

G023 - Problem set 1 - December 10, 2007

1. Suppose that you estimate the model $Y_t = \phi Y_{t-1} + \varepsilon_t$ by OLS and that ε_t is an AR(1) process.

- (a) Write the expression for the OLS estimator $\hat{\phi}$ for a sample of size T
- (b) Show that the OLS estimator $\hat{\phi}$ is inconsistent

2. You run a regression of Y_t on X_t and obtain the following results

$$Y_t = .45X_t + err, R^2 = .72, t - statistic = 60.8 .$$

You then run the regression of ΔY_t on ΔX_t and obtain the following results

$$\Delta Y_t = .01X_t + err, R^2 = .04, t - statistic = 0.15$$

What do you conclude about the time series properties of the two series and their relationship? Explain

3. Find $E(Y_t)$ and $Var(Y_t)$ for the process $Y_t = c + Y_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim iidN(0, \sigma^2)$ and the initial value $Y_0 \sim N(0, 1)$, independent of ε_t . Is the process covariance-stationary? (Hint: derive $E(Y_1)$, $Var(Y_1)$, $E(Y_2)$, $Var(Y_2)$ and generalize to $E(Y_t)$ and $Var(Y_t)$)
4. Consider the following model

$$Y_t = \varepsilon_t + 2.4\varepsilon_{t-1} + 0.8\varepsilon_{t-2}$$

where ε_t is iid $N(0,1)$

- (a) Is this process covariance stationary?
- (b) Is it invertible?
- (c) Calculate the autocovariance function γ_j for $j = 1, 2, \dots$