University College London
Department of Economics

G023: Econometric Theory and Methods *
Exercise 2: Sketch answers

1. (a) In each case the coefficients after the unit changes are denoted by α_a, β_a, σ_a^2.
   i. $\alpha_a = \alpha$, $\beta_a = 1000\beta$, $\sigma_a^2 = \sigma^2$.
   ii. $\alpha_a = \alpha/1000$, $\beta_a = \beta$, $\sigma_a^2 = \sigma^2/10^6$.
   iii. $\alpha_a = \alpha/12$, $\beta_a = \beta/12$, $\sigma_a^2 = \sigma^2/144$.

   (b) This question asks you to think about the role of binary indicator ("dummy") variables in econometric models.
   i. Let $d = 1$ for northern households, $d = 0$ for southern households and extend the model thus.
   $$Y = \alpha + \delta d + \beta x + \varepsilon$$
   ii. You extend the model further as follows
   $$Y = \alpha + \delta d + \beta x + \gamma d \times x + \varepsilon$$
   and test the hypothesis $H_0 : \gamma = 0$.

   (c) A question about setting models up so that a hypothesis can be tested.
   i. Let male income be x_M and female income be x_F. Use the following extended model.
   $$Y = \alpha + \beta_M x_M + \beta_F x_F + \varepsilon$$
   ii. $\beta_M = \beta_F$.
   iii. Test the hypothesis above - either by directly looking at $\hat{\beta}_M - \hat{\beta}_F$, or by fitting the extended model and the restricted model and comparing the sum of squared residuals.

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iv. If $x_M = x_F$ for every household then the matrix of values of the covariates is deficient in rank.

2. The issue of identification is fundamental in econometrics.

(a) Substitute for ε_2 in the W equation giving

$$W = (\alpha_0 - \lambda\beta_0) + (\alpha_1 + \lambda)S + (\alpha_2 - \lambda\beta_1)X + \varepsilon_1$$

and note that both sets of parameter values give

$$W = S + X + \varepsilon_1.$$

For any particular set of values of S, X and ε_1 the two sets of parameter values yield identical sets of values of W. Clearly no amount of data could ever tell us which set of parameter values generated the data. The coefficients α_0, α_1, α_2 and λ cannot be identified.

(b) We now suppose we know that $\alpha_2 = 0$ and $\beta_1 \neq 0$. In the context of the question this could arise if X measured some home background characteristic. In this case, substituting for ε_2 in the W equation gives (setting $\alpha_2 = 0$)

$$W = (\alpha_0 - \lambda\beta_0) + (\alpha_1 + \lambda)S - \lambda\beta_1 X + \varepsilon_1$$

and setting parameters equal to the values proposed:

$$W = S + \varepsilon_1.$$

Consider another set of parameter values, α_0^*, α_1^*, λ^*, β_0^*, β_1^* (and note that α_2 is required to be zero in all admissible sets of parameter values, so α_2^* must be zero). This set generates W data using

$$W = (\alpha_0^* - \lambda^*\beta_0^*) + (\alpha_1^* + \lambda^*)S - \lambda^*\beta_1^* X + \varepsilon_1.$$

At the proposed set of parameter values the S data is generated by

$$S = X + \varepsilon_2$$

and at the alternative set of values it would be generated by

$$S = \beta_0^* + \beta_1^* X + \varepsilon_2.$$

If X takes two or more distinct values we can only obtain the same S data if $\beta_0^* = 0$ and $\beta_1^* = 1$, so these two parameters are identified. The alternative $(*)$ set of parameter values therefore generate W data using

$$W = \alpha_0^* + (\alpha_1^* + \lambda^*)S - \lambda^* X + \varepsilon_1$$

(using $\beta_1 = 1$). But W data generated by this equation will vary with X, unlike the W data generated by the proposed set of parameter values, if $\lambda^* \neq 0$, and from that we can conclude that we must have $\lambda^* = 0$ and then also that $\alpha_0^* = 0$ and $\alpha_1^* = 1$ if the variation with S and the general level of the W data is to be as in the data generated at the proposed set of parameter values.

(c) If X took only one value then there would be no variation in the X data and we could not exclude the possibility that $\lambda^* \neq 0$.

3. OLS estimates and estimated standard errors in parentheses.

$\hat{\alpha}$	$\hat{\beta}$
42.52	0.0129
(8.33)	(0.0043)

In this model the fixed cost of electricity distribution is α.

(a) i. We use a large sample approximation (but note the sample size is very small) and so the standard $N(0,1)$ distribution. If $Z \sim N(0,1)$ then $P[-1.96 \le Z \le 1.96] = 0.95$, giving the 95% confidence interval $[26.19, 58.85]$.

ii. The test statistic is $(42.52 - 25)/8.33 = 2.10$. For a test with size 0.05, we use the probability statement in (i) above and note that $2.10 > 1.96$, so we reject $H_0 : \alpha = 25$.

(b) With the two "efficient companies eliminated we get the following estimates.

$\hat{\alpha}$	$\hat{\beta}$
26.08	.0239
(7.06)	(0.0040)

i. Again we use a large sample approximation and the $N(0,1)$ reference distribution giving the 95% confidence interval $[12.24, 39.92]$.

ii. The test statistic is $(26.08 - 25)/7.06 = 0.15$. For a test with size 0.05, we use the probability statement in (i) above and note that $0.15 < 1.96$, so we do not reject $H_0 : \alpha = 25$. The new estimate is very close to 25.

(c) Let $d = 1$ for the two efficient companies and 0 for the other 12 companies and estimate the extended model

$$C = \alpha + \beta x + \gamma dx + \varepsilon.$$

The slope of OFGEM's efficient frontier is $\beta + \gamma$ ($= b$ in the question) and its intercept is α (a in the question).

i. We get the following OLS estimates.

$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$
26.18	0.0238	-0.0095
(6.67)	(0.0038)	(0.0023)

ii. Again we use a large sample approximation and the $N(0,1)$ reference distribution giving the 95% confidence interval $[13.11, 39.25]$.

iii. The estimated slope of the efficient frontier is $\hat{b} = 0.0238 - 0.0095 = 0.0143$. The estimated standard error for this linear combination of the estimated coefficients is 0.0028. The "large sample approximate" 95% confidence interval is $[0.0088, 0.0197]$.

(d) A question about investigating nonlinearity.

i. Zero, as long as $\phi > 0$.

ii. $\frac{\partial C}{\partial x} = \exp(\theta + \upsilon)\phi x^{\phi - 1}$ which is a decreasing function of x when $\phi < 1$. The estimated coefficients (estimated standard errors in parentheses) are shown below.

$\hat{\theta}$	$\hat{\phi}$
1.331	0.382
(0.711)	(0.095)

iii. Yes. The relevant statistic is $(1 - .382)/.095 = 6.50$. Under $H_0 : \phi = 1$, this statistic is approximately (again the $N(0, 1)$ large sample approximation) realisation of a $N(0, 1)$ random variable. We use a test with size 0.05. If $Z \sim N(0, 1)$ then $P[Z \geq 1.64] = 0.05$ and the value we have obtained is greatly in excess of this so we reject H_0. Note this is a "one-sided" test.