phi(t)) dF(w; t) = \int_{\phi(t)}^{\infty} \bar{F}(w; t) dw \quad \text{with } \bar{F}(w; t) = 1 - F(w; t).$$

If one or more of the exogenous variables are discontinuous in t at some point, then the right-hand side of (2) *gives the right-hand derivative of* ϕ *with respect to t at that point.*

The left-hand derivative can be calculated by replacing the values of the exogenous variables at t in the right-hand side of (2) *by their left-hand limits at that discontinuity point.*

Proof. See Appendix 1. ||

The differential equation (2) is also given by Mortensen (1986). However, in Mortensen's model the exogenous variables are forced to have very simple functional forms; in fact the only departure from stationarity is (in terms of our model) a simultaneous discrete change in λ and b when the unemployment duration equals T time-units. This change is interpreted to be a consequence of liquidity constraints.

In order to get an intuitive feeling for equation (2) we rewrite it in terms of the optimal present value of search $R(t)$ at time t. From Appendix 1, $\phi(t)=\rho\cdot R(t)$ so $\phi(t)$ is the wage rate which makes the individual indifferent between working and being unemployed at t. It follows,

$$\rho R(t)dt=\frac{\partial R}{\partial t}dt+b(t)dt+\lambda(t)dt\cdot\int_{\phi(t)}^{\infty}\left(\frac{w}{\rho}-R(t)\right)dF(w;t). \tag{3}$$

Suppose the optimal value R is an asset which can be traded in a perfect capital market with an interest rate that equals the discount rate ρ. In equilibrium the return from the asset value in a small time interval $[t, t+dt\rangle$, which is $\rho R(t)dt$, must equal what one expects to get from holding the asset in that period. The latter consists of three parts: first, the appreciation of the asset value in the time interval; second, the benefits flow in the interval; and third, the expected gain of finding a job during the period (see Pissarides (1985) for other examples of such an interpretation).

Another way to look at equation (2) requires the introduction of a function $\phi_0(t)$, giving the optimal reservation wage at time t if the environment remains stationary after t, i.e. from equation (1), $\phi_0(t)$ is the unique finite solution of

$$\phi_0(t)=b(t)+\frac{\lambda(t)}{\rho}\cdot Q(\phi_0(t);t), \qquad t\geqq 0. \tag{4}$$

Suppose we want to compare $\phi(t)$ and $\phi_0(t)$. Of course, $\phi(t)=\phi_0(t)$ implies $\phi'_R(t)=0$ (a subscript R denotes a r.h.d.). Further, from equation (2), for every $\phi(t)$,

$$\frac{\partial\phi'_R(t)}{\partial\phi(t)}=\rho+\lambda(t)\bar{F}(\phi(t);t)=\rho+\theta(t)>0 \tag{5}$$

in which $\theta(t)$ denotes the exit rate out of unemployment at time t (see Subsection 3.4). Consequently,

$$\phi(t)\lesseqgtr\phi_0(t) \quad\Leftrightarrow\quad \phi'_R(t)\lesseqgtr 0. \tag{6}$$

Let $R_0(t)$ denote the optimal value of search at t in case $\phi_0(t)$ is the optimal reservation wage. It is clear that $\phi_0(t)=\rho\cdot R_0(t)$. Using $R_0(t)$ and $R(t)$ it can be shown that relationship (6) is perfectly plausible. If for example $\phi(t)>\phi_0(t)$ then $R(t)>R_0(t)$ which means that there are future changes in the values of the exogenous variables that altogether benefit the value of search $R(t)$ as compared to the "stationary state" value of search $R_0(t)$. As time proceeds, these future changes come nearer. Both because future income is discounted by a positive rate ρ and because the probability of not finding a job before the changes take place (following the optimal strategy) increases as time proceeds, this implies that R will rise at t (compare equation (5)). So the right-hand derivative of R with respect to time at t is positive and consequently $\phi'_R(t)>0$. Note that the argument applies to every two possible reservation wages at t, in the sense that it makes clear that given the values of the exogenous variables at t, $\phi_1(t) > \phi_2(t)$ implies $\phi'_{1R}(t) > \phi'_{2R}(t)$. In Section 4, where we make an additional assumption concerning the exogenous variables, we return to the interrelations between ϕ, ϕ' and ϕ_0.

Theorem 1 can be used in order to determine ϕ as a function of time. First solve for ϕ at the point T after which all exogenous variables are constant (this is easily done using equation (1)). $\phi(t)$ is a continuous function of t. Therefore $\phi(T)$ serves as an initial condition for the differential equation (2) in the time interval ending at T within which the exogenous variables are continuous. Thus $\phi(t)$ can be solved for every t in this interval. Backward induction leads to the solution $\phi(t)$ for every $t \geqq 0$.

If restrictions are placed on the way that the exogenous variables may vary over time, then sometimes qualitative conclusions can be drawn concerning the time path of ϕ. In the remainder of this subsection sufficient conditions are given for the reservation wage to be strictly decreasing. Consider models in which one of the exogenous variables is time dependent in a way that is described by one of the following four cases, while the others are constant on the interval $[0, \infty\rangle$.

K1. $\forall t \in [0, T\rangle, \forall \tau > 0, b(t) > b(t+\tau)$.

K2. $\forall t \in [0, T\rangle, \forall \tau > 0, \lambda(t) > \lambda(t+\tau)$.

K3. $\forall t \in [0, T\rangle, \forall \tau > 0$, $F(w; t)$ first order stochastically dominates $F(w; t+\tau)$, that is, $\forall w \in \langle \alpha(t+\tau), \beta(t)\rangle$ $\bar{F}(w; t) > \bar{F}(w; t+\tau)$.

K4. $\forall t \in [0, T\rangle, \forall \tau > 0$, $F(w; t)$ is a mean preserving spread of $F(w; t+\tau)$, that is, $E(w; t) = E(w; t+\tau)$ and

$$\forall x \in \langle \alpha(t), \beta(t)\rangle, \int_{\alpha(t)}^{x} F(w; t)dw > \int_{\alpha(t)}^{x} F(w; t+\tau)\, dw.$$

Note that in all cases we allow the exogenous variable to be discontinuous in a finite number of points. In order to rule out uninteresting situations in which decreasing exogenous variables do not make the reservation wage time dependent, we impose the restrictions that in case K2 for every $t \in [0, T\rangle$ $\phi(t) < \beta(t)$ has to hold, whereas in case K3 for every $t \in [0, T\rangle$ $\phi(t) < \beta(t)$ and $\lambda > 0$ have to hold and in case K4 for every $t \in [0, T\rangle$ $\alpha(t) < \phi(t) < \beta(t)$ and $\lambda > 0$ have to hold. These restrictions can be characterized by the following restrictions on the exogenous variables in each case.

K2. $b < \beta$.

K3. $b < \beta_L(T), \lambda > 0$.

K4. $\alpha_L(T)\left(1+\dfrac{\lambda}{\rho}\right) - \dfrac{\lambda}{\rho} \cdot E(w; t) < b < \beta_L(T), \lambda > 0$.

in which $f_L(a)$ denotes the left-hand limit of $f(x)$ at $x = a$ if it exists. Note that if for every $t \geqq 0$, $\lambda(t) > 0$, $\alpha(t) = 0$ and $\beta(t) = \infty$ then these restrictions are always satisfied. Also note that a decreasing location or scale of the wage offer distribution are special cases of K3 and K4, respectively.

Theorem 2. *Let Assumptions* 1-5 *be satisfied. In addition, let one exogenous variable be time dependent according to* K1, K2, K3 *or* K4 *while the others are constant on the time interval* $[0, \infty\rangle$. *Then*

(i) $\forall t \in [0, T\rangle$ $\phi(t) < \phi_0(t)$ *with* $\phi_0(t)$ *as defined in equation* (4).

(ii) $\forall t \in [0, T\rangle$ $\phi'(t) < 0$ *if this derivative exists. At points t where* $\phi'(t)$ *does not exist (i.e. points at which one of the exogenous variables is discontinuous), both* $\phi'_L(t) < 0$ *and* $\phi'_R(t) < 0$ *hold. If an exogenous variable is discontinuous at T, then* $\phi'_L(T) < 0$, *otherwise* $\phi'(T) = 0$.

Proof. See Appendix 2. ||

Note that simultaneous occurrence of some K1, K2, K3, K4 can be examined by sequential application of Theorem 2. Lippman and McCall (1976*b*) consider a generalization of case K3 for which they derive a result similar to Theorem 2 in a discrete-time model in which exactly one job offer arrives per period.

Clearly, the results make economic sense. Any future decrease in b, λ or the mean or variance of F will make the value of search in the present smaller than it would have been if the exogenous variables were constants. From the discussion of equations (5) and (6) this means that $\phi(t) < \phi_0(t)$ for every $t \in [0, T\rangle$ and that ϕ decreases as lower values of the exogenous variables come nearer.

In the basic job search model that is described in Section 2 and Subsection 3.1 it is assumed that once a job offer is accepted it will be held forever. The model equations become intractable if one tries to relax this assumption by allowing individuals to quit or to be laid off, because nonstationarity in future spells of unemployment influences the optimal strategy in the present spell. However, Burdett and Sharma (1988) argue that the no-quits assumption is unduly strong if there is duration dependence in unemployment according to K1. Basically, the argument is that rejecting a job offer is sub-optimal to accepting it and quitting immediately thereafter, because in the latter case one starts with a fresh spell of unemployment and as a result one obtains a higher level of benefits. However, such behaviour is very unlikely to occur in practice since generally there are effective legal barriers that discourage individuals to act that way. For instance in The Netherlands individuals who quit voluntarily do not get any benefits at all. Moreover, usually the level of benefits in the first period of unemployment is positively related to the pre-unemployment wage if one has had a job before becoming unemployed. Since a model in which quits and layoffs are allowed and in which each and every feature of the benefits system is incorporated would be too complicated to analyze we prefer to stick to the assumption that jobs are held forever.

3.3. *Comparative dynamics*

In this subsection we examine the consequences for the optimal reservation wage path when some particular time path of an exogenous variable is replaced by a different (higher) path. For the sake of convenience we will be using the term "reference model" in the case where every exogenous variable follows the reference path, while the term "alternative model" denotes cases in which one exogenous variable does not follow its reference path while the others do. Variables in the reference model will be labelled with a subscript r. Consider two arbitrary points in time t_1 and t_2, such that $0 \leqq t_1 < t_2 \leqq \infty$. We consider four different departures from the reference model:

C1. $\forall t \in [t_1, t_2\rangle$, $b(t) > b_r(t)$.

C2. $\forall t \in [t_1, t_2\rangle$, $\lambda(t) > \lambda_r(t)$.

C3. $\forall t \in [t_1, t_2\rangle$, $F(w; t)$ first order stochastically dominates $F_r(w; t)$, that is, $\forall w \in \langle \alpha_r(t), \beta(t)\rangle$, $\bar{F}(w; t) > \bar{F}_r(w; t)$.

C4. $\forall t \in [t_1, t_2\rangle$, $F(w; t)$ is a mean preserving spread of $F_r(w; t)$, that is, $E(w; t) = E_r(w; t)$ and

$$\forall x \in \langle \alpha(t), \beta(t)\rangle, \int_{\alpha(t)}^{x} F(w; t)dw > \int_{\alpha(t)}^{x} F_r(w; t)dw.$$

It is important to remark that in every case above, for every exogenous variable the time paths in the reference model and the alternative model are equivalent outside the interval $[t_1, t_2\rangle$. Notice that changing the location and scale of the wage offer distribution are special cases of C3 and C4, respectively.

Theorem 3. *Consider one of the deviations* C1, C2, C3 *or* C4 *from a reference model. Let the exogenous variables of both the reference model and the alternative model satisfy Assumptions* 1-5. *In addition, assume that in cases* C2, C3 *and* C4 there is a $t_3 \in [0, t_2\rangle$ *such that* $\forall t \in [t_3, t_2\rangle$, $\phi_r(t) < \beta(t)$, *while in case* C4 *also* $\forall t \in [t_3, t_2\rangle$, $\phi_r(t) > \alpha(t)$. *Moreover, in cases* C3 *and* C4 $\forall t \in [t_3, t_2\rangle$ $\lambda(t) > 0$ *has to hold. Then as a result*

(i) $\forall t \in [0, t_2\rangle$, $\phi(t) > \phi_r(t)$.
(ii) $\forall t \in [t_2, \infty\rangle$, $\phi(t) = \phi_r(t)$.
(iii) $\forall t \in [0, t_1\rangle$, $\phi'(t) > \phi_r'(t)$ *if t is a point at which* ϕ *and* ϕ_r *are differentiable with respect to time. If they are not differentiable at some point* $t \in [0, t_1\rangle$ *then the inequality still holds in that point for the left- and right-hand derivatives. Further,* $\phi_L'(t_1) > \phi_{rL}'(t_1)$. *(A subscript L denotes left-hand derivatives)*
(iv) $\phi_L'(t_2) \leqq \phi_{rL}'(t_2)$.

Proof. See Appendix 3. ||

By reversing the reference model and the alternative model, we obtain the results in case of "downward" shifting exogenous variables. Simultaneous occurrence of some C1, C2, C3, C4 can be examined by sequential application of Theorem 3. In Theorem 3, the inequality restrictions concerning $\phi_r(t)$ and $\lambda(t)$ are imposed only for expositional elegance; they rule out uninteresting cases in which changing exogenous variables do not influence the reservation wage path. Sufficient conditions in terms of the exogenous variables are given in Appendix 3. If for every $t > 0$, $\alpha(t) = 0$, $\beta(t) = \infty$ (which holds for example in case of lognormally distributed wages) and $\lambda(t) > 0$, then the restrictions are always satisfied.

The intuition behind (i) and (ii) is straightforward. Any future shift in the time path of exogenous variables that benefits the expected discounted lifetime income induces job searchers to be more selective in their search process. As for the period up to t_1, the shift in exogenous variables after point t_1 becomes more important when going forward in time. This implies that $\phi(t)$ shifts away from $\phi_r(t)$ when t comes closer to t_1. However, it is not always true that $\forall t \in \langle t_1, t_2\rangle$, $\phi'(t) < \phi_r'(t)$, if properly defined. It is easy to find time paths of the exogenous variables in the alternative model that cause $\phi'(t) > \phi_r'(t)$ for some $t \in \langle t_1, t_2\rangle$.

Mortensen (1986) gives the signs of the derivatives of the reservation wage with respect to exogenous variables in a stationary model. Those results are in accordance with Theorem 3 (take the reference model and the alternative model to be stationary, so $t_1=0$, $t_2=\infty$).

3.4. *The unemployment duration distribution*

Given the results concerning the time path of the reservation wage, we can construct the unemployment duration distribution in a nonstationary job search model and extend the comparative dynamics analysis using this distribution. Define the hazard $\theta(t)$ of leaving unemployment at time t as

$$\theta(t)=\lambda(t)\cdot\bar{F}(\phi(t);t). \tag{7}$$

By virtue of Assumption 1, $\theta(t)$ is a continuous function of $\phi(t)$. From Theorem 1 and Assumption 3 then, $\theta(t)$ is a continuous function of t except for points of time at which $\lambda(t)$ or $F(w;t)$ are discontinuous functions of t.

The unemployment duration density is given by the well-known equation

$$h(t)=\theta(t)\cdot\exp\left\{-\int_0^t\theta(u)du\right\}. \tag{8}$$

From the continuity of $\phi(t)$ and the piece-wise continuity of $\bar{F}$ and λ as a function of t and from the boundedness of $\bar{F}$ and λ, it is clear that the integral in equation (8) exists for every $t\geqq 0$. For points of time at which $\theta(t)$ is a continuous function of t, $h(t)$ is continuous as well, and vice versa. Note that though $h(t)$ is discontinuous at points where $\lambda(t)$ or $F(w;t)$ are discontinuous functions of t, the distribution function associated with $h(t)$ is a continuous function of t on the whole interval $[0,\infty\rangle$.

The expected duration of unemployment can be written as

$$E(t)=\int_0^\infty\exp\left\{-\int_0^t\theta(u)du\right\}dt. \tag{9}$$

Note that this expression may not exist. E.g. if for every $t\geqq 0$, $\lambda(t)=0$ then also $\theta(t)=0$ and people remain unemployed forever. Sufficient for existence is that $\lambda(T)>0$ and $b(T)<\beta(T)$ for then $\theta(T)>0$ and $E(t)\leqq T+(1/\theta(T))$. From (7) we infer that if for some t, $\alpha(t)<\phi(t)<\beta(t)$ and $\lambda(t)>0$ then shifts in benefits that cause a rise of $\phi(t)$ also cause a fall of $\theta(t)$. Because of the continuity to the right of $\phi(t)$, $\lambda(t)$ and $F(w;t)$ as a function of t, $\theta(t)$ will fall in a neighbourhood of t. Consequently, we have as a corollary from Theorem 3.

Corollary. *Let Assumptions* 1-5 *be satisfied. If* $b(t)$ *is raised for every* $t\in[t_1,t_2\rangle$ *with* $0\leqq t_1<t_2\leqq\infty$ *such that the new* $b(t)$ *also satisfies Assumptions* 2-4 *and if there is a point* t_3 *with* $0\leqq t_3<t_2$ *at which* $\alpha(t_3)<\phi(t_3)<\beta(t_3)$ *and* $\lambda(t_3)>0$ *then the expected duration of unemployment increases if it exists.*

In Appendix 3 sufficient conditions for $\alpha(t_3)<\phi(t_3)<\beta(t_3)$ are given.

4. EXOGENOUS VARIABLES AS STEP FUNCTIONS OF TIME

In the sequel we adopt an additional assumption, namely:

Assumption 6. $F(w;\, t)$, $\lambda(t)$ and $b(t)$ are step functions of t on $[0, \infty\rangle$.

For $b(t)$ in particular this is what is often seen in practice.

We can now split the positive real axis on which time is measured into intervals within which the exogenous variables are constant. On such an interval, equation (2) reduces to a constant coefficient differential equation. Moreover, this differential equation has a stationary solution (i.e. the solution for which $\phi'(t)=0$) which is constant on that interval. This solution corresponds to ϕ_0 as it is defined by equation (4) in a more general setting. In Subsection 3.2 it was shown that $\phi'_R(t)$ can be considered to be a monotonically increasing function of $\phi(t)$. This also holds for $\phi'_L(t)$. Further, if in a model that satisfies Assumptions 1-6, $\phi'(t)$ exists for some t, then so does $\phi''(t)$. By differentiating the constant coefficient differential equation with respect to t we find that $\phi'(t)$ and $\phi''(t)$ have equal sign. Thus we have the following information about the shape of $\phi(t)$ within intervals on which the exogenous variables are constant:

Theorem 4. *Let Assumptions* 1-6 *be satisfied. Let the exogenous variables be constant on an interval* $[t_*, t^*\rangle$, $0 \leqq t_* < t^* \leqq \infty$. *Then for every* $t \in \langle t_*, t^*\rangle$

$$\phi(t) \lesseqgtr \phi_0 \quad \Leftrightarrow \quad \phi'(t) \lesseqgtr 0 \quad \Leftrightarrow \quad \phi''(t) \lesseqgtr 0 \quad \Leftrightarrow$$
$$\phi(t_*) \lesseqgtr \phi_0 \quad \Leftrightarrow \quad \phi'_R(t_*) \lesseqgtr 0 \quad \Leftrightarrow \quad \phi(t^*) \lesseqgtr \phi_0 \quad \Leftrightarrow \quad \phi'_L(t^*) \lesseqgtr 0.$$

Deviations of $\phi(t)$ from ϕ_0 arise because of anticipations of future changes of the values of exogenous variables. As time proceeds, these changes come nearer. Now the rate of discount is positive and the probability of finding a job before the end of the present interval when following the optimal strategy decreases when t rises. Therefore anticipations become stronger and ϕ shifts away further from ϕ_0. As ϕ is the only variable that changes within the interval, this in turn implies that ϕ' increases in absolute value, which explains the sign of ϕ''. Note that the sign of $\phi - \phi_0$ at the end point of an interval can be thought of as determining the sign of the slope of ϕ within the interval.

Now suppose there is only one point in time T at which exogenous variables are allowed to change values. In addition, suppose that only one exogenous variable changes in value at T, according to one of the following four rules: (if necessary, values of the exogenous variables before and after T will be distinguished by subscripts 1 and 2 respectively).

D1. $b_1 > b_2$.

D2. $\lambda_1 > \lambda_2$ while $\phi(T) < \beta$.

D3. F_1 first order stochastically dominates F_2 while $\phi(T) < \beta_1$ and $\lambda > 0$.

D4. F_1 is a mean preserving spread of F_2 while $\alpha_1 < \phi(T) < \beta_1$ and $\lambda > 0$.

Let ϕ, and ϕ_2 denote the stationary solutions on the time intervals $[0, T\rangle$ and $[T, \infty\rangle$, respectively. Whether $\phi_1 \lesseqgtr \phi_2$ follows from the well-known comparative statics results in a stationary model. E.g. in a stationary model an increase in b implies an increase in the stationary reservation wage, so in case D_1 $\phi_1 > \phi_2$ and consequently $\phi(T) < \phi_1$. Theorem 4 can then be applied in order to obtain the following.

Corollary. *Let Assumptions 1-6 be satisfied. Let T be the only one point in time at which exogenous variables are allowed to change values, according to* D1, D2, D3 *or* D4. Then $\phi_1 > \phi_2$ *and*

(i) *for every* $t \in [0, T\rangle$, $\phi_2 < \phi(t) < \phi_1$, $\phi'(t) < 0$, $\phi''(t) < 0$.
(ii) $\phi(T) = \phi_2$, $\phi'_L(T) < 0$, $\phi'_R(T) = 0$.

Again, simultaneous occurrence of some D1, D2, D3, D4 can be examined by sequential application of the corollary. Note that a part of this corollary can also be proved using Theorem 3. Burdett (1979) and Mortensen (1977) proved that in case D_1, $\phi'(t) < 0$ for every $t \in [0, T\rangle$, in a model in which time devoted to search is endogenous. Mortensen (1986) also proved that for every $t \in [0, T\rangle$, $\phi'(t) < 0$ if, in terms of our model, both λ and b decrease at T.

The results in Sections 3 and 4 hold for the basic job search model as outlined in Section 2. However, similar results can be obtained for models that are more realistic in some respects. For instance, in reality one generally knows the wage rate associated with a vacancy before one responds to that vacancy, i.e. before the job is actually offered. Narendranathan & Nickell (1985) constructed a search model that deals with this. In van den Berg (1988) it is shown that such a model can be rewritten as the model described in Section 2 though of course the interpretation of some of the variables changes. Some of the papers in which stationary structural search models are estimated assume utility maximization instead of income maximization (Narendranathan and Nickell (1985), van den Berg (1988)), so it may be worthwhile to examine in what sense the results are affected if utility is nonlinear. Assume that utility is intertemporally separable, the instantaneous utility function being $u(w)$ in case one works at a wage rate w and $v.u(b(t))$ in case one is unemployed for t periods, receiving benefits $b(t)$. The parameter v represents the non-pecuniary component of instantaneous utility in unemployment relative to employment. In order to obtain elegant results and in order to restrict attention to economically meaningful cases, it is assumed that u is a differentiable function on $\langle 0, \infty\rangle$ with for every $t \in \langle 0, \infty\rangle$ $u'(t) > 0$ and that $E_{w;t}(u(w))$ is a uniformly bounded function of t. Further, v has to be positive. If $u(0)$ is not defined then $b(t)$ has to be positive for every $t \geqq 0$. It can be proved that Theorem 1 holds in such a model with $\phi(t)$ satisfying

$$u'(\phi(t)) \cdot \phi'(t) = \rho \cdot u(\phi(t)) - \rho \cdot v \cdot u(b(t)) - \lambda(t) \cdot \int_{\phi(t)}^{\infty} u'(w)\bar{F}(w;t)dw \qquad (10)$$

in all points at which $\phi'(t)$ is defined. Again, (10) can be used to calculate the right-hand derivative and left-hand derivative in points at which $\phi'(t)$ is not defined. The model can be rewritten in terms of the basic model, defining e.g. a transformed level of benefits $b^*(t)$ as $v \cdot u(b(t))$. After doing so the other theorems can be applied to obtain results for the model with utility maximization.

5. AN EMPIRICAL ILLUSTRATION

5.1. *Introduction*

In this section we present the results of the estimation of a nonstationary structural job search model in order to illustrate the importance of allowing for nonstationarity. One of the main items in the applied literature on unemployment duration is the magnitude of the effect of a change in the benefits level on the expected duration (for a survey, see e.g. Atkinson (1988)). However, though generally it is acknowledged that in most countries benefits are a decreasing function of duration, the models used in empirical analyses do not deal with this (for references, see Section 1). The structural models that are used erroneously assume that b is constant throughout duration. Estimated reduced form models of duration generally allow for (parametric) duration-dependence that acts multiplicatively on the hazard θ whereas the observed b is treated as a constant (i.e. non-time-varying) regressor in θ. Moffitt (1985) argued that such a proportional hazard model cannot be a satisfactory representation of the duration-dependence due to decreasing benefits since this acts in a non-proportional way on the hazard. This is an important point because, as will be shown, these decreases can be substantial. It is clear that in a structural nonstationary setting such problems do not exist. Nickell (1979) and Narendranethan, Nickell and Stern (1985) estimate proportional hazard models in which both b and the coefficient in the hazard associated with b are allowed to vary across different periods. Though the models are more general than the proportional hazard models that are commonly used, they are not able to represent some of the essential features of nonstationarity due to decreasing benefits. First, and most important, the models do not allow for anticipation of future changes of the level of benefits. The specified hazard θ at t depends on the present level of benefits $b(t)$ only and is not allowed to depend on future values of b which in fact may have a large influence on the present reservation wage and therefore also on $\theta(t)$. Another objection to these models is that no account is taken of the diminishing influence of the level of benefits within a period, as time proceeds towards the end of that period.

Using micro data from 1983 on unemployed individuals we estimate a nonstationary structural model that allows for decreasing benefits. In The Netherlands at the beginning of the eighties the benefits level during the first years of unemployment was related to the pre-unemployment wage while after that it was determined by the public assistance system. As a consequence, for an individual who has had a job before becoming unemployed benefits generally decrease substantially when duration equals about 2 years. Such a decrease does not occur if the benefits level related to the pre-unemployment wage is below the public assistance level, or if the individual did not have a job before becoming unemployed e.g. because he is a new entrant on the labour market. In those cases he obtains public assistance benefits from the beginning. The data used to estimate the model are obtained from a survey among some 400 males in Amsterdam. The sampling scheme of the survey was meant to over-represent unemployed individuals, but it makes no reference to the benefits paths of the respondents or to factors that determine those paths. As a result the level of benefits at 2 years of unemployment does not decrease for all unemployed individuals in the sample. Though the sample is somewhat small, it contains some interesting information on the labour market environment and the behaviour of the respondents, including subjective responses on reservation wages. This information is extensively used in the analysis. From the estimated model we can calculate sample averages of the elasticities of the expected duration with respect to the levels of benefits before and after 2 years of unemployment. Information on the magnitudes of such elasticities may be valuable for policymakers.

5.2. *The model*

We use the search model described at the end of Section 4. Analogous to Narendranathan and Nickell (1985) and van den Berg (1988), the utility function of income u is logarithmic so we assume that individuals are risk averse. For the wage offer density $f(w)$ the following functional form is chosen

$$f(w)=\begin{cases}\dfrac{1}{w}\dfrac{1}{\log\beta/\alpha} & \alpha\leqq w\leqq\beta,\ 0<\alpha<\beta<\infty\\ 0 & \text{elsewhere.}\end{cases}\tag{11}$$

This distribution is positively skewed and rules out wage offers close to zero or infinity. Moreover, in the stationary version of the model with wage offer density (11) it always holds that $d\theta/d\lambda\geqq 0$ (This inequality does not follow for every conceivable class of wage offer distributions). A particular advantage of specification (11) is that we can use subjective responses on α and β to estimate individual wage offer distributions.

Nonstationarity arises if the level of benefits decreases when unemployment duration t equals T months. After $t=T$ the reservation wage is constant and can be calculated by imposing $\phi'(t)=0$ in equation (10). Before $t=T$, $\phi(t)$ follows the differential equation (10). For the functional forms of u and $F(w)$ mentioned above this is a first-order nonlinear differential equation in $\log\phi(t)$ with constant coefficients. It can be solved using the boundary condition $\phi(T)$.

5.3. *The data and the empirical implementation of the model*

The data were obtained from a survey among some 400 males in Amsterdam who were at the date of the interview between 30 and 55 years old. A descriptive analysis of these data can be found in Ridder (1987). The respondents were asked to reconstruct their labour market histories over the past 10 years until the date of the interview which was between October 20 and December 18, 1983. In addition they were asked to provide information on income variables and personal characteristics. The respondents were drawn from three different sampling schemes. In all cases males in Amsterdam aged between 30 and 55 were sampled. The first subsample (RS) is a random sample from this group of individuals. From this we selected 22 individuals who were unemployed at the date of the interview. The second subsample (SS) is a sample of individuals who were unemployed at September 1, 1983. From this we selected 137 individuals. The third subsample (FS) is a sample of the inflow into unemployment around September 1, 1983. From this we selected 41 individuals, which gives a total number of 200 individuals. For RS we determined t_b, the elapsed duration of unemployment at the date of the interview. For SS we determined t_b, the elapsed duration of unemployment at September 1, 1983, and t_f, the duration of unemployment after that date. Finally, for FS we determined t_c, the duration of the spell of unemployment starting around September 1, 1983. All t_f and most t_c are censored. Because of the lack of information on income variables for past spells of unemployment, such spells could not be used. The individual log-likelihood contribution of t_c for FS is

$$\mathscr{L}_{FS}(t_c)=(1-c_1)\log(\theta(t_c))-\int_0^{t_c}\theta(t)\,dt\tag{12}$$

in which $c_1=1$ if t_c is censored and 0 elsewhere. Assume that the individual entry rate into unemployment is constant before the moment of the interview. Then for RS,

$$\mathscr{L}_{RS}(t_b)=-\int_0^{t_b}\theta(t)\,dt-\log E(t)\tag{13}$$

while for SS

$$\mathscr{L}_{SS}(t_b, t_f) = -\int_0^{(t_b+t_f)} \theta(t)dt - \log E(t) \tag{14}$$

(see e.g. Ridder (1984)). Recall that t_f in (14) is censored. $E(t)$ in (13) and (14) follows from equation (9).

Individuals who were unemployed at the date of the interview were asked for their lowest acceptable net wage in a job at that date. These "observed" reservation wages $\tilde{\phi}(t)$ may differ from the true reservation wages,

$$\tilde{\phi}(t) = \phi(t) + \mathscr{E} \tag{15}$$

$\mathscr{E}$ is an error term which is interpreted as a measurement error that is i.i.d. across individuals and independent of duration t. Consequently, individuals use $\phi(t)$ instead of $\tilde{\phi}(t)$ as their strategy at t so $\theta(t)$ depends on $\phi(t)$ instead of $\tilde{\phi}(t)$ and equations (12), (13) and (14) do not depend on $\mathscr{E}$. Further, by assuming that $\mathscr{E}$ has a normal distribution with mean zero and variance σ^2 we have that, conditional upon the elapsed duration t (t_c in case of FS, t_b in case of RS, $t_b + t_f$ in case of SS), $\tilde{\phi}(t)$ has a normal distribution with mean $\phi(t)$ and variance σ^2. The total log-likelihood contribution of an individual can be written as the sum of the marginal contribution of the duration variables (see equations (12)-(14)) and the conditional contribution of the observed reservation wage. The latter equals

$$(1-c_2)\left\{-\tfrac{1}{2}\log(2\pi) - \log\sigma - \tfrac{1}{2}.\left(\frac{\tilde{\phi}(t)-\phi(t)}{\sigma}\right)^2\right\} \tag{16}$$

in which $c_2 = 1$ if $\tilde{\phi}(t)$ is missing (16 individuals) and 0 elsewhere.

In order to be able to estimate the model additional information is required concerning $F(w)$ (see Flinn and Heckman (1982)). It seems natural to use post-unemployment wages because these are random drawings from $F(w)$ truncated at $\phi(t)$. However, our sample is basically restrospective and only in FS are a few post-unemployment wages observed. Moreover, $F(w)$ as specified is not recoverable from the truncated $F(w)$. Therefore we take a totally different route in estimating $F(w)$. Analogous to Lancaster and Chesher (1983) and Lynch (1983) we use subjective responses on characteristics of $F(w)$. Unemployed respondents were asked what the minimal and maximal wages were of those employed in their occupation. The questions make no references to the strategy actually used to locate potential wage offers, so the answers can be interpreted as "observed" minimal and maximal wage offers $\tilde{\alpha}$ and $\tilde{\beta}$, respectively. Again we postulate that the true α and β are imperfectly observed by $\tilde{\alpha}$ and $\tilde{\beta}$ because of non-systematic measurement errors. We ran an OLS regression of log $\tilde{\alpha}$ and $\tilde{\beta}$ on observed personal characteristics, using data from individuals who responded on $\tilde{\alpha}$ and $\tilde{\beta}$ (134 and 128 observations, respectively). For all 200 individuals, α and β can be predicted using these estimated relationships. Analogous to Narendranathan and Nickell (1985) and van den Berg (1988), the predicted $F(w)$ are substituted in when estimating the structural model.

In order to estimate the model the whole benefits path $b(t)$, $0 \leqq t < \infty$ has to be known rather than just the level of benefits at the moment of interview. If an individual has had a job before becoming unemployed then during the first half year of unemployment his benefits level equals 80% of the previous wage while during the next 1·5 to 2 years it equals about 70% of the previous wage. After about 2 years of unemployment he obtains public assistance benefits depending on household composition and the financial characteristics of other household members. (The exact unemployment duration at which

b decreases from 70% of the previous wage to the public assistance level depends on the individual's labour market history, but is generally close to 2 years. In order to keep the exposition simple, we take T to be equal to 2 years for every individual in the sample. Sensitivity checks show that the results are robust with respect to small changes in T.) The decrease from 80% to 70% is not very substantial and is generally much smaller than the decrease at 2 years of unemployment, so in order not to complicate the empirical analysis we will concentrate on the latter decrease and assume that during the first two years 70% of the previous wage is obtained. If this 70% is below the public assistance benefits level then the individual obtains the latter and the model reduces to a stationary model. If the individual did not have a job before becoming unemployed (e.g. because he is a new entrant on the labour market) then he obtains public assistance benefits from the beginning and the model is stationary. As a result, for 136 of the 200 individuals the model is nonstationary. Using survey information on the (inflation-corrected) previous wage and on the level of benefits at the date of the interview and applying the rules of the public assistance system in 1983 in the Netherlands the variables $b(0)$ and $b(T)$ were constructed.

The job offer arrival rate λ and the relative disutility of being unemployed v are parameterized as exponential functions of observable characteristics x_1 and x_2, respectively,

$$\lambda = \exp(x_1'\beta_1), \qquad v = \exp(x_2'\beta_2).$$

The vector x_1 contains possible indicators of λ e.g. because they give an indication of the productivity of the searcher. Note that the sample is homogeneous with respect to sex and geographic area and fairly homogeneous with respect to age so these are not included in x_1 and x_2. The vector x_2 contains possible indicators of v e.g. because they give an indication of the attitude towards work of people in the direct environment of the searcher.

The estimation we have employed was ML using the BHHH algorithm. Because we can solve analytically for $\phi(t), \int_0^t \theta(u)du$ and $E(t)$ as functions of t, $b(t)$ $(0 \leqq t < \infty)$, $F(w)$, λ, v, ρ and σ^2 it follows that the likelihood can be written analytically as a (very complicated) function of the unknown parameters β_1, β_2, ρ and σ^2.

5.4. *The results*

The parameter estimates for the nonstationary structural model described in Subsections 5.2 and 5.3 are presented in Table 1. The unit time period is one month. For education the reference category is level 1. Generally, the results seem to be in accordance with intuition. Because this is merely an empirical illustration and because our main interest is in the elasticities of duration with respect to benefits we will not give a lengthy account of these results. Also, we are not particularly interested in search characteristics of unemployed individuals whose environment is stationary so the results below are only for individuals whose environment does change. Given the parameter estimates, the main variables of the search process can be estimated. Table 2 presents sample averages of the estimates of λ, $\bar{F}(\phi(0))$ and $\bar{F}(\phi(T))$ for different levels of education. The acceptance probability increases by about 0·1 from the moment that one becomes unemployed until the moment that one is unemployed for 2 years. After 2 years of unemployment most job offers are acceptable. Rejection of an offer may well imply a waiting time of more than a year before the next offer arrives. (This result is also found in other studies on unemployment in The Netherlands at the beginning of the eighties, see van den Berg

TABLE 1

Parameter estimates for the search model

	Variable/Parameter	Coefficient	(t-ratio)
(i)	Job offer arrival rate		
	Constant	−3·72	(18·5)
	Dutch (0/1)	0·18	(1·0)
	Education: level 2 (0/1)	0·18	(0·8)
	Education: level 3 (0/1)	0·42	(1·9)
	Married (0/1)	0·48	(2·6)
	Partner has paid job (0/1)	0·22	(0·9)
(ii)	Relative disutility of unemployment		
	Constant	−0·01	(0·5)
	Education: level 2 (0/1)	−0·02	(0·6)
	Education: level 3 (0/1)	−0·12	(1·9)
	Partner has paid job (0/1)	0·01	(0·3)
(iii)	Subjective rate of discount (in percent per year)	12%	(3·4)
(iv)	Standard deviation of measurement error of reservation wage	469	(29·9)
Log likelihood = −2095·65			

TABLE 2

Probabilities and expectations

Level of Education	1	2	3
λ (expected number of offers)	0·040	0·047	0·060
$\bar{F}(\phi(0))$(proportion of offers acceptable at $t=0$)	0·78	0·68	0·87
$\bar{F}(\phi(T))$(proportion of offers acceptable after 2 years)	0·88	0·83	0·95

(1988)). In the meantime the only source of income is public assistance benefit which generally is much smaller than α. Moreover, because $v<1$ one also dislikes being unemployed for non-pecuniary reasons.

The results so far enable us to investigate a number of questions related to the effectiveness of policies aimed at a reduction of unemployment durations. Table 3 presents for different levels of education, sample averages of the elasticities of the reservation wages $\phi(0)$ and $\phi(T)$ and the expected duration $E(t)$ with respect to the levels of benefits $b(0)$ and $b(T)$. The effects of a simultaneous proportional change of $b(0)$ and $b(T)$ are found by summing the elasticities in part (i) and part (ii) of Table 3. Of course the elasticity of $\phi(T)$ with respect to $b(0)$ is identically zero: the optimal strategy does not depend on past income. What strikes one is that all other elasticities for the highest level of education are smaller than the corresponding elasticities for levels 1 and 2. Highly educated individuals dislike being unemployed for non-pecuniary reasons more than others do. Further, the job offer arrival rate and the difference between the mean wage offer and the level of benefits are larger for them. Consequently, the expected duration is much shorter and the expected present value of search is not dominated by the prospect of being dependent on benefits for a long time. For levels of education 1 and 2 the most striking feature of Table 3 is that the elasticity of the expected duration with respect to the benefits level after 2 years of unemployment (the public assistance benefits level) is

TABLE 3

Elasticities with respect to benefits

	Level of Education	1	2	3
(i)	With respect to the level of benefits before 2 years			
	$\frac{\partial \log \phi(0)}{\partial \log b(0)}$ (reservation wage at $t=0$)	0·15	0·14	0·11
	$\frac{\partial \log \phi(T)}{\partial \log b(0)}$ (id. after 2 years)	0	0	0
	$\frac{\partial \log E(t)}{\partial \log b(0)}$ (expected duration)	0·14	0·16	0·07
(ii)	With respect to the level of benefits after 2 years			
	$\frac{\partial \log \phi(0)}{\partial \log b(T)}$	0·09	0·08	0·04
	$\frac{\partial \log \phi(T)}{\partial \log b(T)}$	0·23	0·21	0·14
	$\frac{\partial \log E(t)}{\partial \log b(T)}$	0·47	0·59	0·06

much larger than the corresponding elasticity with respect to the level before 2 years (the pre-unemployment-wage-related benefits level). This implies that a decrease of $b(T)$ would be much more effective in reducing durations than a decrease of $b(0)$ would be. Note that changing the value of $b(T)$ affects the reservation wage $\phi(t)$ on the whole time interval $[0, \infty\rangle$ whereas changing the value of $b(0)$ only affects $\phi(t)$ on $[0, T\rangle$. Moreover, the influence of $b(0)$ on $\phi(t)$ is diminishing at t proceeds on $[0, T\rangle$. For levels of education 1 and 2 the anticipation on $t < T$ of the decrease of the benefits level at T is quite strong because the probability of getting a job during the first 2 years of unemployment is rather small. In other words, the short-term unemployed individuals' strategy is sensitive with respect to changes in the benefits level for the long-term unemployed because they know they may well become long-term unemployed themselves.

Information on the magnitudes of such elasticities may be valuable for policymakers. E.g. shifting $b(t)$ on $t \geqq T$ is almost as effective in reducing duration of individuals with level of education 1 or 2 as shifting the whole benefits path. The estimated model can be used for simulating alternative benefits policies. For every alternative benefits path the optimal strategy can be solved from equation (10). Note that all these results can not be obtained by using stationary models.

The empirical model used in this section may be restrictive in some respects. For instance, it was assumed that λ and $F(w)$ are stationary. Moreover, we did not allow for transitions into a third state, say non-participation. These features can be implemented but the empirical analysis of such extended models requires more data and is a task for further research.

When deriving the likelihood no account has been taken of unobserved heterogeneity in the sample, which may bias the results. However, from a numerical point of view, the inclusion of a random heterogeneity term would complicate things enormously even in a stationary model, so it would be beyond the scope of this illustration to do so.

6. CONCLUSION

In this paper we have examined nonstationarity in job search theory. The optimal reservation wage path over time has been derived under weak assumptions concerning the exogenous variables. We also have given comparative dynamics results. Furthermore, by assuming the exogenous variables to be step functions of time we were able to derive additional properties of the reservation wage path. Generally these properties are in accordance with economic intuition. As an empirical illustration we estimated a nonstationary structural job search model. The model allows for the level of benefits to be a decreasing function of unemployment duration, which is a stylized fact in most countries. It appeared that generally the elasticity of the expected duration with respect to the level of benefits after two years of unemployment is much larger than the elasticity with respect to the level of benefits that is obtained in the first two years of unemployment.

There are some straightforward directions for further research. Instead of assuming that unemployed individuals have perfect foresight with respect to the future time paths of b, λ and $F(w)$, it might be more realistic to allow for stochastic changes in these. These may be due to such things as unforeseen changes in aggregate macroeconomic conditions or changes in personal circumstances. It then seems reasonable to assume that individuals are aware of these additional elements of uncertainty and derive their optimal strategies given some (subjective) assessment of the probabilities that such changes occur. The analysis of the optimal strategy is much more complicated in such nonstationary models because $\phi(t)$, if it exists, is not only a function of time but also of the realizations at t of the stochastic elements in b, λ and $F(w)$. Also, the empirical analysis will be much harder because the probability assessments of the changes generally appear explicitly in the structural model.

At the end of the empirical illustration in Section 5 we mentioned some apparent rigidities of the model specification used. A task for future empirical research is to relax those rigidities.

APPENDIX.

Proof of Theorem 1.

For a derivation of the properties of the optimal strategy it is necessary to examine in detail the expected present value of income when unemployed. Individuals who are unemployed for t units of time are assumed to maximize the following expression.

$$E\int_t^\infty e^{-\rho(\tau-t)}y(\tau)d\tau \tag{A1}$$

in which $y(\tau)$ denotes the income flow at τ and expectation is taken over job offer arrival times and wage offers. Let $R(t)$ denote the expected present value of income at t when following the optimal strategy. Then $R(t)$ is the supremum of expression (A1) over all admissible policies. For nonstationary decision processes, a recursive equation in terms of the optimal value generally does not follow trivially from some optimality principle. Indeed the derivation of such an equation would need a rather heavy measure-theoretic apparatus (see e.g. Hinderer (1970)) and the optimal control literature on such problems in a continuous-time, nonstationary context is not very well developed yet (see Whittle (1983)). Therefore such a task is beyond the scope of the paper and a different route is followed: the recursive relation is stated and it is proved that there exists a unique solution $R(t)$ which is bounded and continuous in t and which can be differentiated with respect to t almost everywhere. Using the relation between $R(t)$ and the reservation wage, the desired properties of the latter can be deduced.

$R(t)$ is written recursively as a function of $R(\tau)$, $\tau > t$, in which τ is interpreted as the point of time at which the next offer arrives (so $\tau - t$ is the waiting time until the next offer). First the distribution of τ given t has to be derived. The job offer probability in a small interval $[\tau, \tau + d\tau)$ conditional on not having received an offer between t and τ and conditional on being unemployed at t, is $\lambda(\tau)d\tau$. Defining $G(\tau; t)$ to be the distribution function of τ for someone whose elapsed duration equals t, we have the familiar result

$$G(\tau; t) = 1 - \exp\left\{-\int_t^\tau \lambda(s)ds\right\} \qquad \tau \geqq t. \tag{A2}$$

Because λ is uniformly bounded and continuous almost everywhere in t, equation (A2) properly defines $G(\tau; t)$ for every $t \geqq 0$. Note that assumption 2 allows for $\lambda(t) = 0$ for every t. In such a case the state of unemployment is absorbing because job offers never arrive; $G(\tau; t) = 0$ for every τ so τ has a defective distribution.

Now $R(t)$, $0 \leqq t < \infty$, can be written recursively as

$$R(t) = \int_t^\infty \left[\int_t^\tau b(s)e^{-\rho(s-t)}ds + e^{-\rho(\tau-t)}E_{w;\,\tau} \max\left(\frac{w}{\rho}, R(\tau)\right)\right] dG(\tau; t) \tag{A3}$$

From t to τ the individual receives benefits; at τ he has to choose between acceptance of a job offer (present value w/ρ) and rejection of it (present value $R(\tau)$). From Assumption 4 it follows that if duration t exceeds T then the model breaks down to a stationary model. Therefore $R(t)$ is constant for $t \geqq T$ and, as has been shown often before, $R(T)$ is the unique finite solution to

$$\rho R(T) = b(T) + \lambda(T)\, E_{w;\,T} \max\left(\frac{w}{\rho} - R(T), 0\right) \tag{A4}$$

if the assumptions on boundedness hold. Consequently, further analysis of equation (A3) can be restricted to $t \in [0, T]$.

It is rather straightforward but tedious to show that if the assumptions on semi-continuity and boundedness hold, then the right-hand side of equation (A3) is a mapping $M(R)$ which maps the space of continuous functions on $[0, T]$ into itself, and the integrals in (A3) are well-defined (see e.g. Haaser and Sullivan (1971)). Let C[0, T] denote the space of continuous functions on $[0, T]$, normed with the sup-norm. Then $C[0, T]$ is a Banach space. We now show that M is a contraction mapping, i.e. that there is an $\alpha \in \langle 0, 1\rangle$ such that for every $R, R^* \in C[0, T]$ it holds that $\|M(R) - M(R^*)\| \leqq \alpha.\ \|R - R^*\|$. We have

$$\begin{aligned}
\|M(R) - M(R^*)\| &= \sup_{0 \leqq t \leqq T} |M(R)(t) - M(R^*)(t)| \\
&= \sup_{0 \leqq t \leqq T} \left| \int_t^\infty e^{-\rho(\tau-t)} E_{w;\tau} \left(\max\left(\frac{w}{\rho}, R(\tau)\right) - \max\left(\frac{w}{\rho}, R^*(\tau)\right)\right) dG(\tau; t)\right| \\
&\leqq \sup_{0 \leqq t \leqq T} \int_t^\infty e^{-\rho(\tau-t)} E_{w;\tau} \left|\max\left(\frac{w}{\rho}, R(\tau)\right) - \max\left(\frac{w}{\rho}, R^*(\tau)\right)\right| dG(\tau; t).
\end{aligned} \tag{A5}$$

Because for every $x, y, z \in \mathbb{R}$, $|\max(x, y) - \max(x, z)| \leqq |y - z|$ the expression above is bounded by

$$\begin{aligned}
&\sup_{0 \leqq t \leqq T} \int_t^\infty e^{-\rho(\tau-t)} |R(\tau) - R^*(\tau)| dG(\tau; t) \\
&\leqq \sup_{0 \leqq t \leqq T} \int_t^\infty e^{-\rho(\tau-t)} (\sup_{0 \leqq \tau \leqq T} |R(\tau) - R^*(\tau)|) dG(\tau; t) \\
&= \sup_{0 \leqq t \leqq T} |R(t) - R^*(t)| \cdot \sup_{0 \leqq t \leqq T} \int_t^\infty e^{-\rho(\tau-t)} dG(\tau; t).
\end{aligned}$$

The second part of the right-hand side of the last equation does not depend on R or R^* so it is now sufficient to prove that

$$\sup_{0 \leqq t \leqq T} \int_t^\infty e^{-\rho(\tau-t)} dG(\tau; t) \in [0, 1\rangle. \tag{A6}$$

One sees immediately that the supremum lies in the interval $[0, 1]$. It remains to prove that 1 is never obtained. If $\forall t \geqq 0$, $\lambda(t) = 0$ then expression (A6) equals zero. If there is a $t \geqq 0$ with $\lambda(t) > 0$ then, from the semi-continuity of λ as a function of time and from the positiveness of ρ, it follows that the expression is strictly bounded from above by the supremum over $0 \leqq t \leqq T$ of $1 - G(t; t)$, which never exceeds 1. Consequently, (A6) holds and M is a contraction mapping. From Banach's Theorem (Wouk (1979)) it follows that M which is defined on $C[0, T]$ has a unique fixed point. So, from equation (A3), the function $R(t)$ on $[0, T]$ exists and is the unique continuous function that solves equation (A3). Because of the stationarity after T this result can be extended to $R(t)$ on $[0, \infty\rangle$.

Equation (A3) can be used to derive the derivative of $R(t)$ with respect to t. It follows that

$$R'(t)=\rho R(t)-b(t)-\frac{\lambda(t)}{\rho}\int_{\rho R(t)}^{\infty}(w-\rho R(t))dF(w;t) \tag{A7}$$

in which the integral can be simplified to

$$\int_{\rho R(t)}^{\infty}\bar{F}(w;t)dw$$

by partial integration. Differentiation is only allowed if λ, b and $F(w)$ are continuous functions of time at t. However, because these functions are always continuous from the right, the right-hand side of (A7) gives the right-hand derivative of $R(t)$ with respect to t at points at which the exogenous variables are discontinuous. Similarly, because the left-hand limits of these variables exist, the left-hand side of $R(t)$ with respect to t at such discontinuity points is defined by

$$R'_L(t)=\rho.R(t)-\lim_{\tau\uparrow t}b(\tau)-\lim_{\tau\uparrow t}\lambda(\tau)\lim_{\tau\uparrow t}\int_{\rho R(t)}^{\infty}\bar{F}(w;\tau)dw. \tag{A8}$$

It is clear from equation (A3) that the optimal policy is to accept a wage offer w at time t if and only if w/ρ exceeds $R(t)$. In other words, the present value of working at a wage w has to exceed the present value of searching further in the optimal way. Consequently the optimal policy can be characterized by a reservation wage $\phi(t)$ defined by

$$\phi(t)=\rho\cdot R(t). \tag{A9}$$

and the theorem follows from the results on $R(t)$. If $\phi(t)$ does not lie between the upper and lower bound of the interval on which $F(w;t)$ increases then there are many other reservation wages that are able to characterize the optimal strategy. Still, $\phi(t)$ as defined by (A9) can be used any time to describe optimal behaviour.

Finally, it should be noted that Theorem 1 can be proved without using Assumption 4.

Proof of Theorem 2.

The structure of the proof is as follows. First we restrict attention to an unspecified time interval within which the exogenous variables are continuous. In Lemma A1 we show that sufficient for (i) and (ii) to hold in the interval is that, loosely speaking, $\phi_0(t)$ is strictly decreasing within that interval. The remainder of the proof is concerned with finding conditions that impose the required property to $\phi_0(t)$ for every interval, using backward induction.

We split the time axis into a finite number of intervals, within which every exogenous variable is continuous in time. The intervals are closed to the left side and open to the right. The last interval is $[T,\infty\rangle$. Now consider one such interval, say $[t_*,t^*\rangle$. From theorem 1, ϕ is a differentiable function of t and ϕ_0 is a continuous function of t on $[t_*,t^*\rangle$. Further, $\phi_L(t^*)=\phi(t^*)$ but it may be that $\phi'_L(t^*)\neq\phi'_R(t^*)$ or $\phi_{0L}(t^*)\neq\phi_0(t^*)$.

Lemma A1. *Let Assumptions 1-5 be satisfied. Consider the interval $[t_*,t^*\rangle$ as defined before. If $\phi(t^*)\leqq\phi_{0L}(t^*)$ and if*

$$\forall t\in[t_*,t^*\rangle\quad\forall\tau\in\langle 0,t^*-t\rangle\qquad\phi_0(t+\tau)<\phi_0(t) \tag{A10}$$

then $\forall t\in[t_*,t^*\rangle$ $\phi(t)<\phi_0(t)$, $\forall t\in\langle t_*,t^*\rangle$ $\phi'(t)<0$; $\phi'_R(t_*)<0$ and if $\phi_{0L}(t^*)>\phi(t^*)$ then $\phi'_L(t^*)<0$ while if $10\phi_{0L}(t^*)=\phi(t^*)$ then $\phi'_L(t^*)=0$.

Proof. Suppose that at some $t\in[t_*,t^*\rangle$ $\phi_0(t)\leqq\phi(t)$ holds. Then, from the discussions of equations (5) and (6) in subsection 3.2, $\phi'(t)\geqq 0$ if $t>t_*$, while $\phi'_R(t)\geqq 0$ if $t=t_*$. On the other hand, $\phi(t^*)\leqq\phi_{0L}(t^*)$. ϕ and ϕ_0 are continuous functions of t on $[t_*,t^*\rangle$ and ϕ_0 is decreasing in t. Therefore $\phi_0(t)\leqq\phi(t)$ cannot hold for any $t\in[t_*,t^*\rangle$. If $\phi_0(t)>\phi(t)$ for every $t\in[t_*,t^*\rangle$ then, again from Subsection 3.2, $\phi'(t)<0$ for every $t\in\langle t_*,t^*\rangle$ and $\phi'_R(t_*)<0$. Furthermore, if $\phi_{0L}(t^*)>\phi(t^*)$ then $\phi'_L(t^*)<0$ while if $\phi_{0L}(t^*)=\phi(t^*)$ then $\phi'_L(t^*)=0$. This completes the proof of Lemma A1. ||

Basically, we now only have to prove that ϕ_0 is decreasing in t. Consider case K2. For every $t\geqq T$, $\phi(t)=\phi_0(t)$ holds, due to the stationarity after T. If λ is discontinuous at T, then $\lambda_L(T)>\lambda(T)$. Because $b<\beta$ holds, we have for every $t\geqq 0$ that $\phi_0(t)<\beta$ holds (see equation (4)). Consequently, $Q(\phi_0(t))>0$ and therefore $\lambda_L(T)>\lambda(T)$ implies $\phi_{0L}(T)>\phi_0(T)$, as can be seen from equation (4). If $\lambda_L(T)=\lambda(T)$, then

$\phi_{0L}(T) = \phi_0(T)$, so in any case $\phi_0(T) \leqq \phi_{0L}(T)$. Now consider the interval $[t_*, t^*\rangle$ with $t^* = T$. Take a $t \in [t_*, t^*\rangle$ and a $\tau > 0$. Then, because b and $F(w)$ are constant in case K2,

$$\phi_0(t+\tau) - \phi_0(t) = \frac{\lambda(t+\tau)}{\rho}\{Q(\phi_0(t+\tau) - Q(\phi_0(t))\}$$

$$+\frac{1}{\rho}\cdot Q(\phi_0(t))\{\lambda(t+\tau) - \lambda(t)\} \tag{A11}$$

Again, $Q(\phi_0(t)) > 0$. Further, $\lambda(t+\tau) < \lambda(t)$. Inspection of (A11) shows that therefore $\phi_0(t+\tau) \geqq \phi_0(t)$ cannot hold. Because this is true for every $t \in [t_*, t^*\rangle$ and for every $\tau > 0$, we infer that (A10) holds for the interval ending at T.

So the conditions of Lemma A1 are satisfied and we can apply it, noting that $\phi_{0L}(t^*) > \phi_0(t^*)$ if $\lambda_L(t^*) > \lambda(t^*)$ while $\phi_{0L}(t^*) = \phi_0(t^*)$ if λ is continuous at t^*. In the latter case $\phi'_L(t^*) = 0$ of course implies $\phi'(T) = 0$.

As for the interval $[u, t_*\rangle$ before $[t_*, t^*\rangle$ with $t^* = T$, we can go through the same lines of argument. We have seen that $\phi(t_*) < \phi_0(t_*)$. Again λ may be discontinuous at t_*. In that case it follows that $\phi_{0L}(t_*) > \phi_0(t_*)$. So $\phi(t_*) < \phi_{0L}(t_*)$ holds in any case. Further, ϕ_0 decreases in t on $[u, t_*\rangle$ and Lemma A1 can be applied again. Going backward in time, one thus obtains Theorem 2 for case K2. Proofs of the other cases are analogous.

Proof of Theorem 3.

We split the time axis into a finite number of intervals, within which all exogenous variables from both models are continuous functions of time. The intervals are closed to the left and open to the right. We let t_1 and t_3 be left-hand bounds of an interval and we let t_2 be the right-hand bound of an interval. Now consider one of the intervals, say, $[t_*, t^*\rangle$. From Theorem 1, ϕ and ϕ_r are differentiable functions of t on $[t_*, t^*\rangle$. Further, ϕ and ϕ_r are continuous at t_* and t^* but they may not be differentiable at those points.

We outline the proof of case C2. Just like the proof of Theorem 2, we work backward in time. First, suppose $t_2 < \infty$. For every $t \geqq t_2$ $\phi(t) = \phi_r(t)$ holds, due to the equivalence of the exogenous variables of both models on $[t_2, \infty\rangle$. Consider the interval $[u, t_2\rangle$. (By definition $t_3 \leqq u$). From equation (2), we have for every $t \in [u, t_2\rangle$

$$\phi'(t) - \phi'_r(t) = \rho(\phi(t) - \phi_r(t)) - \lambda(t)\{Q(\phi(t); t) - Q(\phi_r(t); t)\}$$

$$+\{\lambda_r(t) - \lambda(t)\}\cdot Q(\phi_r(t); t). \tag{A12}$$

If $t = u$, we replace $\phi'(t) - \phi'_r(t)$ by $\phi'_R(u) - \phi'_{rR}(u)$. As for every $t \in [u, t_2\rangle$ $\phi_r(t) < \beta(t)$ holds, we have $Q(\phi_r(t); t) > 0$ on $[u, t_2\rangle$. So if there is a $t \in \langle u, t_2\rangle$ at which $\phi(t) \leqq \phi_r(t)$ then it follows from (A12) that $\phi'(t) < \phi'_r(t)$. Also, if $\phi(u) \leqq \phi_r(u)$ then $\phi'_R(u) < \phi'_{rR}(u)$. But $\phi(t_2) = \phi_r(t_2)$ and ϕ and ϕ_r are continuous functions of t. Therefore for every $t \in [u, t_2\rangle$, $\phi(t) > \phi_r(t)$ has to hold. Further, according to Theorem 1,

$$\phi'_L(t_2) - \phi'_{rL}(t_2) = \{\lambda_{rL}(t_2) - \lambda_L(t_2)\}\cdot Q_L(\phi_r(t_2); t_2)$$

which is nonpositive.

Now consider the interval $[y, u\rangle$. We just derived that $\phi(u) > \phi_r(u)$. Going through the same line of argument, it follows that for every $t \in [y, u\rangle$ $\phi(t) > \phi_r(t)$. Whether $t_3 = u$ or $t_3 \leqq y$ does not matter for this result. We can proceed this way until we arrive at the interval of which t_1 is the right-hand bound, say $[v, t_1\rangle$. We now have for every $t \in \langle v, t_1\rangle$

$$\phi'(t) - \phi'_r(t) = \rho(\phi(t) - \phi_r(t)) - \lambda(t)\{Q(\phi(t); t) - Q(\phi_r(t); t)\} \tag{A13}$$

For $t = v$ we have to replace $\phi'(t) - \phi'_r(t)$ by $\phi'_R(v) - \phi'_{rR}(v)$. If there is a $t \in \langle v, t_1\rangle$ at which $\phi(t) \leqq \phi_r(t)$ holds, then it follows from (A13) that $\phi'(t) \leqq \phi'_r(t)$, regardless of $t \lesseqgtr t_3$. Similarly, $\phi(v) \leqq \phi_r(v)$ implies $\phi'_R(v) \leqq \phi'_{rR}(v)$. But $\phi(t_1) > \phi_r(t_1)$ and ϕ and ϕ_r are continuous functions of t. Therefore for every $t \in [v, t_1\rangle$ $\phi(t) > \phi_r(t)$ has to hold. Further,

$$\phi'_L(t_1) - \phi'_{rL}(t_1) = \rho(\phi(t_1) - \phi_r(t_1)) - \lambda_L(t_1)\{Q_L(\phi(t_1); t_1) - Q_L(\phi_r(t_1); t_1)\} \tag{A14}$$

which is positive. Also, from (A13) it follows that for every $t \in \langle v, t_1\rangle$, $\phi(t) > \phi_r(t)$ implies that $\phi'(t) > \phi'_r(t)$ while $\phi(v) > \phi_r(v)$ implies that $\phi'_R(v) > \phi'_{rR}(v)$. Backward induction leads to the results for $t < v$.

If $t_2 = \infty$ we first examine the interval $[T, \infty\rangle$ on which the exogenous variables are constant. Because $T \geqq t_3$ we have $Q(\phi_r(t); t) > 0$ on this interval. Therefore increasing λ in this interval induces an increasing reservation wage. Now we can go through the same line of argument as before concerning the intervals that lie to the left of T. This completes the proof in case C2. Proofs of the other cases are analogous.

We now give sufficient conditions for the inequality restrictions on $\phi_r(t)$ on the interval $[t_3, t_2\rangle$. Without loss of generality we take $t_3 \geqq t_1$. Suppose that for every $t \geqq t_3$ it holds that $b_r(t) < \beta_r(t)$, while $\beta_r(t)$ does not increase as a function of t on $[t_3, \infty\rangle$. Using Theorem 1, we can prove that as a result $\phi_r(t) < \beta_r(t)$ for every $t \in [t_3, t_2\rangle$. In case C2 $\beta_r(t) = \beta(t)$ while in cases C3 and C4 $\beta_r(t) \leqq \beta(t)$ on $[t_1, t_2\rangle$. Further, in all three cases $b_r(t) = b(t)$. This gives the sufficient condition for $\phi_r(t) < \beta(t)$ on $[t_3, t_2\rangle$. Analogously, we can prove that in case C4 sufficient for $\phi_r(t) > \alpha(t)$ on $[t_3, t_2\rangle$ is, that $\alpha_r(t)$ does not decrease on $[t_3, \infty\rangle$ and that for every $t \geqq t_3$

$$b(t) > \alpha_r(t) - \frac{\lambda(t)}{\rho}\{E(w; t) - \alpha_r(t)\}.$$

Acknowledgement. I would like to thank Andrew Chesher, Arie Kapteyn, Peter Kooreman, Tony Lancaster, Robert Moffitt and Geert Ridder for their helpful comments on an earlier version of this paper. Also thanks to John Rust and Hans Nieuwenhuis. Financial support from the Netherlands Organization for the Advancement of Pure Research (ZWO) is acknowledged. A large part of this paper was written while the author worked at Tilburg University.

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