Competitive Screening of Customers with Non-Common Priors

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Abstract

This paper provides an explanation for the variety of contracts offered by competitive firms for seemingly identical products or services, e.g. mobile communication. I show that two competing firms offering menus of non-linear price schedules to customers with mistaken priors will be able to screen these customers on the basis of their priors. Hence firms' menus can be understood as screening devices for boundedly rational consumers.

Key words: competitive price discrimination, bounded rationality
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1 Introduction

The multitude of tariffs for seemingly homogenous goods and services offered by firms in a wide range of competitive industries, e.g. mobile phone contracts, constitutes a theoretical problem. While at first sight one is tempted to infer that firms use this variety of tariffs to price discriminate between customers with different consumption profiles, standard models of competitive price discrimination (Armstrong and Vickers (2001), Rochet and Stole (2002)) tell us that in sufficiently competitive markets with firms facing identical technological constraints and demand conditions, firms loose the ability to engage

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in price discrimination. Competition forces all firms to offer a single two-part tariff which consist of a fixed fee plus costs.

In this paper I will provide an explanation for these large menus of tariffs offered by competitive firms which is based on heterogeneity in cognitive abilities of the customer base. In doing so other possible causes which could potentially account for the phenomenon will be ignored, most prominently among them dynamic considerations like switching costs or asymmetries between firms both in technologies or in consumer preferences. At the center of the analysis will be the customer's ability to anticipate his/her consumption profile for a product which is provided by firms in a competitive market. This ability is important as customers will have to choose between contracts specifying price-quantity schedules before knowing their exact demand for the good. Once signed up to a contract, they will find out about their exact valuation for the good and will choose their optimal consumption level from this specific contract. Crucially the firm can disagree with its customers at the time contracts are signed concerning the customer's consumption profile. One example would be that the firm believes its customer is too optimistic concerning his/her future intensity of usage of the service. In the technical term of the cognitive psychology literature, the firm thinks that its customer's beliefs are *miscalibrated*.

I will show that if firms believe that some of their customers have miscalibrated beliefs but these beliefs are private information, firms will design their tariffs so as to screen customers with respect to these heterogenous beliefs. Competition between firms will not prevent firms from using their menu design to screen customers in this manner.

The results of this model allow some insights into the question of what beneficial effects we can expect from market competition in the presence of boundedly rational market participants. It will be shown that the standard inefficiency that price discrimination creates by distorting optimal quantities for *low types* in order to extract more surplus from *high types* will be eliminated by competition even if some of the market participants commit mistakes in evaluating deals. However firms will not deliver the efficient quantity schedule to all customers, as some customers' perceived surplus is not their actual surplus. This will induce firms to create what in the behavioral economics literature is sometimes called *fictitious surplus*, surplus which only exists in the customers' imagination, will never materialize and therefore can be costlessly provided by the firm. Competition will nevertheless ensure that firms do not make any additional profits by creating this kind of surplus. Standard customers, i.e. customers that shares common prior beliefs with the firms, will choose a cost plus fixed fee tariff. This tariff turns out to be the same tariff firms would offer to this customer if no *miscalibrated* customers were present in the market. It follows that in this model the presence of boundedly rational customers does not exert an externality on the fully rational customers.

Empirical Evidence on Miscalibration Calibration measures the agreement between the objective and subjective assessment of validity of a statement. A person is therefore perfectly calibrated if the subjective probability she assigns to any event matches the long run frequency of occurrences of this event. In our setting a perfectly calibrated customer can correctly quantify the probability that his marginal utility from consuming a certain quantity of the good is below a given level. [2]

The notion of calibration has been extensively discussed in the experimental psychology literature (Lichtenstein et al. (1982), McClelland and Bolger (1994)). A robust result in this literature is that for a large variety of situations, experimental subjects show a significant degree of miscalibration. A well know example is the tendency of experimental subjects to overestimate their ability to correctly answer general knowledge question.

More recently economists have begun to elicit subjective probability distribution which matter for specific economic problems and check the goodness of calibration of people's beliefs. Dominitz and Manski (2005) analyze probabilistic beliefs concerning equity returns using data from the Michigan Survey of Consumers and the Survey of Economic Expectations. They find significant interpersonal variation but intrapersonal stability in beliefs. When compared to the long run average of equity returns, subjects tend to be miscalibrated, on average overestimating both the mean and the volatility of returns. Similar studies have been conducted for job losses, eligibility for social security or income uncertainty with similar results. Manski (2004) provides a survey of this literature.

Non-common Prior Approach At the time of contracting between customer and firm, the exact utility the customer will derive from the good is uncertain. This uncertainty will be indexed by a one-dimensional random variable which parameterizes the customer's utility function. Call this random variable θ and let its probability distribution be given by $F(\theta)$. In the following model a person will be called miscalibrated if his/her prior beliefs concerning the random variable θ do not agree with the objective probability distribution $F(\theta)$.

It will be assumed that firms are perfectly calibrated, i.e. that their prior beliefs concerning θ are given by $F(\theta)$. Firms know that their customers are all identical and have a consumption profile which is determined by $F(\theta)$. Customers' prior beliefs can divert from $F(\theta)$. To solve the competitive model I will not have to restrict the way in which customers' beliefs divert from this probability distribution. All that is necessary is that firms know in which ways their customers' beliefs can differ from $F(\theta)$. In order to solve the monopolistic case, I will have to impose more structure on the set of possible deviations. I will use two criteria to order customers' beliefs. Customers' prior beliefs will be allowed to divert from $F(\theta)$ either in a first order stochastic dominance sense, or in a mean preserving spread sense. In psychological terms, customers will

[2] This obviously presumes the existence of an objectively valid probability distribution for these marginal utilities.

be allowed to be either overpessimistic (overoptimistic), or underconfident (overconfident).

The above description of the situation with calibrated firms and miscalibrated customers is only one possible interpretation one could give to this setup. What is crucial is that all firms share a common prior belief concerning θ and that firms and customers possibly disagree concerning the distribution of this variable. Another possibility would be to interpret firms as boundedly rational and customers to be perfectly calibrated. However it is not possible to interpret this in the sense that customers are better informed concerning their tastes. Such an interpretation would alter firms' strategies as compared to our analysis. Firms would now gain additional information through customers' choices which would have to be taken into account. Disagreement about beliefs is central to the analysis.

Related Literature There is a fast growing literature on optimal contract design in the presence of consumer biases. Two papers are particularly close in spirit to our analysis. Della Vigna and Malmendier (2004) analyse the profit-maximizing contract when customers have time-inconsitent preferences. They show that goods with immediate costs but delayed benefits (investment goods) are priced below marginal cost, while goods with immediate benefits but delayed costs (leisure goods) are priced above marginal costs. Consumers that face self control problems and are (partially) naive about it will overestimate their demand in the former case and underestimate demand in the latter case. Firms will exploit these biases by readjusting the price profile, i.e. raising (lowering) the fixed fee and adjusting the per usage price accordingly in order to create what they call *fictitious surplus*. DellaVigna and Malmendier show that these pricing patterns are observable in a wide variety of industries ranging from health clubs to credit card companies.

Eliaz and Spiegler (2005) extend the above analysis to a contracting environment in which consumers have time inconsistent preferences but vary in their degree of awareness concerning this inconsistency. While firms know that consumption of their good will induce a change of customers' preferences, customers assign a probability between zero and one to this shift, where this probability describes the customer's type unkown to the firm. Elias and Spiegler show that three part tariffs are necessary to implement the optimal screening contract. While sufficiently naive customers will sign exploitative contracts, in the sense that under the correct prior the contract offers less utility than their outside option, the contract offers a commitment device for sufficiently sophisitcated customers helping them to overcome their self-control problems. Our modeling approach differs from the above papers in that it focuses on the effects of varying degrees of demand uncertainty on the side of customers on contract design.

In Della Vigna and Malmendier (2004) there is no demand uncertainty. Partially naive customers simply misconceive of their future demand, judging it too high or too low depending on the cost structure of the problem. Eliaz and

Spiegler (2005) introduce demand uncertainty into their framework as partially naive consumers are unsure about future changes in their preferences. However, as there are only two possible alternative preference structures, their model does not allow to disentangle the effects of overestimating or underestimating demand from increases in demand uncertainty without changes in the mean valuation for the service provided. [3]

The structure of the model is very close to the setup of the Sequential Screening literature (Armstrong (1996), Courty and Li (2000)). Here a monopolist tries to screen customers with differing demand patterns through contracts that are offered before customers know their actual demand. Types are indexed by their distribution functions over ex-post demand realization. Firms and customers ex-ante hold identical priors for these distribution functions. Thus customer' types can be interpreted as high demand / low demand types if priors are order by first order stochastic dominance, or risky / safe customers if ordered by mean preserving spreads.

2 The Model

Firms

There are two firms A and B, which are situated at the opposite end of a segment of unit length.

Each firm produces a single good. The costs of providing a quantity q of this good to a customer are $C(q)$, where we assume that $C'(q), C''(q) \geq 0$.

Firms offer a menu of (non-linear) price schedules to their customers. Firm i's menu of contracts J_i is given by

$$\{\mathcal{P}^i(q,j)\}_{j \in J_i}. \tag{1}$$

A customer who has signed contract j at firm i pays $\mathcal{P}^i(q,j)$ for a quantity q of the good.

Consumers

Consumers are located on the line between firm A and B. A customer who has signed contract $j \in J_A$ at firm A and is situated at x on the line derives a utility of

$$u(q,\theta) - \mathcal{P}^A(q,j) - \tau x \tag{2}$$

from consuming a quantity q of the good. The same consumer on contract $k \in J_B$ at firm B would derive a utility of

$$u(q,\theta) - \mathcal{P}^B(q,k) - (1-\tau)x \tag{3}$$

[3] The advantage of this approach is that the way in which preferences can change is completely unrestricted. In our model ex-post preferences are indexed by a one-dimensional parameter and in addition to that have to satisfy a single-crossing property in this parameter.

from consuming q.
$\theta \in [\theta_L, \theta_H]$ is a one-dimensional preference shifter. It is assumed that $u_q(q,\theta) > 0$, $u_\theta(q,\theta) > 0$, and that the utility function satisfies a standard *single-crossing* property in θ, i.e.

$$u_{q\theta}(q,\theta) > 0.$$

τ is the consumer's per-unit travel cost.

Information Structure
Both consumer location x and the consumer's demand type θ are private information. While the consumer knows x at the time of signing a contract, she does not know θ. She only finds out about her exact demand, once she has signed a contract. A consumer has prior beliefs concerning the possible realizations of θ which are given by the probability distribution $F(\theta,\alpha)$. α will be called the consumer's *ex-ante type*. For the moment no restrictions are imposed on the parameter α. [4] Let A designate the set of *ex-ante types* that are active in the market. There is a measure one of each type $\alpha \in A$, and for each type α this mass is distributed uniformly on the segment between firm A and B.
Firms do not know the exact type of their customers. They only know the locational distribution of types, the set A of active *ex-ante types* [5], and all firms share a common prior belief about the distribution of θ which we will designate by $F(\theta)$.
Under my intended interpretation of the model, $F(\theta)$ will be the "true" distribution of θ, and the *ex-ante type* $\hat{\alpha}$ such that $F(\theta,\hat{\alpha}) = F(\theta)$ will be called the *fully rational type*.

2.1 Competition in the Utility Space

Each price schedule in the firm's menu can be considered as a *deal* of a certain value that is offered by the firm to its customer. Firms compete over customers by trying to offer them better deals, i.e. higher utility levels, through their choice of price schedules. This idea of formalizing competition in multi-dimensional objects as competition in the utility space has been first put forward by Bliss (1988) and further developed by Armstrong and Vickers (2001).

Suppose firm A intended to offer a customer of type α a utility level of u. Obviously firm A would provide this utility level in a way that maximizes its own profits. Thus the following profits maximizing problem implicitly deter-

[4] When solving the monopoly case, α will be one-dimensional and α will order the set of distributions $\{F(\theta,\alpha)\}$ by first order stochastic dominance or mean preserving spreads.
[5] For the monopolistic case, the firm will need to know the distribution of α not only its support.

mines the quantity schedule firm A would provide to a customer of type α if it wanted to guarantee him a utility level u net of travel costs:

$$\pi^A(u,\alpha) = \max_{\mathcal{P}^A,\{q(\theta,\alpha)\}} \int_{\theta_L}^{\theta_H} \left(\mathcal{P}^A(q(\theta,\alpha),\alpha) - C(q(\theta,\alpha))\right) f(\theta,\hat{\alpha})d\theta$$
$$\text{subject to } \int_{\theta_L}^{\theta_H} s^A(\theta,\alpha)f(\theta,\alpha)d\theta \geq u$$

where $s^A(\theta,\alpha) = u(q(\theta,\alpha),\theta) - \mathcal{P}^A(q(\theta,\alpha),\alpha)$.
Any chosen quantity schedule $\{q(\theta,\alpha)\}$ has to be ex-post incentive compatible which is equivalent to imposing the usual Envelope condition $s^A_\theta(\theta,\alpha) = u_\theta(q(\theta,\alpha),\theta)$ and requiring $q(\theta,\alpha)$ to be weakly increasing in θ.
This allows us the rewrite the constraint on consumers' expected utility under a quantity schedule $\{q(\theta,\alpha)\}$ as

$$s(\theta_L,\alpha) + \int_{\theta_L}^{\theta_H} u_\theta(q(\theta,\alpha),\theta)[1-F(\theta,\alpha)]d\theta \geq u$$

Using the definition of consumer surplus to replace $\mathcal{P}^A$ in firm A's objective function and integrating by parts we get

$$\int_{\theta_L}^{\theta_H} \left(\mathcal{P}^A(q(\theta,\alpha),\alpha) - C(q(\theta,\alpha))\right) f(\theta,\hat{\alpha})d\theta =$$
$$\int_{\theta_L}^{\theta_H} \left\{u(q(\theta,\alpha),\theta) - s(\theta_L,\alpha) - u_\theta(q(\theta,\alpha),\theta)\left(\frac{1-F(\theta,\hat{\alpha})}{f(\theta,\hat{\alpha})}\right) - C(q(\theta,\alpha))\right\} f(\theta,\hat{\alpha})d\theta$$

We can now substitute the constraint on consumers' expected utility into this expression upon noting that in any profit maximizing solution firm A will provide consumers with an expected utility of exactly u.

$$\pi(u,\alpha) = \max_{\{q(\theta,\alpha)\}} \int_{\theta_L}^{\theta_H} \Lambda^c(q(\theta,\alpha),\theta,\alpha)f(\theta,\hat{\alpha})d\theta - u \tag{4}$$

where

$$\Lambda^c(q,\theta,\alpha) = u(q,\theta) + u_\theta(q,\theta)\left[\frac{F(\theta,\hat{\alpha}) - F(\theta,\alpha)}{f(\theta,\hat{\alpha})}\right] - C(q) \tag{5}$$

We get a candidate solution for the profit-maximizing schedule by maximizing the objective function pointwise with respect to q for each θ:

$$q(\theta,\alpha) = \arg\max_{q\geq 0} \Lambda^c(q,\theta,\alpha) \tag{6}$$

If Λ^c is strictly quasi-concave in q, this candidate is given by the first order condition

$$\Lambda^c_q(q(\theta,\alpha),\theta,\alpha) = 0 \tag{7}$$

If furthermore Λ^c is supermodular in (q, θ), $q(\theta, \alpha)$ will be (weakly) increasing in θ and the above candidate is indeed the profit maximizing quantity schedule.
Notice that this schedule is independent of the level of utility u provided to a customer of type α. Differences in utility are not provided by altering the quantity schedule but are undertaken through changes in $s(\theta_L, \alpha)$, i.e. through changes in the "fixed fee" associated with the deal.

To construct the price schedule $\mathcal{P}^*(\cdot, \alpha)$ that implements the profit maximizing quantity schedule $\{q(\theta, \alpha)\}$ we can follow the usual steps. First note that

$$s(\theta_L, \alpha) = u - \int_{\theta_L}^{\theta_H} u_\theta(q(\theta, \alpha), \theta)\, d\,\theta$$

This allows us to recover the surplus function

$$s(\theta, \alpha) = s(\theta_L, \alpha) + \int_{\theta_L}^{\theta} u_\theta(q(x, \alpha), x)\, d\,x$$

Now from the definition of $s(\theta, \alpha)$ we can construct the optimal price schedule:

$$\mathcal{P}^*(q(\theta, \alpha), \alpha) = u(q(\theta, \alpha), \theta) - s(\theta, \alpha) \tag{8}$$

We will separate $\mathcal{P}^*$ into a usage charge $\mathcal{P}(\cdot, \alpha)$ such that $\mathcal{P}(q(\theta_L, \alpha), \alpha) = 0$ and a fixed fee $t(u, \alpha)$. This fee will adjust the value of the deal to the required utility level u.
For future reference let $c(\alpha)$ designate the expected cost of implementing the profit maximizing quantity schedule net of the fixed fee, i.e.

$$c(\alpha) = \int_{\theta_L}^{\theta_H} \{C(q(\theta, \alpha)) - \mathcal{P}(q(\theta, \alpha), \alpha)\}\, f(\theta, \hat{\alpha}) d\theta \tag{9}$$

Note that for the fully rational type $\alpha = \hat{\alpha}$ this cost is obviously zero, as the profit maximizing price schedule equals actual costs.
Furthermore we designate by $S(\alpha)$ the total surplus excluding the fixed fee a customer of type α will receive from the profit maximizing quantity schedule $q(\theta, \alpha)$:

$$S(\alpha) = \int_{\theta_L}^{\theta_H} \{u(q(\theta, \alpha), \theta) - \mathcal{P}(q(\theta, \alpha), \alpha)\}\, f(\theta, \alpha) d\theta \tag{10}$$

2.2 Hotelling Competition over a Single Type

Suppose for the moment that only a single ex-ante type α is on the market. As described above a measure one of them is distributed uniformly between the two firms A and B. The following proposition characterizes the equilibrium of the standard Hotelling game:

Proposition 1: *Provided that $\tau \le 2/3[S(\alpha) - c(\alpha)]$, offering the fixed fee $t(\alpha) = \tau + c(\alpha)$ and the price schedule $\mathcal{P}(q,\alpha)$ is an equilibrium of the above Hotelling game.*

Proof: Suppose firm B offers the suggested schedule. Then firm A's market share, given that it offers a surplus of S (net of travel costs) to its customers, is

$$x = \min\left\{\frac{1}{2}\left(1 + \frac{S - S(\alpha) + \tau + c(\alpha)}{\tau}\right), \frac{S}{t}\right\}$$

Obviously firm A will provide the surplus level S through the profit maximizing quantity schedule. Customers' surplus from buying at firm A net of travel costs will then be $S(\alpha) - k$, where k is the fixed fee firm A will charge in addition to the price schedule $\mathcal{P}(q,\alpha)$.
Under the assumption that $\tau \le 2/3[S(\alpha) - c(\alpha)]$ the market is fully covered and profits for firm A when charging a fixed fee k are

$$[k - c(\alpha)]\frac{1}{2}\left(1 + \frac{\tau + c(\alpha) - k}{\tau}\right)$$

The fixed fee that maximizes these profits is given by $k = \tau + c(\alpha)$ which proves the claim. ■

2.3 Hotelling Competition with Multiple Unobservable Types

Now let us go back to the initially described setting in which the two firms face a variety of ex-ante types α, where the exact type of each customer is unknown to the firm. A measure one of each α type is distributed uniformly between A and B.
The following proposition states that the competitive situation with multiple unobservable types is separable. Offering the equilibrium derived for the single type setting to every type α present in the market is an equilibrium of this more complex game. The incentive constraints that ensure self-selection will turn out to be non-binding.

Proposition 2: *Suppose $\tau \le 2/3[S(\alpha) - c(\alpha)]$ for all α present in the market. Then each firm offering a menu of contracts in which each α receives his contract from Proposition 1 is an equilibrium.*

Proof: The following proof adopts an argument from the proof of Proposition 5 in Armstrong and Vickers (2001). Suppose firm B offered such a menu of contracts and that furthermore this menu was ex-ante incentive compatible in the sense that each α would indeed choose his contract from Proposition 1. Now suppose that firm A could actually observe the types α. This gives us an upper bound on the profits firm A could make. Then it is a best reply for

firm A to offer each type α the contract derived in Proposition 1.
If we can show that this menu of contracts is also incentive compatible, that is that no type α would like to deviate to any other contract originally intended for $\alpha' \neq \alpha$, then it is also a best reply by firm A if it cannot observe the types. Ex-ante incentive compatibility of contracts requires that

$$\int_{\theta_L}^{\theta_H} \Big[u(q(\theta,\alpha),\theta) - \mathcal{P}(q(\theta,\alpha),\alpha)\Big] f(\theta,\alpha)\, d\theta \\ \geq \int_{\theta_L}^{\theta_H} \Big[u(q(\theta,\alpha'),\theta) - \mathcal{P}(q(\theta,\alpha'),\alpha')\Big] f(\theta,\alpha)\, d\theta$$

for all $\alpha' \neq \alpha$.
Upon substituting in $t(\alpha) = \tau + c(\alpha)$ and rearranging this is equivalent to

$$c(\alpha') - c(\alpha) \geq \int_{\theta_L}^{\theta_H} \Big[u(q(\theta,\alpha'),\theta) - u(q(\theta,\alpha),\theta)\Big] f(\theta,\alpha)\, d\theta$$

Now suppose there existed an $\alpha' \neq \alpha$ for which this inequality would be violated. Then the costs of moving a customer of type α from the quantity schedule $q(\theta,\alpha)$ to $q(\theta,\alpha')$ would be lower than the consumer surplus (real and fictitious) created through this reallocation. Thus a firm could raise its profits from a type α by such a move, raising or lowering the fixed fee in order to keep the type's expected utility from the contract unaltered. But this contradicts the condition that $q(\theta,\alpha)$ is the profit maximizing quantity schedule for type α. ■

The proof shows that incentive compatibility under competition is a direct consequence of profit maximization by firms.
Furthermore the proof does not require us to impose any structure on the set of priors. Thus Proposition 2 holds for any kind of differing priors held by consumers in the market.

3 The Effects of Competition

To get an understanding of what competition can and cannot achieve in the presence of customers with mistaken priors, I will contrast the above result with two settings. Firstly, I will analyze the optimal tariff design of a monopolist where consumers are as described above.
Secondly I will analyze the case where consumers do not have mistaken priors, but differ in their true consumption profiles. That is for two ex-ante types $\alpha \neq \alpha'$, we will have $F(\theta,\alpha) \neq F(\theta,\alpha')$, but firms and consumers have common priors. In other words, I will analyze a competitive version of the standard Sequential Screening problem (Armstrong (1996), Courty and Li (2001)).

3.1 Monopoly

Suppose that instead of having a duopoly, there is only a single firm providing the good to customers. Customers are as described in the above setting except that I will ignore travel costs here.
Again, the firm offers its customers a menu of price schedules $\{\mathcal{P}(q,j)\}_{j\in J}$ before these customers know their type θ. Customers will have ex-ante types $\alpha \in A$, but now in order to solve the problem, I will have to assume that the monopolist knows the population distribution of ex-ante types $G(\alpha)$. Here it will furthermore be necessary to be impose more structure on the set of priors $\{F(\theta,\alpha)\}_{\alpha\in A}$. While in the competitive case, the ex-ante incentive constraints were not binding, in any profit-maximizing solution of the monopolist, some ex-ante constraint will have to be binding. In order to know which are these, I will solve the problem making two possible assumptions on the set of priors, Assumption 1 or alternatively Assumptions 2A and 2B.

The condition imposed under Assumption 1 is that high α types always associate a higher probability with having a high demand in the second period than low α types. Formally for every $\alpha' > \alpha$, the distribution $F(\theta,\alpha')$ first order stochastically dominates $F(\theta,\alpha)$.

Assumption 1 $F_\alpha(\theta,\alpha) \leq 0$ *for all* θ

Under the conditions imposed by Assumption 2A, the high α types will always be less confident to accurately predict their demand in the second period than the low α type. We will formalize this idea by imposing that for any $\alpha' > \alpha$, the distribution $F(\theta,\alpha')$ is a mean preserving spread of $F(\theta,\alpha)$.
Assumption 2B will only be used to derive sufficient conditions for an optimum. It implies that for all $\alpha' > \alpha$, $F(\theta,\alpha) < F(\theta,\alpha')$ if $\theta < z$, and $F(\theta,\alpha) > F(\theta,\alpha')$ if $\theta > z$.

Assumption 2A $\int_{\theta_L}^{\theta} F_\alpha(x,\alpha)dx \geq 0$ *for all* θ *and* $\int_{\theta_L}^{\theta^H} F_\alpha(x,\alpha)dx = 0$.

Assumption 2B *The distribution functions* $F(\theta,\alpha)$ *cross in one and only one point* $\theta = z$ *for all* α.

Define $\Lambda^m(q,\theta,\alpha)$ as follows

$$\Lambda^m(q,\theta,\alpha) = u(q,\theta) + u_\theta(q,\theta)\left[\frac{F(\theta,\hat{\alpha}) - F(\theta,\alpha)}{f(\theta,\hat{\alpha})}\right] - C(q) + u_\theta(q,\theta)\frac{F_\alpha(\theta,\alpha)}{f(\theta,\hat{\alpha})}\left(\frac{1-G(\alpha)}{g(\alpha)}\right) \quad (11)$$

The following propositions characterizes the monopolist's profit-maximizing menu design.

Proposition 3a: *Suppose the set of priors* $\{F(\theta,\alpha)\}$ *satisfies Assumption 1. Then then optimal quantity schedule* $\{\{q(\theta,\alpha)\}_{\theta\in\Theta}\}_{\alpha\in A}$ *is given by*

$$q(\theta,\alpha) = arg\max_q \Lambda^m(q,\theta,\alpha)$$

if $q(\theta,\alpha)$ *is (weakly) increasing in both* θ *and* α.

Proof: see Appendix

Proposition 3b: *Suppose the set of priors* $\{F(\theta,\alpha)\}$ *satisfies Assumptions 2A and 2B. Then then optimal quantity schedule* $\{\{q(\theta,\alpha)\}_{\theta\in\Theta}\}_{\alpha\in A}$ *is given by*

$$q(\theta,\alpha) = arg\max_q \Lambda^m(q,\theta,\alpha)$$

if $q(\theta,\alpha)$ *is (weakly) increasing in* θ*, (weakly) decreasing in* α *on* $\theta \leq z$*, and (weakly) increasing in* α *on* $\theta > z$.

Proof: see Appendix

These results allow us to compare the monopoly outcome to the menu of tariffs supplied by firms in a duopoly with differentiated brands.
We have seen that the optimal quantity schedules $\{q(\theta,\alpha)\}$ can be derived by pointwise maximization of Λ^m as defined by (11) in the monopoly case, and of Λ^c defined by (5) in the duopoly case. These two functions differ in one term

$$u_\theta(q,\theta)\frac{F_\alpha(\theta,\alpha)}{f(\theta,\hat{\alpha})}\left(\frac{1-G(\alpha)}{g(\alpha)}\right)$$

which is absent in the competitive setting. This term reflects the usual inefficiency a monopolists creates by distorting the quantity schedules of the low α types in order to extract more surplus from the high α types. For the highest α type this deviation from the efficient quantity schedule disappears. As in in the standard models of competitive price discrimination, this inefficiency is eliminated by competition. Firms loose the ability to distort the quantity schedules of some ex-ante type in order to extract more surplus from another ex-ante type. As we have seen, the competitive solution is separable, ex-ante incentive constraints are not binding.

3.2 Common Priors but Different Consumption Profiles

Now, I will analyze a setting with competitive firms which face consumers with real differences in their consumption profiles, e.g. frequent v. infrequent

users. It will be shown that in such a setting firms will not be able to use the design of their menu in order to screen customers with different consumption profiles.
Let us return to the previous duopoly setting, but suppose that $F(\theta, \alpha)$ describes the "true" consumption profile of type α. Again, there will be a heterogenous population of α types belonging to some some set A, but firms and consumers agree that consumption profiles are actually different. That is if a firm knew the α type of its customer, the firm and this customer would agree that the customer's consumption profile is given by $F(\theta, \alpha)$. However, as before, α will be private information of the consumers and if firms want to discriminate between consumers, they will have to do so in an incentive compatible way.

Consider a firm that provides a utility level u (net of travel costs) to a customer of type α. The solution to this problem is given by

$$\pi^A(u, \alpha) = \max_{\mathcal{P}^A, \{q(\theta,\alpha)\}} \int_{\theta_L}^{\theta_H} \left(\mathcal{P}^A(q(\theta, \alpha), \alpha) - C(q(\theta, \alpha))\right) f(\theta, \alpha) d\theta$$

$$\text{subject to} \quad \int_{\theta_L}^{\theta_H} s^A(\theta, \alpha) f(\theta, \alpha) d\theta \geq u$$

Notice that the only difference to the situation with non-common priors is that the firm's expectation is taken over the same distribution as the consumer's evaluation of the deal.
Carrying out the same steps as before we find that the firm's profits as a function of the type α and the provided utility level u are

$$\pi(u, \alpha) = \max_{\{q(\theta,\alpha)\}} \int_{\theta_L}^{\theta_H} [u(q(\theta, \alpha), \theta) - C(q(\theta, \alpha))] f(\theta, \alpha) d\theta - u \qquad (12)$$

Now obviously the optimal quantity schedule is the efficient one, that is where marginal utility equals marginal cost, $u_q(q, \theta) = C_q(q)$. Clearly, the optimal quantity schedule does not depend on α. The firm provides the same quantity schedule $\{q(\theta)\}$ to all its customers irrespective of their type α. Furthermore, the single crossing property $u_{q\theta} > 0$ ensures that $q(\theta)$ is increasing in θ and thus implementable.
The above solution is obvious. The profit-maximizing deal maximizes total surplus and extracts any surplus that the firm wants to extract through a fixed fee ex-ante.

Let us now analyze competition by firms over (unobservable) types $\alpha \in A$. The following Proposition follows the line of argument of Proposition 5 in Armstrong and Vickers (2001), and establishes that firms will only offer one contract. This contract will be a fixed fee plus cost contract. Thus firms will not be able to screen customers by the α type.

Proposition 4: *Suppose $\tau \leq 2/3\, S(\alpha)$ for all α present in the market. Then it is an equilibrium for both firms to offer a single contract with a fixed fee equal to τ and usage charge $\mathcal{P}(q) = C(q)$.*

Proof: Suppose firm B offered this contract. Assume firm A could actually observe α, which gives us an upper bound on its profits. Then firm A's market share, given that it offers a surplus of S (net of travel costs) to its customers, is

$$x = \min \left\{ \frac{1}{2}\left(1 + \frac{S - S(\alpha) + \tau}{\tau}\right), \frac{S}{t} \right\}$$

Obviously firm A will provide the surplus level S through the profit maximizing quantity schedule. Customers' surplus from buying at firm A net of travel costs will then be $S(\alpha) - k$, where k is the fixed fee firm A will charge in addition to the price schedule $\mathcal{P}(q)$.
Under the assumption that $\tau \leq 2/3\, S(\alpha)$ the market is fully covered and profits for firm A when charging a fixed fee k are

$$k\,\frac{1}{2}\left(1 + \frac{\tau - k}{\tau}\right)$$

The fixed fee that maximizes these profits is given by $k = \tau$ which proves the claim. ■

Thus, firms in this setting will provide all customers with the efficient quantity schedule irrespective of their consumption profile. If we compare this outcome to the outcome when firms compete over consumers with mistaken priors, we see that in this latter settings firms do not maximize total "real" surplus when designing the quantity schedule for type α, but total real and *fictitious* surplus. *Fictitious* surplus appears in the firm's design problem through an additional term in Λ^c

$$u_\theta(q, \theta)\left[\frac{F(\theta, \hat{\alpha}) - F(\theta, \alpha)}{f(\theta, \hat{\alpha})}\right]$$

This term reflects the disagreement between firms and customers concerning the surplus provided by any given quantity schedule. When designing the optimal contract, firms do so by providing customers with "what customers want" in a cost minimizing way. If customers misperceive the surplus implied by a deal, even competitive firms will not be able to correct this misperception. Firms will compete over customers by delivering what customers want, not by delivering what firms think is best for them.
However firms will not be able to raise their profits by dealing with mistaken customers. Firms make a profit which is equal to τ on each customer irrespective of their type α. All additional profits that could be made by creating

fictitious surplus are competed away. Firms do not have a comparative advantage in exploiting mistaken priors. They only have a comparative advantage in terms of brand preferences of customers. This is why their profit levels are linked to travel costs, i.e. the intensity of brand preferences.
Finally, note that because the competitive problem with mistaken customers is separable in the ex-ante types, the fully rational customer $\hat{\alpha}$ gets the efficient quantity schedule. He does not suffer or enjoy any externalities from the presence of mistaken customers in the market.

4 Conclusion

This paper has shown that menus of contingent contracts by competitive firms can be understood as screening devices for customers whose beliefs differ from firms' beliefs. In a setting where consumers' tastes are private information but firms and customers agree on the value implied by any offered contract for a given value of the private information, firms facing competition will not be able to screen customers with respect to their tastes. Here however, firms and customers can disagree about the implied value of any given deal and this disagreement allows firms to screen customers through menu design even if the exact reason for disagreement is private information of the customer.
It turns out that competition prevents firms from exploiting customers with mistaken beliefs through contract design. The profits a firm makes on a customer are independent of the beliefs this customer holds. Also flawed evaluation of deals by some customers do not influence that kind of deals offered to other customers. The presence of boundedly rational customers in the market does not exert an externality on other customers as long as this market is sufficiently competitive.
Empirically, these results are meaningful if there are a fixed number of psychological types present in the market. In this case this model would predict that all firms offer the same number of contracts, which should all be identical, and each tailored towards a specific psychological type. If this is the case, one could try to recover these types from the shape of contracts of firms and consumption data [6]. Because of the separability result, each contract schedule should only contain information on one specific type and hence such an exercise is possible.

[6] Miravete (2004) shows how to recover the distribution of types in a standard non-linear pricing problem from the shape of tariffs. In our context the shape of tariffs alone would not be enough. One would also need information on individual consumption to recover the true distribution of ex-post types.

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Appendix

A1 Monopolistic Price Discrimination with Non-Common Priors

The following section closely follows Armstrong (1996) who analyzes a very similar setting in which customers differ in their probability distributions for future demand, but the firm and customers have identical priors. Armstrong solves for the monopolist's optimal menu of tariffs under Assumption 1.

The monopolist offers his customers a menu of price schedules $\{\mathcal{P}(q,j)\}_{j\in J}$ before these customers know their type θ. Customers will therefor have to forecast their type and thus their demand for the good in order to pick the individually optimal price schedule $\mathcal{P}(q,j)$ from the offered menu J.
Consumers differ in their prior over the ex-post type θ. In particular the prior of a consumer of ex-ante type α is the distribution function $F(\theta,\alpha)$.
The firm has a prior $F(\theta,\hat{\alpha})$ over the ex-post types θ which is identical for each of its customers. It cannot observe the ex-ante types α but knows their population distribution $G(\alpha)$, where $\alpha \in [\alpha_L, \alpha_H]$.

The firm chooses a menu of price schedules $\{\mathcal{P}(q,j)\}$ that maximizes expected profits given that each customers picks the price schedule which maximizes her expected utility (where expectations are take over the respective priors). Appealing to the revelation principle the problem is to maximize profits over a menu of price schedules $\{\mathcal{P}(q(\theta,\alpha),\alpha)\}$, such that these schedules are both ex-ante and ex-post incentive compatible, and that they are ex-ante individually rational.

A consumer's surplus with ex-post type θ, who has chosen the price schedule $\mathcal{P}(q,\alpha)$ will be defined as

$$s(\theta,\alpha) = \max_{q\geq 0} \; u(q,\theta) - \mathcal{P}(q,\alpha) \tag{13}$$

The ex-post implementability of each price schedule $\mathcal{P}(q,\alpha)$ is ensured by the usual Envelope condition $s_\theta(\theta,\alpha) = u_\theta(q(\theta,\alpha),\theta)$ and the requirement that $q(\theta,\alpha)$ be non-decreasing in θ.

These conditions allow us to write the expected utility of a type α consumer from choosing the tariff $\mathcal{P}(q,\tilde{\alpha})$ as

$$v(\alpha,\tilde{\alpha}) = s(\theta_L,\tilde{\alpha}) + \int_{\theta_L}^{\theta_H} u_\theta(q(\theta,\tilde{\alpha}),\theta)[1 - F(\theta,\alpha)]d\theta \tag{14}$$

Define the maximum of this function as

$$V(\alpha) = \max_{\alpha_L \leq \tilde{\alpha} \leq \alpha_H} \; v(\alpha,\tilde{\alpha}) \tag{15}$$

Then to implement the menu of tariffs we need to ensure that $V(\alpha)$ is maximized at $\tilde{\alpha} = \alpha$. This yields a second Envelope condition

$$V'(\alpha) = -\int_{\theta_L}^{\theta_H} u_\theta(q(\theta,\alpha),\theta)F_\alpha(\theta,\alpha)d\theta \geq 0 \tag{16}$$

The fact that $V'(\alpha)$ is non-negative follows from the the condition that $u_\theta > 0$ and $F_\alpha \leq 0$ in case of Assumption 1A.
Under Assumption 2A it follows from the fact that $q(\theta,\alpha)$ is non-decreasing in θ, the single-crossing property $u_{q\theta} > 0$, and the conditions imposed on the integral of F_α.

Now as $V(\alpha)$ is weakly increasing in α, if the participation constraint is satisfied for the lowest type α_L, it is necessarily satisfied for all other $\alpha > \alpha_L$. Thus it is optimal for the firm to set $V(\alpha_L)$ equal to the outside option which we will normalize to zero, i.e. $V(\alpha_L) = 0$.

Therefore the rent of a type α consumer under any incentive compatible scheme will be

$$V(\alpha) = -\int_{\alpha_L}^{\alpha}\int_{\theta_L}^{\theta_H} u_\theta(q(\theta,\tilde{\alpha}),\theta)F_\alpha(\theta,\tilde{\alpha})d\theta d\tilde{\alpha} \tag{17}$$

and by (14)

$$s(\theta_L,\alpha) = V(\alpha) - \int_{\theta_L}^{\theta_H} u_\theta(q(\theta,\alpha),\theta)[1 - F(\theta,\alpha)]d\theta \tag{18}$$

Lemma 1 (Armstrong (1996)) *Under Assumption 1, if the function $s(\theta_L,\alpha)$ in* (14) *is given by* (18)*, then the type α consumer will choose $\tilde{\alpha} = \alpha$ in* (14) *provided that $q(\theta,\alpha)$ is (weakly) increasing in α.*
Using the expression in (14) and differentiating with respect to $\tilde{\alpha}$ yields

$$v_{\tilde{\alpha}}(\alpha,\tilde{\alpha}) = \int_{\theta_L}^{\theta_H} u_{q\theta}(q(\theta,\tilde{\alpha}),\theta)q_\alpha(\theta,\tilde{\alpha})[F(\theta,\tilde{\alpha}) - F(\theta,\alpha)]d\theta$$

Under Assumption 1A, $v(\alpha,\tilde{\alpha})$ is increasing for all $\tilde{\alpha} < \alpha$ and decreasing for all $\tilde{\alpha} > \alpha$ as long as $q_\alpha \geq 0$. Thus a sufficient condition for ex-ante implementability under Assumption 1A is that $q(\theta,\alpha)$ be weakly increasing in α. ∎

Lemma 2 *Under Assumption 2A and 2B, if the function $s(\theta_L,\alpha)$ in* (14) *is given by* (18)*, then the type α consumer will choose $\tilde{\alpha} = \alpha$ in* (14) *provided that $q(\theta,\alpha)$ is weakly decreasing in α for all $\theta \leq z$, and weakly increasing in α for all $\theta \geq z$.*

To see this first look at the case $\tilde{\alpha} < \alpha$. In this case $F(\theta,\tilde{\alpha}) - F(\theta,\alpha)$ is less or equal to zero for all $\theta \leq z$ and greater or equal to zero for all $\theta \geq z$. Thus under the assumptions imposed on q_α, $q_\alpha(\theta,\tilde{\alpha})[F(\theta,\tilde{\alpha}) - F(\theta,\alpha)]$ will be greater or equal to zero on $\theta \in [\theta_L, \theta_H]$, and therefore $v(\alpha, \tilde{\alpha})$ will be weakly increasing in $\tilde{\alpha}$ for all $\tilde{\alpha} < \alpha$. The reverse holds true for $\tilde{\alpha} > \alpha$, i.e. $v(\alpha, \tilde{\alpha})$ will be weakly decreasing on this interval. It follows that $v(\alpha, \tilde{\alpha})$ reaches its maximum at $\tilde{\alpha} = \alpha$. ■

Firm's profits from a customer of type α are

$$\int_{\theta_L}^{\theta_H} [u(q(\theta,\alpha),\theta) - C(q(\theta,\alpha))] f(\theta,\hat{\alpha}) d\theta - \\ \int_{\theta_L}^{\theta_H} u_\theta(q(\theta,\alpha),\theta)[1 - F(\theta,\hat{\alpha})] d\theta - s(\theta_L, \alpha)$$

Add and subtract $V(\alpha)$ from this expression to get

$$\int_{\theta_L}^{\theta_H} \left\{ u(q(\theta,\alpha),\theta) + u_\theta(q(\theta,\alpha),\theta) \left[\frac{F(\theta,\hat{\alpha}) - F(\theta,\alpha)}{f(\theta,\hat{\alpha})} \right] - C(q(\theta,\alpha)) \right\} f(\theta,\hat{\alpha}) d\theta - V(\alpha)$$

Total profits are

$$\pi = \int_{\alpha_L}^{\alpha_H} \left\{ \int_{\theta_L}^{\theta_H} [u(q(\theta,\alpha),\theta) + u_\theta(q(\theta,\alpha),\theta) \left[\frac{F(\theta,\hat{\alpha}) - F(\theta,\alpha)}{f(\theta,\hat{\alpha})} \right] \right. \\ \left. - C(q(\theta,\alpha))] f(\theta,\hat{\alpha}) d\theta - V(\alpha) \right\} dG(\alpha)$$

Integrating by parts using (16) we have

$$\int_{\alpha_L}^{\alpha_H} V(\alpha) dG(\alpha) = -\int_{\alpha_L}^{\alpha_H} \int_{\theta_L}^{\alpha_H} u_\theta(q(\theta,\alpha),\theta) F_\alpha(\theta,\alpha)[1 - G(\alpha)] d\theta d\alpha$$

Substituting this expression back into the firm's profits yields

$$\pi = \int_{\alpha_L}^{\alpha_H} \int_{\theta_L}^{\theta_H} \Lambda^m(q(\theta,\alpha),\theta,\alpha) f(\theta,\hat{\alpha})\, g(\alpha)\, d\theta\, d\alpha \tag{19}$$

where

$$\Lambda^m(q,\theta,\alpha) = u(q,\theta) + u_\theta(q,\theta) \left[\frac{F(\theta,\hat{\alpha}) - F(\theta,\alpha)}{f(\theta,\hat{\alpha})} \right] \\ - C(q) + u_\theta(q,\theta) \frac{F_\alpha(\theta,\alpha)}{f(\theta,\hat{\alpha})} \left(\frac{1 - G(\alpha)}{g(\alpha)} \right) \tag{20}$$

To maximize profits, we can maximize Λ^m for each combination of (θ, α) pointwise with respect to q, i.e.

$$q(\theta,\alpha) = \arg\max_{q \geq 0} \Lambda^m(q,\theta,\alpha) \tag{21}$$

If Λ^m is strictly quasi-concave in q, the optimal quantity schedule is given by the first order condition:

$$\Lambda_q^m(q(\theta,\alpha),\theta) = 0 \tag{22}$$

If furthermore Λ^m is supermodular in (q,θ), then $q(\theta,\alpha)$ will be (weakly) increasing in θ.

For optimality we also require $q_\alpha(\theta,\alpha) \geq 0$ under Assumption 1. Under Assumptions 2A and B we require that $q_\alpha(\theta,\alpha) \leq 0$ for all $\theta < z$ and $q_\alpha(\theta,\alpha) \geq 0$ for all $\theta > z$.

The quantity schedule defined by (22) deviates from the efficient quantity schedule which equates marginal utility to marginal cost, i.e $u_q(q(\theta,\alpha),\theta) = C'(q(\theta,\alpha))$, in two terms. The term $u_{q\theta}\,[F(\theta,\hat{\alpha}) - F(\theta,\alpha)]/f(\theta,\hat{\alpha})$ arises from the firm's attempt to create *fictitious surplus*. The second source of inefficiency is the usual distortion imposed on low α types in order to extract more surplus from high α types.

While the first distortion disappears for the fully rational type $\alpha = \hat{\alpha}$, the second only disappears for the highest type $\alpha = \alpha_H$. This obviously implies that the quantity schedules $q(\theta,\alpha)$ will be inefficient for all types α, unless $\alpha_H = \hat{\alpha}$ in which case there will be no distortions at the top.