

The 3-Equation New Keynesian Model — a Graphical Exposition

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Abstract

We develop a graphical 3-equation New Keynesian model for macroeconomic analysis to replace the traditional IS-LM-AS model. The new graphical IS-PC-MR model is a simple version of the one commonly used in central banks and captures the forward-looking thinking engaged in by the policy maker. We show how it can be modified to include a forward-looking IS curve and how it relates to current debates in monetary macroeconomics, including the New Keynesian Phillips Curve and the Sticky Information Phillips Curve models.

1 Introduction¹

Much of modern macroeconomics is inaccessible to the non-specialist. There is a gulf between the simple models found in principles and intermediate macro textbooks — notably, the *IS-LM-AS* approach — and the models currently at the heart of the debates in monetary macroeconomics in academic and central bank circles that are taught in graduate courses. Our aim is to show how a graphical approach can help bridge this divide.

Modern monetary macroeconomics is based on what is increasingly known as the 3-equation New Keynesian model: *IS* curve, Phillips curve and interest rate-based monetary policy rule (*IS-PC-MR*). This is the basic analytical structure of Michael Woodford’s book *Interest and Prices* published in 2003 and, for example, of the widely cited paper “The New Keynesian Science of Monetary Policy” by Clarida et al. published in the *Journal of Economic Literature* in 1999. An earlier influential paper is Goodfriend and King (1997). These authors are concerned to show how the equations can be derived from explicit optimizing behaviour on the part of the monetary authority, price-setters and households in the presence of some nominal imperfections. Moreover, “[t]his is in fact the approach already taken in many of the econometric models used for policy simulations within central banks or international institutions” (Woodford, 2003, p.237).

Our contribution — motivated by the objective of making modern macroeconomics accessible — is to provide a graphical presentation of the 3-equation *IS-PC-MR* model. The *IS* diagram is placed vertically above the Phillips diagram, with the monetary rule shown in the latter along with the Phillips curves. We believe that our *IS-PC-MR* graphical analysis is particularly useful for explaining the optimizing behaviour of the central bank. Users can see and remember readily where the key relationships come from and are therefore able to vary the assumptions about the behaviour of the policy-maker or the private sector. In order to use the model, it is necessary to think about the economics behind the processes of adjustment. One of the reasons *IS-LM-AS* got a bad name is that it too frequently became an exercise in mechanical curve-shifting: students were often unable to explain the economic processes involved in moving from one equilibrium to another. In the framework presented here, in order to work through the adjustment process, the student has to engage in the same forward-looking thinking as the policy-maker.

David Romer took some steps toward answering the question of how modern macroeconomics can be presented to undergraduates in his paper “Keynesian Macroeconomics without the *LM* Curve” published in the *Journal of Economic Perspectives* in 2000. His alternative to the standard *IS-LM-AS* framework follows earlier work by Taylor (1993) in which instead of the *LM* curve, there is an interest rate based monetary policy rule.² While our approach is a little less simple than Romer’s, it has the advantage of greater transparency.

In this paper, we focus on the explicit forward-looking optimization behaviour of the central bank. Monetary policy makers must diagnose the nature of shocks affecting the economy and forecast their impact. In sections 1 and 2, the basic graphical analysis for doing this in the *IS-PC-MR* model is set out. The way that central banks adjust the interest rate in response to current information about inflation and output is summarized by a so-called Taylor rule. In section 3, we show how

¹We are grateful for the advice and comments of Christopher Allsopp, John Driffill, Andrew Glyn, Matthew Harding, Campbell Leith, Colin Mayer, Edward Nelson, Terry O’Shaughnessy, Nicholas Rau, Daniel Rogger and David Vines.

²Other presentations of “macroeconomics without the *LM*” are provided in Allsopp and Vines (2000), Taylor (2000) and in Walsh (2002).

a Taylor rule can be derived graphically. A major pre-occupation in monetary macroeconomics in the past twenty years has been the design of a policy framework to ensure that policy is “time consistent”, i.e. that the policy maker will not have an incentive to deviate from the optimal policy after private sector agents have made commitments based on the assumption that the central bank will stick to its rule. The logic of the time-inconsistency problem and the associated problem of inflation bias are illustrated graphically in section 4.

In order to introduce the graphical *IS-PC-MR* model and demonstrate its versatility, we begin with a standard *IS* curve without a forward-looking component and a simple ‘backwards-looking’ Phillips curve. In section 5, we provide a graphical explanation of how forward-looking household behaviour alters the traditional interpretation of the *IS* curve by including expected future excess demand in the *IS* equation (e.g. McCallum and Nelson, 1999). We show how a forward-looking *IS* curve, when combined with a monetary policy rule dampens the response of the economy to shocks. The discussion of agent optimization in the Phillips curve is postponed to section 6.

While the analysis of central bank and household behaviour is widely accepted, the nature of the Phillips curve remains the subject of sharp disagreement in the literature. Although there is strong empirical evidence that inflation is highly persistent, it has proved challenging to provide an explanation for this consistent with optimizing agents, even in the presence of sticky prices (see for example, Ball, 1994, Fuhrer and Moore, 1995, Nelson, 1998, and Estrella and Fuhrer, 2002). Walsh summarizes the nature of the inflation persistence that is at issue: “In response to serially uncorrelated monetary policy shocks (measured by money growth rates or by interest rate movements), the response of inflation appears to follow a highly serially correlated pattern.” (2003, p.223). Staiger, Stock and Watson (1997), Mankiw (2001), and Eller and Gordon (2003) provide overviews of the evidence. There are two main contending theories of the Phillips curve based on optimizing behaviour, the so-called New Keynesian Phillips curve (Clarida et al., 1999) where price-setters are constrained by sticky prices, and the Sticky Information Phillips curve (Mankiw and Reis, 2002) where they are constrained by sticky information. In section 6, the graphical analysis and some simplified maths is used to explain both. The paper concludes with a comparison between the base-line *IS-PC-MR* model and the model when modified either by the use of a forward-looking *IS* curve or a rational expectations-based Phillips curve with price or information stickiness.

2 The *IS-PC-MR* model

We take as our starting point an economy in which policy-makers are faced with a vertical Phillips curve in the medium run and by a trade-off between inflation and unemployment in the short run. In setting out the 3-equation model, we make two ad hoc but empirically based assumptions: the first relates to the persistence of inflation and the second to the time lags in the reaction of the economy. At this stage, we simply assume that the inflation process is persistent, in line with a wealth of empirical evidence. In terms of adjustment lags, we assume that it takes one year for monetary policy to affect output and a year for a change in output to affect inflation. This accords, for example, with the view of the Bank of England³:

³Bank of England (1999) *The Transmission of Monetary Policy* p.9
<http://www.bankofengland.co.uk/montrans.pdf>

The empirical evidence is that on average it takes up to about one year in this and other industrial economies for the response to a monetary policy change to have its peak effect on demand and production, and that it takes up to a further year for these activity changes to have their fullest impact on the inflation rate.

The first step is to present two of the equations of the 3-equation model. The standard *IS* curve is shown in the top part of the diagram (Fig.1) as a function of the real interest rate. The real interest rate is the short-term real interest rate, r . The central bank can set the nominal short-term interest rate directly, but since the expected rate of inflation is given in the short run, the central bank is assumed to be able to control r indirectly. In the lower part of the diagram the vertical Phillips curve at the equilibrium output level, y_e , is shown. We think of labour and product markets as being imperfectly competitive so that the equilibrium output level is where both wage- and price-setters make no attempt to change the prevailing real wage or relative prices. For convenience, the ‘short-run’ Phillips curves are shown as linear. Each Phillips curve is indexed by the pre-existing or inertial rate of inflation, $\pi^I = \pi_{-1}$. They take the standard simple form in which inflation this period is equal to lagged inflation plus a term that depends on the difference between the current level of output and that at which the labour market is in equilibrium, i.e. $\pi = \pi^I + \alpha \cdot (y - y_e)$, where y_e is output at the equilibrium rate of unemployment. Given π^I , firms faced with excess demand will be trying to raise relative prices and wage-setters, relative wages.

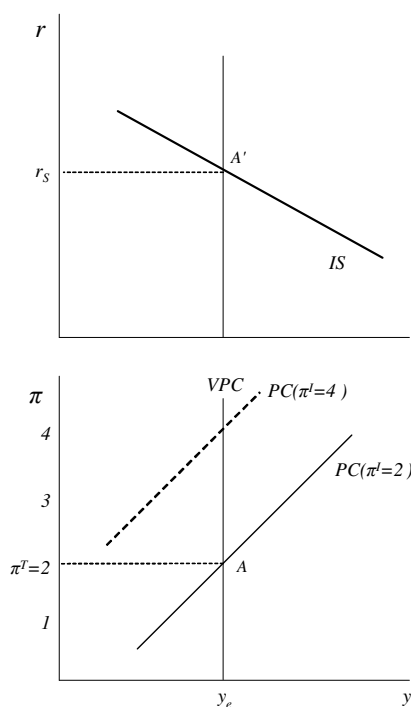


Figure 1: *IS* and *PC* curves

If it is so desired, these Phillips curves can be interpreted as expectations-augmented Phillips curves in the traditional way where expectations are adaptive. Alternatively, the presence of lagged

inflation in the Phillips curve could be the outcome of the imperfect availability of information or of institutional arrangements in a world where agents have rational expectations. For this reason, we prefer the more general term of inertial or backwards-looking Phillips curves since the key assumption relates to the persistence of inflation rather than to a specific expectations hypothesis. As shown in Fig.1, the economy is in a constant inflation equilibrium at the output level of y_e ; inflation is constant at the target rate of π^T and the real interest rate required to ensure that aggregate demand is consistent with this level of output is r_S , where the ‘S’ stands for the ‘stabilizing’ interest rate.⁴

As Romer argues, monetary policy is now usually thought about in terms of a reaction function that the central bank uses to respond to shocks to the economy and steer it toward an explicit or implicit inflation target. The first task of the reaction function is to provide a nominal anchor for the medium run, which is defined in terms of an inflation target. The second task of the reaction function is to provide guidance as to how the real interest rate should be adjusted in response to different shocks hitting the economy so that the medium-run objective of stable inflation is met while minimising output fluctuations. This broad structure for monetary policy can be formalized as an optimal monetary policy rule in the sense that the monetary rule can be derived as the solution to the problem faced by the central bank in minimizing the costs of achieving its objectives given the constraints it faces from the private sector.

To derive the monetary rule graphically, we need to consider how the central bank behaves. In Fig.2, we assume that the economy is initially at point B with high but stable inflation (on $PC(\pi^I = 4\%)$). We assume that the central bank wishes to reduce inflation to its target rate of $\pi^T = 2\%$. One plausible scenario would be that after a period of higher inflation, a new government is elected, which charges the central bank with the task of bringing inflation down to the new 2% target rate. The Phillips curve ($PC(\pi^I = 4\%)$) shows — given last period’s inflation — the feasible inflation and output pairs faced by the central bank. The only points on the curve with inflation below 4% are to the left of B , i.e. with lower output and hence higher unemployment. With Phillips curves like this, disinflation will always be costly. This result comes from the assumption that last period’s inflation always has some influence on inflation this period.

Let us assume that the central bank has chosen to reduce output to point C . In order to do this by using monetary policy, it must raise the real interest rate to r' . Inflation falls and a new Phillips curve constraint faces the central bank. The central bank will adjust the interest rate downwards as inflation falls. The economy moves along the IS curve from C' to A' and along the line labelled MR for ‘monetary rule’ from C to A . Eventually, the objective of inflation at $\pi^T = 2\%$ is achieved and the economy is at equilibrium unemployment, where it will remain until a new shock or policy change arises. The MR line shows the level of output the central bank will choose, given the Phillips curve constraint that it faces. To implement its output choice, the central bank sets the appropriate interest rate as shown in the IS diagram. As inflation gradually falls, the Phillips curve shifts down and the central bank chooses an output level closer to the equilibrium: this traces out the path down the MR along which the economy moves back to equilibrium (i.e. along the MR from C to $D \dots$ to A in the Phillips diagram; along the IS from C' to $D' \dots$ to A' in the IS diagram).

⁴Woodford (2003) calls this the Wicksellian or natural rate of interest. We do not follow his usage because r_S changes whenever the IS curve shifts.

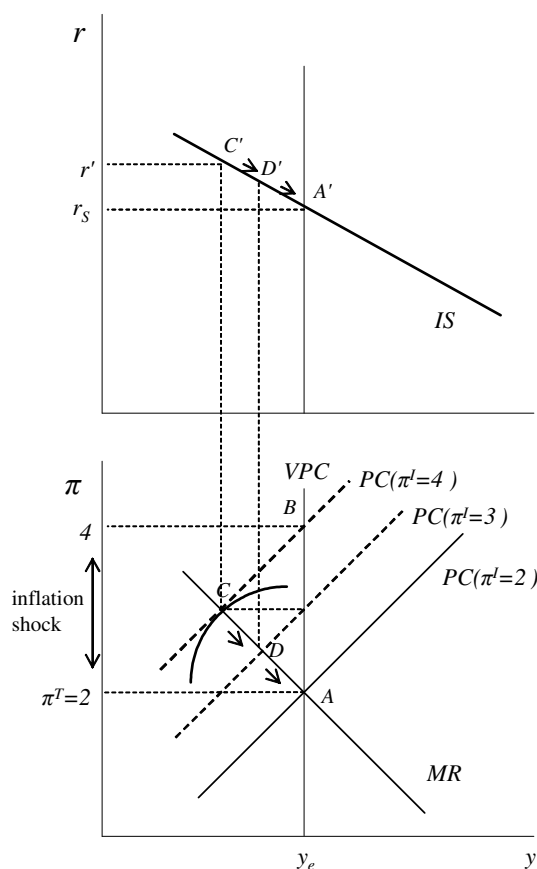


Figure 2: IS , PC and MR curves

By presenting the Phillips curve explicitly as a constraint facing the central bank, the role of its preferences in shaping the monetary rule arises naturally. An indifference curve is shown in Fig.2. The shape of the indifference curve reflects the view of the central bank about the costs of trading off a cut in inflation for a rise in unemployment. The central bank is shown as optimizing by choosing the tangency between the indifference curve and the Phillips curve constraint it faces.

More precision about these ‘indifference curves’ comes from specifying the central bank’s problem more tightly. A simple way to do this is to assume that the central bank minimizes a loss function in which it suffers disutility from deviations in inflation from target (π^T) and in output from equilibrium (y_e). If we assume that such disutility is symmetric in relation to positive and negative deviations and that the loss rises more than in proportion to the size of the deviation, a natural way to model the loss function is in terms of the squared deviation of output and inflation from y_e and π^T . This produces a set of loss ellipses centred on (y_e, π^T) , which will be circles if the same weight is placed on output and inflation deviations. Only portions of the ellipses or circles are shown so as to avoid cluttering the diagrams.

The central bank’s preferences can be presented in this simple graphical way since as long as the central bank can re-optimize each period, future developments are not relevant to the optimiza-

tion problem. Thus we are implicitly assuming that the central bank has ‘discretion’ to choose the interest rate each period. This means that it cannot commit to future levels of the interest rate even though it is concerned about future losses. So although the central bank may currently have a low inflation target for the future, it cannot bind the hands of future central bank decision-makers to this target. We return to the discussion of discretion in the context of the problem of time inconsistency in section 4.

The indifference curves of two different central banks are shown in Fig.3. The more inflation-averse central bank has a set of relatively “flat” indifference curves since such a central bank is prepared to sacrifice a larger fall in output to deliver a given reduction in inflation, whereas the less inflation-averse one has “steeper” ones. In the former case, the long axis of the ellipses is horizontal; in the latter, it is vertical.

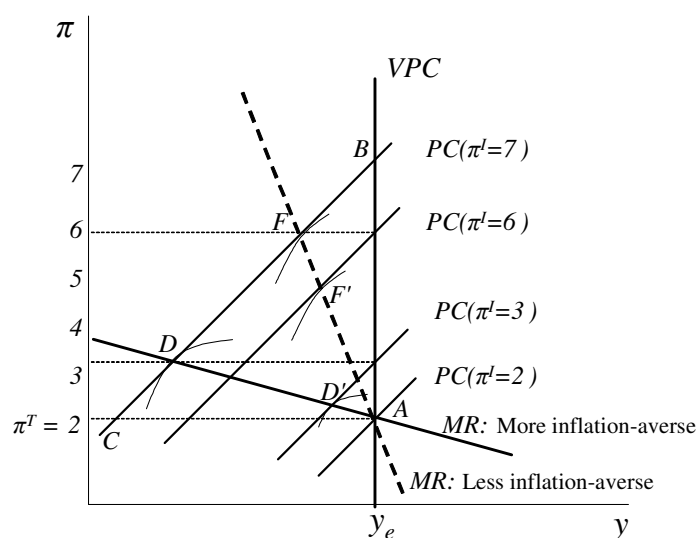


Figure 3: The monetary rule and central bank preferences

We assume that there is an inflation shock to the economy that takes inflation to 7%, i.e. to point B and each central bank is faced with the Phillips curve, $PC(\pi^I = 7)$. The more inflation-averse central bank chooses point D and guides the economy down the monetary rule path from D to A . In exactly the same way, the less inflation-averse central bank with steeper indifference curves guides the economy down its MR path from F to A . For both central banks, since their most preferred position is with $\pi = \pi^T$ and $y = y_e$, the indifference ellipses shrink to a point at A . The initial rise in unemployment is greater for the more inflation-averse central bank but disinflation is faster: in fact, with linear Phillips curves, the cumulative output (or employment) loss required to

reduce inflation back to target is independent of the degree of inflation-aversion.⁵

The MR -curve is shown in the Phillips rather than in the IS -diagram because the essence of the monetary rule is to identify the central bank's best policy response to any shock. Both the central bank's preferences between output and inflation deviations and the objective trade-off between output and inflation appear in the Phillips diagram. Moreover, by working in the Phillips diagram, the impact on the monetary rule of the structure of the supply side, which determines both the position of the vertical Phillips curve and the slope of the inertia-augmented Phillips curves is kept to the forefront. Once the central bank has calculated its desired output response by using the relevant Phillips curve and indifference curve, it is straightforward to go to the IS -diagram and discover what interest rate must be set in order to achieve this output level. For completeness, it is important to note that the LM curve depicting the interest rate-output combinations at which the demand for and supply of money are equal has not literally disappeared from the model. We can think of there being a 'shadow' LM curve that needs to intersect the IS curve at the interest rate chosen by the central bank in order for that interest rate to be sustained. The use of an interest-rate based monetary policy rule implies that shocks to the demand for money will be automatically offset by the central bank in order to maintain its interest rate at target.

Romer (2000), Taylor (2000), Allsopp and Vines (2000) and Walsh (2002) combine the IS and the MR into a single 'aggregate demand-inflation curve' in the Phillips diagram. There are three reasons why we prefer to show the IS explicitly and thereby provide a direct graphical correspondence with the 3-equation model. First it reveals a key element of structure by allowing aggregate demand shocks to be clearly identified as ' IS shocks'. It is possible to see directly whether a particular kind of shock requires a change in the interest rate relative to the stabilizing interest rate, in the stabilizing interest rate only, or in both. Second, it separates the steps in the central bank's decision process: what is the optimal output response to any shock given its preferences and the constraints it faces; how is it to be achieved? Finally, as we have seen above, by keeping the monetary rule separate from the IS , the MR only shifts when there is a change in the inflation target or in the output target, and its slope reflects only the inputs to the central bank's monetary policy decision, i.e. the slope of the Phillips curve and the central bank's preferences.

3 Aggregate demand and supply shocks

We have already seen how an inflation shock is handled in the $IS-PC-MR$ framework. We now look briefly at aggregate demand and supply shocks to illustrate the roles played in the transmission of these shocks by inflation inertia and by lags. It is assumed that the economy starts off with output at equilibrium and inflation at the target rate of 2%. First, we take a positive aggregate demand shock such as improved buoyancy of consumer expectations: the IS moves to IS' (Fig.4).

The consequence of output above y_e is that inflation will rise above target — in this case to 4%. This defines the Phillips curve ($PC(\pi^I = 4)$) along which the central bank must choose its preferred point for the next period: point C . By going vertically up to point C' in the IS -diagram, the central bank can work out that the appropriate interest rate to set is r' . The subsequent

⁵This can be seen geometrically by noting that the period-by-period output gaps as the economy adjusts to equilibrium with any MR curve can be matched one-for-one with segments of the distance AC and shown to sum to that distance. AC is the one period output cost of returning the economy to target inflation when the central bank is fully non-accommodating.

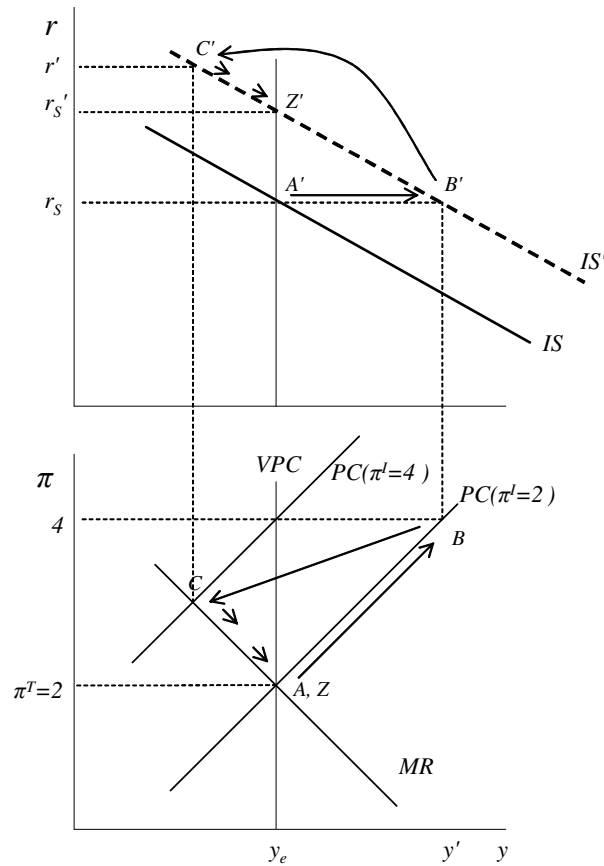


Figure 4: Aggregate demand shock and the monetary policy rule

adjustment path down the MR -curve to point Z is exactly as described in the case of the inflation shock.

This example highlights the role of the stabilizing real interest rate, r_S : following the shift in the IS curve, there is a new stabilizing interest rate and in order to reduce inflation, the interest rate must be raised above the new r_S , i.e. to r' . If the demand shock is only temporary, the IS curve shifts to IS' for only one period before returning to its initial position. In this case, there is no change to the stabilizing interest rate and the central bank simply raises the real interest relative to the original r_S . This example illustrates the importance for the central bank in being able to forecast the persistence of such shocks.

To summarize, the rise in output builds a rise in inflation above target into the economy. Because of inflation inertia, this can only be eliminated by pushing output below and (unemployment above) the equilibrium. The graphical presentation emphasizes that the central bank raises the interest rate in response to the aggregate demand shock because it can work out the consequences for inflation. The central bank is forward-looking and takes all available information into account: its ability to control the economy is limited by the presence of inflation inertia and by the time lag for a change in the interest rate to take effect. When using the model, each of these key elements

is encountered as the nature of the shock is diagnosed, its implications for the future worked out and the central bank's optimal response deduced.

An aggregate demand shock can be fully offset by the central bank even if there is inflation inertia if the central bank's interest rate decision has an immediate effect on output. The economy then remains at A in the Phillips diagram in which points A and Z coincide and goes directly from A' to Z' in the IS -diagram. This highlights the crucial role of lags and hence of forecasting for the central bank: the more timely and accurate are forecasts of shifts in aggregate demand, the greater is the chance that the central bank can offset such shocks and prevent the impact of inflation from being built into the economy.

One of the key tasks of a basic macroeconomic model is to help illuminate how the main variables are correlated following different kinds of shocks. We can appraise the usefulness of the $IS-PC-MR$ model in this respect by looking at a positive aggregate supply shock and comparing the optimal response of the central bank and hence the output and inflation correlations with those above. A supply shock results in a change in the equilibrium rate of unemployment and therefore a shift in the vertical Phillips curve. It can arise from changes that affect wage- or price-setting behaviour such as a structural change in wage-setting arrangements, a change in taxation or in unemployment benefits or in the strength of product market competition, which alters the mark-up.

Fig.5 shows the analysis of a positive supply-side shock, which reduces the equilibrium rate of unemployment and therefore increases the level of output at which inflation is constant to y'_e .

The vertical Phillips curve shifts to the right as does the short-run Phillips curve corresponding to inflation equal to the target (shown by the $PC(\pi^I = 2, y'_e)$). The first consequence of the supply shock is a fall in inflation (from 2% to zero) as the economy goes from A to B . To decide how monetary policy should respond to this, the central bank locates the appropriate Phillips curve constraint ($PC(\pi^I = 0, y'_e)$) and chooses its optimal level of output as shown by point C . To raise output to this level, it is necessary to cut the interest rate to r' as shown in the IS diagram. (Note that the stabilizing interest rate has fallen to r'_S .) The economy is then guided along the MR' curve to the new equilibrium at Z . The positive supply shock is associated initially with a fall in inflation and a rise in output — in contrast to the initial rise in both output and inflation in response to the aggregate demand shock. When examining an aggregate demand shock, we saw that even with inflation inertia, such a shock could be fully offset if the central bank is able to affect output immediately, i.e. without a lag. This is also the case if the central bank correctly anticipates a supply shock.

4 A Taylor Rule in the $IS-PC-MR$ model

A Taylor Rule is a policy rule that tells the central bank how to set the current interest rate in response to shocks that result in deviations of inflation from target or output from equilibrium or both. In other words, $(r_t - r_S)$ responds to $(\pi_t - \pi^T)$ and $(y_t - y_e)$. On the basis of an empirical analysis of the behaviour of the US Federal Reserve, Taylor put the weights on the two deviations equal to 0.5. So a widely discussed version of the rule takes the form:

$$r_t - r_S = 0.5(\pi_t - \pi^T) + 0.5(y_t - y_e).$$

Based on the timing of events we have used so far, we can show how a Taylor Rule is derived

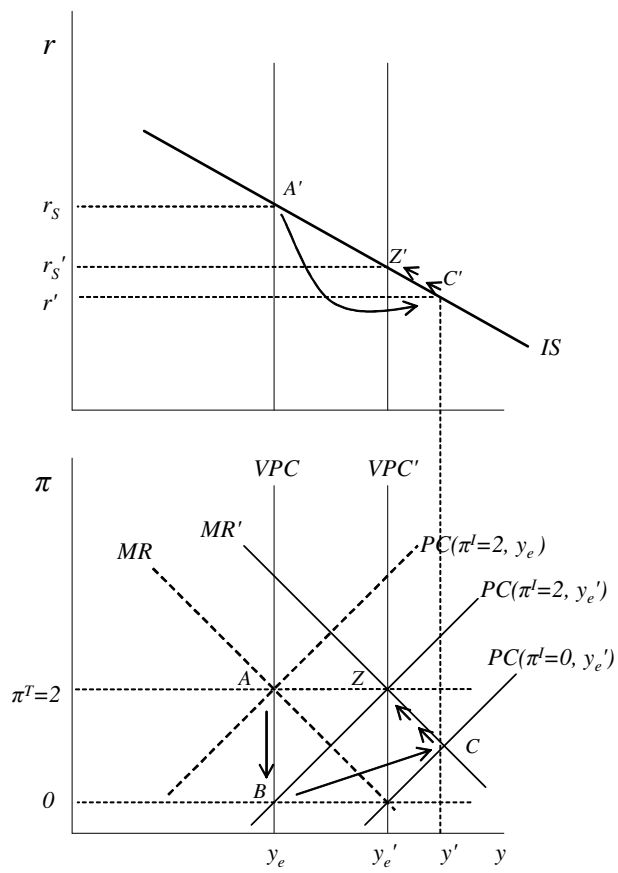


Figure 5: Aggregate supply shock and the monetary rule

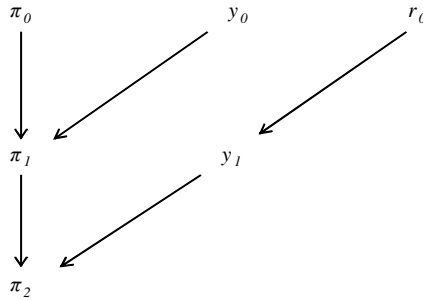


Figure 6: Lag structure in the *IS-PC-MR* model

geometrically from the *IS-PC-MR* model. Specifically, we can investigate how the coefficients on the inflation and output deviations depend on the slopes of the three curves: we shall see that if the absolute value of the slope of the *IS*, the Phillips curves and the *MR* are each equal to one, then the weights in the Taylor rule are 0.5 and 0.5. This helps bring out the role that differences in economic structure (demand and supply sides) and in central bank preferences can have on the coefficients of Taylor Rules.

To see how the central bank should react now to a signal from current economic data about inflation and output, it is necessary to state clearly the lags between the variables. It is assumed that there is no observational time lag for the monetary authorities, i.e. the central bank can set the interest rate (r_0) as soon as it observes current data (π_0 and y_0). However, the interest rate set now only has an effect on output next period, i.e. r_0 affects y_1 . This is because it takes time for a change in the interest rate to feed through to consumption and investment decisions. It is also the case that inflation is affected by output with a lag: i.e. output level y_1 affects inflation a period later, π_2 . The lag structure is shown in Fig.6 and highlights the fact that a decision taken today by the central bank to react to a shock will only affect the inflation rate π_2 . When the economy is disturbed in the current period (period zero), the central bank looks ahead to the implications for inflation and sets the interest rate so as to determine y_1 ; which in turn determines the central bank's desired value of π_2 . As the diagram illustrates, action by the central bank in the current period has no effect on output or inflation in the current period or on inflation in a year's time.

In Fig.7, the initial observation of output and inflation in period zero is shown by the large cross, \times . To work out what interest rate to set, the central bank notes that in the following period, inflation will rise to π_1 and output will still be at y_0 since a change in the interest rate can only affect y_1 . The central bank therefore knows that the constraint it faces is the *PC*(π_1) and it chooses its best position on it to deliver π_2 . This means that output must be y_1 and therefore that the central bank sets r_0 in response to the initial information shown by point \times . This emphasises that the central bank is forecasting what inflation will be in period one: its only observed information is inflation and output at time zero, i.e. point \times .

This reasoning is by now familiar. As shown in the left hand panel of Fig.7, the two components

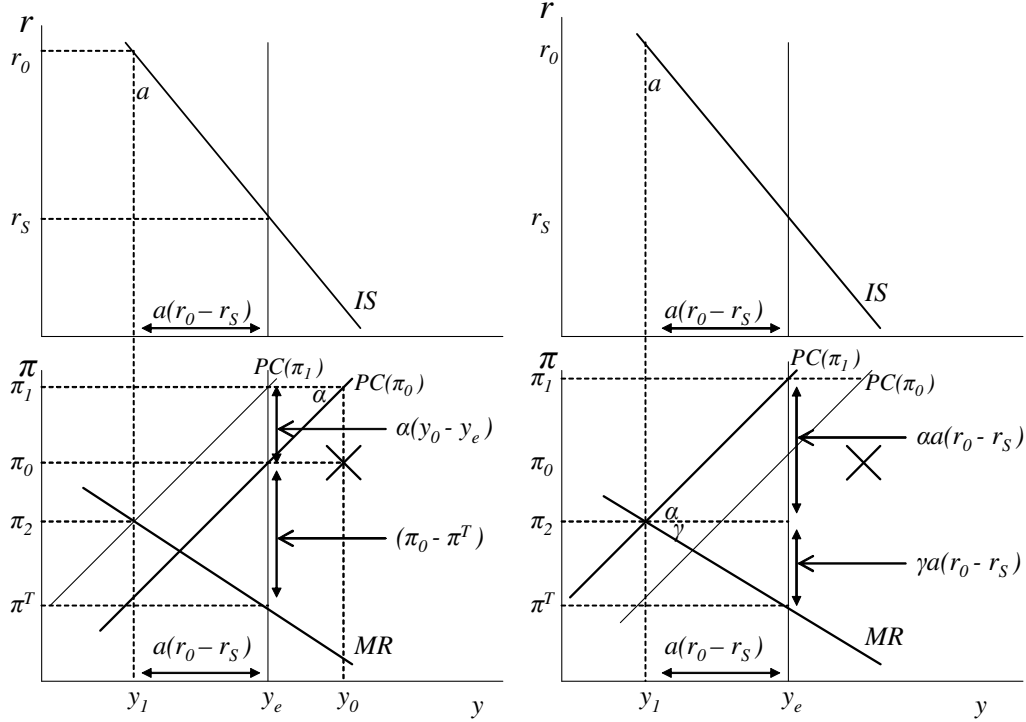


Figure 7: Deriving a Taylor Rule

of our Taylor Rule are shown by the vertical distances equal to $\alpha(y_0 - y_e)$ and $\pi_0 - \pi^T$, where α is the slope of the Phillips curve. If these are added together, we have the forecast of $\pi_1 - \pi^T$. Just one more step is needed to express this forecast in terms of $(r_0 - r_s)$ and therefore to deliver a Taylor Rule. As shown in the right hand panel of Fig.7, the vertical distance $\pi_1 - \pi^T$ can also be expressed as $(\alpha + \gamma) \cdot a(r_0 - r_s)$, where α and γ reflect the slopes of the Phillips curve and the monetary rule curve, respectively and a is the reciprocal of the slope of the IS curve.⁶

Thus, we have

$$(\alpha + \gamma) \cdot a(r_0 - r_s) = (\pi_0 - \pi^T) + \alpha(y_0 - y_e)$$

and by rearranging to write this in terms of the interest rate, we have a Taylor Rule:

$$\begin{aligned} r_0 - r_s &= \frac{1}{(\alpha + \gamma)a} [(\pi_0 - \pi^T) + \alpha(y_0 - y_e)] && \text{(Taylor Rule)} \\ &= 0.5(\pi_0 - \pi^T) + 0.5(y_0 - y_e) \end{aligned}$$

if $\alpha = \gamma = a = 1$.

It is important to note that in the case of the three kinds of shocks examined above, i.e. an inflation shock that shifts the Phillips curve (or the analytically identical case in the $IS-PC-MR$ model of a change in monetary policy that shifts the inflation target), an aggregate demand shock

⁶Note that in the diagram, a , α and γ refer to the angle shown and in the algebra to the gradients i.e. to the tans of the relevant angles.

that shifts the IS or an aggregate supply shock that shifts the equilibrium level of output, the period zero effect is either a deviation of output from equilibrium or a deviation of inflation from target, but not both. What the Taylor Rule does is to provide the central bank with guidance as to its optimal response should the economy be characterized by any of these shocks or by a combination of shocks that together produce an output and/or inflation deviation. Implicitly the Taylor Rule incorporates changes in the interest rate that are required as a result of a change in the stabilizing interest rate (in the case of a permanent shift in the IS or of a supply-side shift): r_S in the rule must be interpreted as the post-shock stabilizing interest rate.

One striking aspect of the graphical derivation is that it helps to dispel a common confusion about Taylor Rules. It is often said that the relative weights on output and inflation in a Taylor Rule reflect the central bank's preferences for reducing inflation as compared to output deviations. As can be seen from the left hand panel of Fig.7 and from the Taylor Rule equation, this is not the case in the $IS-PC-MR$ model. The *relative* weight on inflation and output in our Taylor Rule depend only on α , the slope of the Phillips curve since the relative weights are used only to forecast next period's inflation. Another way to express this result is to say that the output term only appears in the Taylor Rule because of the lag from a change in output to a change in inflation.⁷ As is clear from the right hand panel of Fig.7 and from Taylor Rule equation, central bank preferences determine the interest rate response to next period's inflation (as embodied in the slope of the MR curve). In the appendix, the monetary rule equation is derived explicitly and the consequences for central bank behaviour are shown when the slopes of the three curves differ from one.

5 Monetary policy rules and time inconsistency

It is straightforward to illustrate the problem of time inconsistency in monetary policy using the $IS-PC-MR$ model. The problem relates to whether the central bank has the incentive to stick to its inflation target once the private sector has committed to acting on the basis that the central bank will do so. In the analysis so far, the problem has not arisen since the central bank's utility is maximized when output is at equilibrium and inflation at target. To demonstrate the source of the time inconsistency problem, we take the 3-equation model and make just one change so that the government's output target is above the equilibrium: $y^T > y_e$. We assume that the government can impose this target on the central bank and that the central bank's loss function is otherwise unaffected. Since with imperfect competition in product and labour markets equilibrium unemployment is higher than that associated with labour market clearing, the government may have a higher target level of output than y_e . As we shall see, inflation in equilibrium is now above the government's inflation target.

As before, the central bank aims to minimize the extent to which the economy deviates from its inflation target and from the output target, so its indifference curves are now centred on (y^T, π^T) rather than on (y_e, π^T) . This is highlighted by showing the full indifference circles. As a consequence, the monetary policy rule is shifted to the right as shown in Fig.8. We can see immediately that the government's target, point A , does not lie on the Phillips curve for inertial

⁷Bean (1998) derives an optimal Taylor rule in a model similar to the $IS-PC-MR$ model. However, in his model, the central bank's preferences do affect the Taylor Rule weights. This arises from his inclusion of lagged output in the IS equation: if the coefficient on lagged output is zero then the difference between the weight on inflation and on output in the Taylor rule only depends on the slope of the Phillips curve and not on preferences.

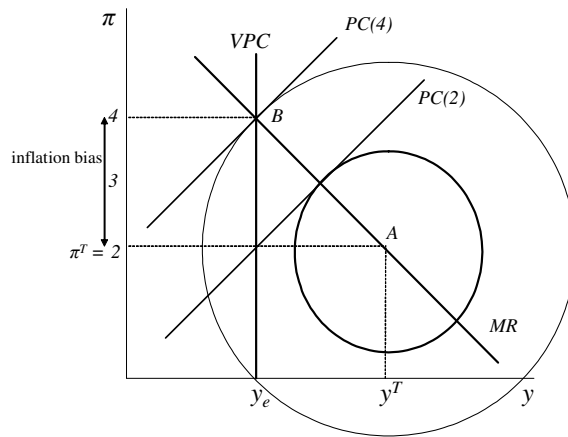


Figure 8: The inflation bias

inflation equal to the target rate of $\pi^T = 2\%$: the economy will only be in equilibrium with constant inflation at point B . This is where the monetary rule (MR) intersects the vertical Phillips curve at $y = y_e$. At point B , inflation is above the target: the target rate is 2% but inflation is 4% : this is called the inflation bias associated with central bank discretion. This example highlights that although the central bank uses a monetary rule — i.e. it uses a reaction function to respond in a systematic way to deviations of inflation and output from target — its behaviour is discretionary because it has the discretion to choose the inflation rate after the private sector has formed its inflation expectations.

The same conclusion is reached if there is no inflation inertia and price-setters form their expectations about inflation rationally so that expected inflation is equal to actual inflation plus a random disturbance, ε_t : $\pi^E = \pi + \varepsilon_t$. The intuition is that price-setters know that whatever their expected rate of inflation, the condition for their inflation expectations to be fulfilled (i.e. $\pi^E = \pi$) is that the economy be at the equilibrium level of output, i.e. $y = y_e$. In the case of central bank discretion, the government chooses the level of output after price-setters have chosen their expected rate of inflation. So in order for price-setters to have correct inflation expectations, they must choose the Phillips curve such that it pays the government to choose $y = y_e$ and that must be where the government's monetary rule cuts the vertical Phillips curve, i.e. at point B . Inflation must be sufficiently high to remove the temptation of the government to raise output toward its target. With $\pi = 4\%$ and $y = y_e$, the temptation has been removed because any increase in output puts the government on an indifference curve more distant from point A and therefore with lower utility. It is the over-ambition of the government that produces the inflation bias (under discretion) in the time-inconsistency model. In short, as long as the policy-maker aims for y_e , there is no time-inconsistency problem.

The graphical presentation also highlights the fact that the steeper is the government's monetary rule, the greater will be the inflation bias. As we have already seen, for a given Phillips curve, the

monetary rule is steeper for a less inflation-averse central bank.

6 The forward looking IS curve

In the traditional IS curve, aggregate demand shocks are amplified through the operation of the multiplier. Once a forward-looking IS curve is introduced, demand shocks are dampened as the behaviour of households interacts with that of the forward-looking central bank. A typical way of introducing forward-looking behaviour in the IS curve is to ignore investment and concentrate attention on consumption behaviour. Households are assumed to make their consumption decisions on the basis of their expected future income in such a way that their life-time utility is maximized. Since it is assumed that households wish to smooth consumption over time, higher expected future output, which entails higher future consumption, will raise current consumption and output. A higher real interest rate depresses consumption because of the household's ability to substitute future for current consumption (it is assumed that the substitution effect outweighs the income effect of an interest rate change). Government expenditure is incorporated in an exogenous demand term. The so-called Euler condition for optimal consumption over time is derived from the household's optimization problem and when combined with the exogenous demand, A_t , implies an equation of the following form for the IS curve⁸:

$$y_t = E_t y_{t+1} + A_t - ar_{t-1}$$

where $E_t y_{t+1}$ is the expectation formed in period t of the value of output in period $t + 1$.

The stabilising short term real rate of interest, r_S , is defined by $y_{e,t} = E_t y_{e,t+1} + A_t - ar_{S,t}$, and since some algebra is necessary, we simplify the notation by defining the gap between actual and equilibrium output as x , 'excess demand': $x_t \equiv y_t - y_e$. The IS equation can be written in terms of deviations from equilibrium as follows:

$$x_t = E_t x_{t+1} - a(r_{t-1} - r_{S,t}). \quad (\text{Forward-looking } IS)$$

In this section, we use the same lag structure as before. We also continue to assume that the Phillips curve is backwards looking, and that the monetary authority adopts a discretionary optimising policy:

$$\pi_t = \pi_{t-1} + \alpha x_{t-1} \quad (\text{Phillips curve})$$

$$x_t = -\frac{1}{\gamma} (E_t \pi_{t+1} - \pi^T). \quad (\text{Monetary policy rule})$$

How is the analysis of inflation and demand shocks affected by the requirement that the authorities take into account the impact of future output on current demand in the IS curve; and that households work out the effect on future output and hence on their current demand of the consequences of shocks for the actions of the central bank? We develop our graphical approach to show that the forward-looking behaviour of households dampens the interest rate consequences of shocks. The intuition is straightforward: households can forecast that a positive inflation shock now will lead to increased (though declining) interest rates over future periods until equilibrium is again restored. Hence households immediately dampen demand by more than the impact of an

⁸See, for example, McCallum and Nelson (1999) for the details.

increased short-term interest rate so as to smooth the effect of the future higher interest rates on their consumption path.

This can be seen by rewriting the forward-looking IS curve as:

$$x_t = -a(r_{t-1} - r_{S,t}) - a(r_t - r_{S,t+1}) - a(r_{t+1} - r_{S,t+2}) - \dots$$

In other words, current demand is a function, not just of the lagged real short-term interest rate, but of all (expected) future real short term interest rates. Thus household demand immediately contracts in response to the full course of expected future real interest rates. This means in turn that the central bank — so long as it works through the implications of forward looking household behaviour — needs to raise interest rates by less than it otherwise would to achieve the desired reduction in inflation. The anticipatory behaviour of households and the central bank are reinforcing, so that output falls and interest rates rise less than in our previous examples (with the traditional IS curve). We illustrate the difference that the forward-looking IS curve makes by looking at an inflation shock (the case of an IS shock is shown in the appendix).

We start in equilibrium (in Fig.9) with $\pi_{-1} = \pi^T$ and $x_{-1} = 0$ (at point A). In period zero there is an inflation shock of ε , so that

$$\pi_0 = (\pi^T + \varepsilon) + \alpha x_{-1} = \pi^T + \varepsilon.$$

The economy moves up the vertical Phillips Curve to $\pi_0 = \pi^T + \varepsilon$ at point B . What happens to excess demand in period zero? As usual, the central bank cannot influence this directly since the effect of its interest rate decision takes effect with a lag. Hence the interest rate that affects output in period zero is $r_{-1} = r_S$. However, the central bank can affect current period output indirectly: the IS equation at period zero says that $x_0 = E_0 x_1 - a(r_{-1} - r_S) = x_1$. From now on we drop the expectations operator. As we shall see, since households at time zero can work out that the central bank will choose to raise r_0 in order to create $x_1 < 0$, households will immediately cut back demand in period zero in anticipation of this.

How precisely do households form expectations of x_1 ? Since the central bank can only influence inflation in period two and output in period one, households know that the central bank chooses the pair jointly at the intersection of the monetary rule line, MR and the period two Phillips curve, $\pi_2 = \pi_1 + \alpha x_1$. But in order to know π_1 the central bank has to work out x_0 , because $\pi_1 = \pi_0 + \alpha x_0$. Since households will set $x_0 = x_1$, the central bank can work out the period two Phillips curve as:

$$\pi_2 = \pi_1 + \alpha x_1 = (\pi_0 + \alpha x_0) + \alpha x_1 = \pi_0 + 2\alpha x_1.$$

This is the steeper Phillips curve shown by the dashed line in Fig.9. Thus households can in turn forecast that the central bank's choice of (π_2, x_1) will be at the intersection of the MR and $\pi_2 = \pi_0 + \alpha x_1$ (point D). It can also be shown that

$$x_0 = x_1 = -\frac{\gamma\varepsilon}{1 + 2\alpha\gamma}.$$

Now Fig.9 can be used to see what happens in periods zero and one. In period zero, households cut demand to x_0 . We know where that is in the diagram since it is equal to x_1 . Since $\pi_1 = \pi_0 + \alpha x_0$, π_1 also falls; we move from point B to point C in the Phillips curve diagram.

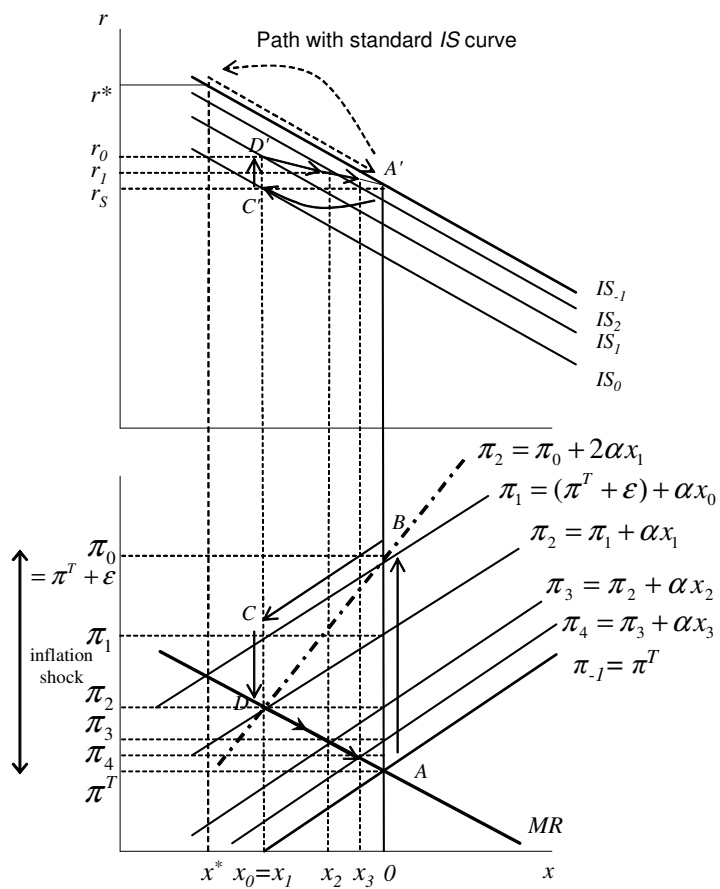


Figure 9: Inflation shock with a forward-looking IS curve

The future path of excess demand and inflation, $(\pi_3, x_2), (\pi_4, x_3), \dots$ is easy to work out. This is because each pair is chosen by the central bank in the relevant time period by the intersection of the MR line with the relevant Phillips curve. So (π_3, x_2) , is the intersection of MR with the PC , $\pi_3 = \pi_2 + \alpha x_2$, and so on. Thus once the economy has reached (π_2, x_1) i.e. point D , the adjustment path down the MR line is the same as in the analysis in section 1.

We now turn to the path of interest rates, and to the IS diagram. Given the time lags between a change in the interest rate and its effect on the output gap and of the output gap to inflation, the central bank chooses r_0 to set (π_2, x_1) (point D'), r_1 to set (π_3, x_2) , r_2 to set (π_4, x_3) , etc. The initial IS curve, IS_{-1} , goes through the vertical Phillips curve at r_S . This is because $E_{-1}x_0 = 0$, so that $x_{-1} = E_{-1}x_0 - a(r_{-2} - r_S) = -a(r_{-2} - r_S)$; and since $r_{-2} = r_S$, $x_{-1} = 0$. In period zero, the IS curve, IS_0 , is given by

$$x_0 = E_0x_1 - a(r_{-1} - r_S).$$

It is easy to see that this IS curve at time zero goes through the intersection of x_1 and r_S (point C'). Since $r_{-1} = r_S$, this confirms that $x_0 = x_1$. In other words, the anticipation by the household of the central bank's action leads it to reduce consumption before the interest rate rises. The IS curve in period 1, IS_1 , is

$$x_1 = E_1x_2 - a(r_0 - r_S),$$

and goes through r_S at x_2 . Hence to ensure that the central bank can hit its x_1 target, it needs to set r_0 above r_{-1} , where x_2 intersects IS_1 . In the same way, IS_2

$$x_2 = E_2x_3 - a(r_1 - r_S)$$

intersects r_S at x_3 . And r_1 is given by the intersection of IS_2 with x_3 . And so on.

Thus, it can be seen, that less excess supply and correspondingly lower interest rates are needed by the central bank to adjust back to equilibrium after an inflation shock with a forward looking IS curve. With an ordinary IS curve, households do not take into account the future pattern of deflation the central bank will impose in the event of a shock of this kind. Therefore the central bank has to impose a bigger recession. With such an IS curve, $x_0 = 0$ since households take no account of the fact that x_1 will be negative. Hence the period two Phillips curve is $\pi_2 = \pi_0 + \alpha x_1$ and the pair (π_2, x_1) is determined by the intersection of the monetary rule line with this Phillips curve with $x_1 = x^*$. Moreover, since there is a unique IS curve (which goes through the vertical Phillips curve at r_S), $r_0 = r^*$ is at the intersection of that IS curve and $x_1 = x^*$. The adjustment process in the IS diagram is shown by the dashed line in Fig.9. The outcome is that cycles are more muted as the forward-looking behaviour of households and the central bank reinforce each other.

7 Debates over Phillips curves: the New Keynesian Phillips Curve versus the Sticky Information Phillips Curve

The $IS-PC-MR$ model provides a simple macro-economic framework for use in analyzing contemporary performance and policy issues. It matches the empirical evidence concerning inflation persistence and the lag structure of key variables. Its main shortcoming is that it rests on ad hoc

assumptions — in particular about the inflation process — rather than being derived from an optimizing micro model of firm behaviour. An important manifestation of this problem relates to the issue of the credibility of monetary policy. We have seen in section 1 that when the central bank announces a lower inflation target, the economy moves only slowly towards this as the Phillips curve shifts period-by-period (as shown in Fig.2). Whether or not the central bank’s announcement is believed by the private sector makes no difference at all to the path of inflation. For this reason, the analysis of an inflation shock and of an announced change in the inflation target is identical in the *IS-PC-MR* model: either way, the inflation that is built into the system takes time (with higher unemployment) to work its way out. The inability of the model to take into account the reaction of price-setters to announced changes in monetary policy is unsatisfactory. Recent developments in modelling the Phillips curve aim to provide a micro-optimizing based model that can produce both costly disinflation and a role for the credibility of monetary policy.

7.1 The New Keynesian Phillips Curve

The New Keynesian Phillips Curve (*NKPC*) is derived from the Calvo model (1983), which combines staggered price-setting by imperfectly competitive firms and the use of rational expectations by private sector agents. Specifically, Calvo assumes that each period a proportion δ of firms, randomly chosen, can reset their prices. Using this assumption, Clarida et al. (1999) show that the Phillips curve — the so-called New Keynesian Phillips curve — then takes a particularly simple form in which inflation depends on the current gap between actual and equilibrium output as in the standard Phillips curve but on expected future inflation rather than on past inflation. The *NKPC* takes the following form:

$$\pi_t = \frac{\alpha\delta}{1-\delta}x_t + \theta E_t\pi_{t+1} \quad (NKPC)$$

where θ is the discount factor ($\theta < 1$) and $E_t\pi_{t+1}$ is the expected value of inflation in $t + 1$ at t . In an appendix we provide a simplified explanation of how the *NKPC* is derived from the sticky price assumption. The larger the per cent of firms that can set their price in the current period, the more important is current excess demand as a determinant of inflation, shown by the term $\delta/(1-\delta)$. The intuition is that current excess demand will be more important than future factors if there is a high chance you can reset your price each period. In terms of the graphical presentation, a higher presence of price-stickiness, i.e. lower δ , implies a flatter Phillips curve; if all firms set prices every period, $\delta = 1$ and the Phillips curve is vertical.⁹ This is of course the case of rational expectations with full price flexibility.

The most important point about the *NKPC* equation is that current inflation depends simply: (i) on the present, i.e. on the current output gap, x_t , and (ii) on the future, embodied in $E_t\pi_{t+1}$. There is no role for last period’s inflation, despite sticky prices. The big advantage of the *NKPC* is that it embodies rational expectations on the part of all agents.

In order to use the *NKPC*, it is necessary to work out how rational agents form their expectations of future inflation, $E_t\pi_{t+1}$. To do this, we need to derive the monetary rule. This is done as before by minimizing the central bank’s loss function subject to the Phillips curve, in this case, the

⁹As is clear from the equation, the *NKPC* delivers a non-vertical long-run Phillips curve because of the presence of the discount factor, θ . This feature of the *NKPC* is unsatisfactory but we do not pursue it here. Since θ will be close to 1, we show it as vertical in the figures.

NKPC. As shown in the appendix, this produces the usual monetary rule, which can be written as:

$$\pi_t = -\gamma x_t, \quad (MR)$$

where $\gamma = \frac{1-\delta}{\delta\alpha\beta}$ and reflects the slope of the Phillips curve and the inflation-aversion of the central bank as in the *IS-PC-MR* model as well as the “Calvo coefficient”, δ . The intuition is that both the *NKPC* and the monetary rule have to hold in each period and this implies that expected inflation is equal to the central bank’s inflation target (for the details, see the appendix). This result is neat and has the attractive property that the credibility of monetary policy matters. In terms of the *IS-PC-MR* diagram, the *NKPC* always intersects the *MR* schedule at ($y = y_e$ and $\pi = \pi^T$), in the absence of unanticipated shocks. Thus an announced reduction in the inflation target would immediately translate into an equivalent reduction in inflation since the *NKPC* would jump to its new position and output would remain unchanged. Inflation depends on future expected inflation and this changes as soon as a new inflation target is announced. Although only $\delta\%$ of firms can change their prices, they alter them taking into account their rational expectation of when they will next be able to optimize and this calculation leads to aggregate inflation jumping to the new inflation target. The *NKPC* has the property that credibility matters but brings with it the disadvantage that there is no inflation persistence and therefore no output cost associated with a change in monetary policy.

Let us check whether the *NKPC* meets the requirement that an unanticipated one-period inflation shock, such as a cost shock, entails a costly disinflation. Such a shock shifts the *NKPC* vertically upwards as shown in Fig.10 by the $NKPC(\pi^T + \epsilon)$. The δ proportion of firms that can reset their prices take this into account and optimize whilst the prices of the other $1 - \delta$ firms remain as determined by their previous pricing decision. Since the latter group cannot change their pricing decision, they react by cutting output by more than do the price-setters. The aggregate result for the economy is shown by the intersection of the *MR* curve and the *NKPC* curve at point *C* in Fig.10. Hence the consequence of the inflation shock is a reduction in activity in the economy below the equilibrium. The next period, however, the economy will once again be at equilibrium with target inflation (point *A*): the cost shock has gone and the inflation outcome the previous period has no lasting effect on either group of price-setters. It is also important to note that a higher degree of price-stickiness is reflected purely in the magnitude of the one-period unemployment cost of disinflation (a higher weight of stickiness means the *NKPC* and *MR* are both flatter and hence the one-period fall in output is higher).

The inability of the *NKPC* to account for the persistence of inflation following a shock is its Achilles heel: there is no inflation persistence following a change in monetary policy and only a single period impact on inflation following an inflation shock (using Fig.10, if the economy is initially in equilibrium at point *B*, it goes straight to point *A* following an announced reduction in the inflation target to π^T). Clarida et al. attempt to build more realistic results into their model by introducing an exogenous “cost push” factor, c , which is an inflation shock, the effect of which is assumed to diminish over time. The mechanics of the *NKPC* with such an autocorrelated cost-push shock added are set out in the appendix. By assumption, this modified model produces inflation inertia, with disinflation taking place over many periods, but in common with the backwards-looking Phillips curve, it lacks micro-foundations.

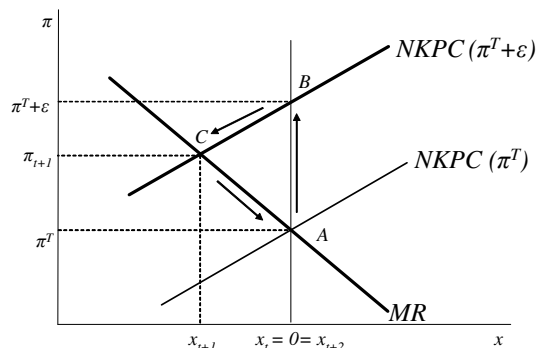


Figure 10: *NKPC*: adjustment to an inflation shock

7.2 The Sticky Information Phillips Curve

The *NKPC* brings back rational expectations into the inflationary process, but it provides only a poor match with the empirical fact of inflation inertia. An important recent development by Mankiw and Reis (2002) argues that this is a consequence of basing the microeconomics on sticky prices. Instead they assume that many price setters may only receive up-to-date information with a lag. Mankiw and Reis call the Phillips curve based on this assumption the Sticky Information Phillips Curve (*SIPC*).

In the Mankiw-Reis formulation, the *SIPC* is somewhat complex mathematically. In part this is because Mankiw and Reis assume that the central bank has a monetary growth target rather than an inflation target and an interest rate-based monetary policy rule. It also follows from their assumption about sticky information, namely that a given percentage δ of price setters acquire up-to-date information each period. We develop instead a simple example based on their model, which uses the interest-rate based monetary rule and assumes that, while δ percent of price setters acquire up-to-date information each period, the remaining $(1 - \delta)$ receive that information exactly one period later. (As with the *NKPC* we continue to assume that there is no time lag from the rate of interest to excess demand or from excess demand to price-setting.)

Leaving aside for a moment the question of just what information is sticky, it is important to note that irrespective of whether a firm has full or only limited information, all firms use rational expectations. So everyone knows that:

$$\pi_t = \delta \pi_t^{FI} + (1 - \delta) \pi_t^{LI}$$

where *FI* and *LI* denote the firms with full and limited information respectively. As in the previous section, when a firm sets its price it will want to raise its relative price when there is excess demand, i.e. $x > 0$ and vice versa. Expressed in terms of inflation, those with full information will choose

the inflation rate

$$\pi_t^{FI} = \pi_t + \alpha x_t$$

since they are assumed to know or be able to work out π_t and x_t . And those with limited information will set

$$\pi_t^{LI} = E_{t-1}(\pi_t + \alpha x_t).$$

Since all firms use rational expectations, they all know the equation

$$\begin{aligned} \pi_t &= \delta(\pi_t + \alpha x_t) + (1 - \delta)(E_{t-1}\pi_t + \alpha E_{t-1}x_t) \\ &= \frac{\alpha\delta}{1 - \delta}x_t + E_{t-1}\pi_t + \alpha E_{t-1}x_t. \end{aligned}$$

Of course those with limited information will not necessarily know π_t and x_t . However, using rational expectations the *LI* firms can deduce

$$E_{t-1}\pi_t = \frac{\alpha\delta}{1 - \delta}E_{t-1}x_t + E_{t-1}\pi_t + \alpha E_{t-1}x_t,$$

which implies that $E_{t-1}x_t = 0$.

To find out $E_{t-1}\pi_t$, the *LI* firms now simply have to use the monetary rule, namely $\pi_t = \pi_{t-1} - \gamma x_t$. This implies $E_{t-1}\pi_t = E_{t-1}\pi_t^T - \gamma E_{t-1}x_t = E_{t-1}\pi_t^T$ since $E_{t-1}x_t = 0$. Going back to the earlier equation for π_t , it can now be rewritten as the Sticky Information Phillips Curve

$$\pi_t = \frac{\alpha\delta}{1 - \delta}x_t + E_{t-1}\pi_t^T. \quad (SIPC)$$

And together with the monetary rule,

$$\pi_t = \pi_t^T - \gamma x_t, \quad (MR)$$

these two equations determine π_t and x_t . The slope of the *SIPC* depends on α and on δ , but in this case unlike the *NKPC*, δ refers to the proportion of price-setters with up-to-date information. As δ tends toward one, the Phillips curve becomes vertical: this is the standard case of rational expectations with flexible prices and full information. If $\delta < .5$, the slope is flatter than α , and vice versa. We need to highlight the critical difference with the *NKPC*. In the *NKPC*,

$$\pi_t = \frac{\alpha\delta}{1 - \delta}x_t + \theta E_t\pi_{t+1}.$$

In the *SIPC*, the $E_t\pi_{t+1}$ term is replaced by $E_{t-1}\pi_t$. In other words, if wage and price setters are rational in an *NKPC* world it is difficult to see how inflation inertia can come about, since $E_t\pi_{t+1}$ depends on expectations at t about *future* inflation. But in a *SIPC* world the most up to date information that $(1 - \delta)$ percent of firms may have in forming their views about π_t at $t - 1$, i.e. $E_{t-1}\pi_t$, may plausibly include past factors that influence π_{t-1} .

Let us assume that the limited information relates to the central bank's inflation target. This key case for monetary policy allows us to show both that disinflation is costly in the *SIPC* model when the central bank lowers its inflation target — in contrast to the *NKPC* modelling of this case — and that credibility matters. The model implies that if those with limited information believe

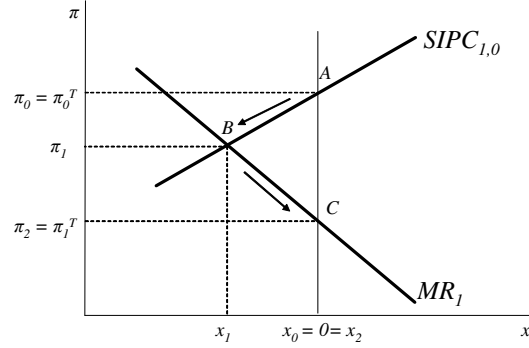


Figure 11: *SIPC*: Reduction of the inflation target; one period delay in information assimilation

(rightly or wrongly) that the central bank's target is π_0^T , and if the central bank reduces its target to π_1^T in period one, the *SIPC* at period one is

$$\pi_1 = \frac{\alpha\delta}{1-\delta}x_1 + \pi_0^T$$

and we call this $SIPC_{1,0}$. As we have seen, it holds as $SIPC_{t,0}$ for each period t in which those with limited information think the target is π_0^T .

In Fig.11, the initial equilibrium is at $(\pi_0^T, x = 0)$, with $\pi_0 = \pi_0^T$ and $x_0 = 0$ (at point A). In period one, the central bank reduces the inflation target to π_1^T , so that the MR_1 goes through $x = 0$ and $\pi = \pi_1^T$ and the *SIPC* becomes $SIPC_{1,0}$. Hence excess supply of x_1 is created and inflation falls to $\pi_1 > \pi_1^T$. In period two all firms have full information and the economy moves to the $(\pi_2 = \pi_1^T, x_2)$ equilibrium at point C .

Two consequences should be noted in this example: disinflation is costly following a change in monetary policy and once those with limited information have understood that the inflation target has fallen, they immediately adjust their behaviour. Thus by contrast with the backwards-looking Phillips curve, the credibility of the central bank's announcement plays a role in adjustment and by contrast with the *NKPC*, there is a cost of disinflation when the inflation target changes.

However, it is not really appropriate to compare the *SIPC* with a one period delay in information assimilation to the *NKPC* because the *NKPC* assumes that there is a distribution over time in the ability of firms to change their price. The appropriate comparison entails allowing information to diffuse more slowly in the *SIPC*. When we allow for more than a one-period lag in information assimilation this has the effect of slowing down the adjustment of the economy back to equilibrium following a shock, with the result that it is able better to predict the empirically observed phenomenon of inflation persistence.

To illustrate this we now assume that δ_1 per cent of firms acquire immediate knowledge of the target reduction in the inflation target, δ_2 per cent cumulatively after one period, δ_3 per cent after

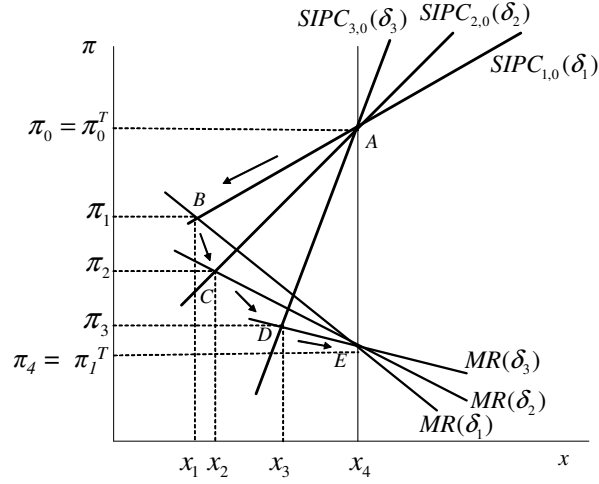


Figure 12: *SIPC*: Reduction of inflation target; information diffusion over several periods

two periods, and all firms after four periods. The reduction of output and inflation in period one (to π_1, x_1) takes place in an identical way to the previous one-lag model with $\delta = \delta_1$. In period two, all that happens is that the percentage of firms with full information rises to δ_2 and therefore of limited information falls to $(1 - \delta_2)$, which implies that $SIPC_{2,0}(\delta_2)$ is given by $\pi_2 = \frac{\alpha\delta_2}{1-\delta_2}x_2 + \pi_0^T$. This is a steeper curve than $SIPC_{1,0}(\delta_1)$, as can be seen in Fig.12. Likewise with $SIPC_{3,0}(\delta_3)$ in period three.

The monetary rule curve, MR_1 , also changes with δ . This is because the central bank, faced with a Phillips curve constraint $\pi = \frac{\alpha\delta}{1-\delta}x + \pi_0^T$ sets an optimal monetary rule under discretion of $(\pi - \pi_1^T) = -\frac{1-\delta}{\delta\alpha\beta}x$. Thus, as δ increases (and the $PC(\delta)$ gets steeper), $MR(\delta)$ gets flatter. It can be seen that inflation falls as the share of firms with full information, δ , rises; and after δ has risen sufficiently the output gap will start to rise towards zero. The economy moves from point A to B to C to D and back to the new equilibrium at E . Hence we get a closer approximation to inflation inertia.¹⁰

However, it could be argued that this assumption of slowly diffusing information is just as ad hoc as the assumption of an exogenously decaying cost shock in the *NKPC*. The *SIPC* model does not provide an explanation for slow information acquisition in terms of the incentives of agents.

¹⁰It can be shown that, if the actual path of inflation is known ex-post to all firms the inflation they set will cancel out previous relative price changes.

8 Conclusion

The graphical *IS-PC-MR* model is a replacement for the standard *IS-LM-AS* model. It conforms with the view that monetary policy is conducted by forward-looking central banks and provides non-specialists with the tools for analyzing a wide range of macroeconomic disturbances. By building on the lag structure consistent with empirical evidence, the model allows a Taylor Rule to be derived graphically.

The *IS-PC-MR* model also provides access to contemporary debates in the more specialized monetary macroeconomics literature. It is straightforward to demonstrate the origin of the time inconsistency problem using the graphical approach. The model is extended to show how replacing the traditional *IS* curve with an *IS* incorporating forward-looking behaviour dampens the effect of shocks on output and inflation.

As demonstrated by the lively debates in the literature and in central banks over recent years, the modelling of the inflation process remains controversial. The table provides a crude summary of the characteristics of the three models of the Phillips curve presented in this paper: the traditional backwards looking or inertial Phillips curve (*BLPC*), the *NKPC* and the *SIPC*. A score of 1 is awarded if the model satisfies a criterion; 2 for partial fulfilment and 3 for failure to fulfil a criterion.

Model is consistent with:	<i>BLPC</i>	<i>NKPC</i>	<i>SIPC</i>
The empirical evidence of inflation inertia	1	3	1
Costly disinflation after inflation shock	1	2	1
Costly disinflation after reduction in inflation target	1	3	1
Rational price-setters	3	1	1
Credibility effect of monetary policy	3	1	1

It is important that the modelling of price-setting is based on rational behaviour but the persuasiveness of the ways in which this has been done remains open to question. The Calvo assumption in the *NKPC* that price-setters are chosen randomly each period has no micro-economic rationale. Although there are other choice-based models incorporating price-stickiness, they neither deliver inflation-inertia nor have the elegance of the Calvo model. Although the *SIPC* delivers inflation inertia, the question remains open as to why it does not pay firms to be better informed, and in what respects firms operating in the context of central banks with monetary rules are inadequately informed. The field seems still to be wide open for further work on the micro-foundations of inflation inertia.

Appendix

Appendix 1. Deriving the Monetary Policy Rule in the *IS-PC-MR* model

The central bank aims to minimize:

$$L = \frac{1}{2} \left[(y - y_e)^2 + \beta (\pi - \pi^T)^2 \right]$$

subject to the Phillips curve: $\pi = \pi_{-1} + \alpha(y - y_e)$. Solving this minimization problem delivers the monetary rule: $y - y_e = -\alpha\beta(\pi - \pi^T)$, which implies that the slope of the monetary rule curve as shown in Fig.7 is $\gamma = \frac{1}{\alpha\beta}$, reflecting both the slope of the Phillips curve and the inflation aversion of the central bank. In its Taylor Rule form, the monetary rule is:

$$r_0 - r_S = \frac{1}{(\alpha + \gamma)a} \left[(\pi_0 - \pi^T) + \alpha(y_0 - y_e) \right].$$

Inflation aversion shows up in the coefficient $\frac{1}{(\alpha + \gamma)a}$: it therefore affects both elements of the Taylor Rule equally.

We can see that Taylor's weights of 0.5 and 0.5 arise when the *IS* curve, the Phillips curves and the *MR* curve all have a slope of one (or more precisely in the case of the *IS* and the *MR* of minus one). To consider the implications for the central bank's reaction to current inflation and output information when the key parameters differ from one, we take each in turn, keeping the other two equal to one. We begin with the monetary rule curve, the slope of which depends on both the slope of the Phillips curve and on the degree of inflation-aversion of the central bank since $\gamma = \frac{1}{\alpha\beta}$. Since β is the weight on inflation in the central bank's loss function and holding $\alpha = 1$, a value of $\beta > 1$ reflects more inflation aversion on the part of the central bank than in our base-line case. Hence the *MR*-curve is flatter. The implications for the central bank are unambiguous and intuitive: the central bank will raise the interest rate by more in the face of a given inflation or output shock.

Turning to the Phillips curve, as we have seen, its slope α affects the relative weight on inflation and output in the Taylor Rule. For $\alpha > 1$, the Phillips curves are steeper and the *MR* curve is flatter. There are two implications, which go in opposite directions. First, a more restrictive interest rate reaction is optimal to deal with any given increase in output because this will have a bigger effect on inflation than with $\alpha = 1$ (this is the result of the flatter *MR*-curve). But on the other hand, a given rise in the interest rate will have a bigger negative effect on inflation. These two effects imply that with $\alpha > 1$, the balance between the coefficients changes: the coefficient on $(\pi_0 - \pi^T)$ goes down — so the central bank reacts less to an inflation shock whereas the coefficient on $(y_0 - y_e)$ goes up — the central bank reacts more to an output shock as compared with the equal weights in the Taylor rule.

Finally, if $a > 1$ this means that the effect on demand of a change in the interest rate increases: the *IS*-curve is flatter. This has a predictable effect on the central bank policy rule: a rise in a above one, reduces the coefficients on both the inflation and output deviations. Since a given interest rate response has a bigger effect, the central bank should react less to any given shock.

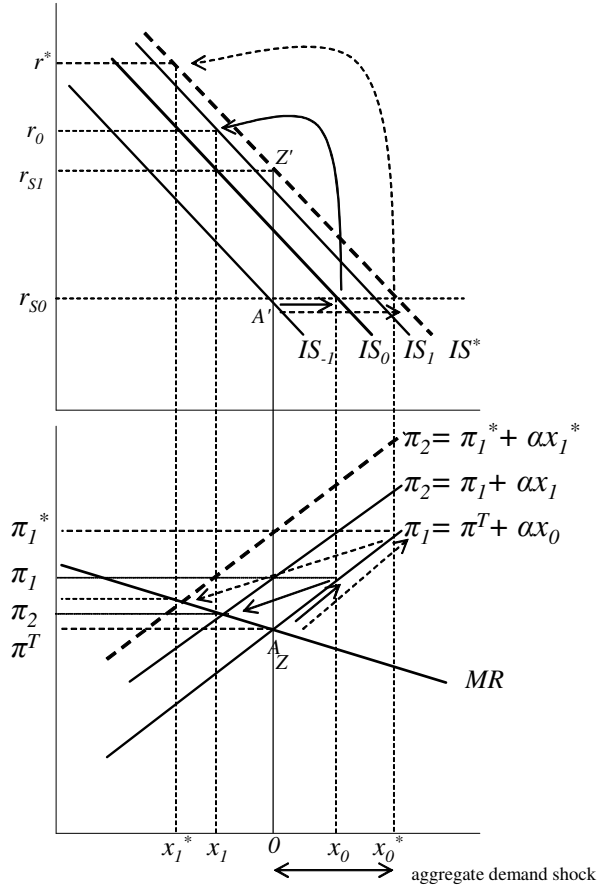


Figure 13: Aggregate demand shock with a forward-looking IS -curve

Appendix 2. An aggregate demand shock with a forward-looking IS curve

As a second exercise, we take the case of an aggregate demand or IS shock. This is illustrated in Fig.13. The initial IS curve is IS_{-1} , where the economy is in equilibrium at $x_{-1} = 0$, $E_{-1}x_0 = 0$, $\pi_{-1} = \pi^T$ and $r_{-1} = r_{S0}$. In period zero there is a permanent demand shock of ϵ that shifts the IS curve rightwards: if E_0x_1 remained zero the new IS curve would be IS^* . Thus when we compare forward and non-forward looking IS curves, IS^* is the new non-forward looking IS curve and we shall use dashed lines for adjustment with the traditional IS curve.

The forward-looking IS curve in period zero is $x_0 = E_0x_1 - a(r_{-1} - r_{S1})$. Since forward-looking households know that the central bank will take action to depress x_1 , IS_0 will be to the left of IS^* by exactly x_1 . For the moment, we assume that x_1 is known — we shall see how it is worked out below — and is correctly embodied in IS_0 (so graphically $x_1 = -(x_0^* - x_0)$). Since $r_{-1} = r_{S0}$, x_0 is given by the intersection of IS_0 and r_{-1} . Dropping down to the bottom diagram, this determines π_1 via the Phillips curve $\pi_1 = \pi^T + \alpha x_0$; this means that the Phillips curve $\pi_2 = \pi_1 + \alpha x_1$ goes through the Vertical Phillips curve at π_1 . The central bank can now choose the optimal pair (π_2, x_1) at the intersection of the π_2 Phillips curve and the MR . From here

it is easy to work out the central bank's choices of the pairs (π_3, x_2) , (π_4, x_3) and so on from the intersections of the MR with the relevant Phillips curves. The path of adjustment is shown by the arrows in the lower panel.

How does the central bank use its choice of the interest rate to produce this adjustment path? We have already worked out the values of x_1, x_2, x_3, \dots which the central bank engineers during the adjustment process. The IS diagram can now be used to find the values of r_0, r_1, r_2, \dots which the central bank needs to set for this pattern of excess supply. Starting with r_0 , the relevant IS curve is IS_0 :

$$x_1 = x_2 - a(r_0 - r_{S,1})$$

and r_0 is now the interest rate at the intersection of IS_0 and the vertical x_1 line. Similarly, since

$$x_2 = x_3 - a(r_1 - r_{S,1})$$

and r_1 is the interest rate at the intersection of IS_1 and the vertical x_2 line. Thus the interest rate is raised from its original level of $r_{-1} = r_{S,0}$ to r_0 and then is gradually adjusted back down to the new equilibrium at $r_{S,1}$. This is shown by the arrow in the IS diagram.

Fig.13 shows clearly that the forward-looking IS curve reduces the amplitude of both output and interest rate changes. Absent its forward-looking component, the shocked IS curve is the dashed line, IS^* , $x_0 = -a(r_{-1} - r_{S,1})$ so that with $r_{-1} = r_{S,0}$ initial excess demand is x_0^* . This generates the dashed π_2 Phillips curve ($\pi_2 = \pi_1^* + \alpha x_1^*$), and hence x_1^* , and in turn $r^* > r_0$. The return to equilibrium is down the dashed IS^* line.

Appendix 3. The New Keynesian Phillips Curve

Deriving the New Keynesian Phillips Curve

The standard derivation of the $NKPC$ is somewhat lengthy. By cutting a few corners, however, there is a much simpler derivation, which makes the intuition behind the equation clearer. Let p^* be the log of the price set by the δ percent of price setters at time t . Hence, since the other $1 - \delta$ firms will retain last period's price level, the current price level is given by

$$p_t = \delta p_t^* + (1 - \delta) p_{t-1}$$

Since $\pi_t \equiv p_t - p_{t-1}$ and the inflation rate of those who set their prices at t is $\pi_t^* \equiv p_t^* - p_{t-1}$, it is easy to see that $\pi_t = \delta \pi_t^*$.

What inflation rate π^* will price setters want in the current period if they have the chance to reset their prices? Given imperfect competition, they will want to raise the relative price of their differentiated products if there is excess demand, i.e. $x > 0$ and vice versa if $x < 0$. Hence, for $x > 0$, they will want an inflation rate above the aggregate: $\pi^* = \pi + \alpha x$. However, the price they set now, p^* , has to last until they next get a chance to reset their prices. There is a $(1 - \delta)$ chance that they will not be able to reset in $t + 1$, a $(1 - \delta)^2$ chance in $t + 2$, etc. In addition they care less about future periods because of the discount factor θ . Thus they attach a value of 1 to having the right inflation rate in the current period, $\theta(1 - \delta)$ in $t + 1$, $\theta^2(1 - \delta)^2$ in $t + 2$, and so on. So their chosen π^* has to be the correct rate for the current period plus the correct rate for $t + 1$ weighted by $\theta(1 - \delta)$, and so on. Hence:

$$\pi_t^* = [(\pi_t + \alpha x_t) + \theta(1 - \delta) E_t \pi_{t+1} + \alpha E_t x_{t+1}) + \theta^2(1 - \delta)^2 E_t \pi_{t+2} + \alpha E_t x_{t+2}) + \dots].$$

Since $\pi_t = \delta\pi_t^*$, we have

$$\pi_t = \delta [(\pi_t + \alpha x_t) + \theta(1 - \delta) E_t \pi_{t+1} + \alpha E_t x_{t+1}] + \dots$$

This enables us to make a simple transformation: leading both sides by one period and multiplying both sides by $\theta(1 - \delta)$ generates

$$\theta(1 - \delta) E_t \pi_{t+1} = \theta(1 - \delta) \delta [(E_t \pi_{t+1} + \alpha E_t x_{t+1}) + \theta(1 - \delta) (E_t \pi_{t+2} + \alpha E_t x_{t+2}) + \dots]$$

and subtracting this from the previous equation implies

$$\pi_t = \delta\pi_t + \alpha\delta x_t + \theta(1 - \delta) E_t \pi_{t+1}.$$

When rearranged, this produces the New Keynesian Phillips Curve:

$$\pi_t = \frac{\alpha\delta}{1 - \delta} x_t + \theta E_t \pi_{t+1}.$$

We turn now to the question of what is the rationally expected value of π_{t+1} . To find this, we need first to derive the monetary rule. Just as before, the central bank minimises a loss function and to simplify the notation, it is assumed that the central bank's inflation target is zero. The loss function can be written explicitly as

$$L = \frac{1}{2} (x_t^2 + \beta\pi_t^2)$$

and is minimized subject to the constraint imposed by the *NKPC*. Since the central bank takes expectations of future inflation by private agents as given, and assuming it can re-maximise each period, this implies that there is an *MR* curve as follows:

$$\pi_t = -\gamma x_t,$$

where $\gamma = \frac{1-\delta}{\delta\alpha\beta}$. For ease of exposition, we follow *NKPC* theorists and assume there are no time lags from the interest rate to output or from output to inflation.

The next step is to use rational expectations to evaluate the expected future inflation rate, $E_t \pi_{t+1}$. It is not difficult to show that expected future inflation is equal to the target, i.e. $E_t \pi_{t+1} = 0$. First, the *NKPC* and the *MR* must both hold in each period: $\pi_t = -\gamma x_t$ and $\pi_t = \frac{1}{\beta\gamma} x_t + \theta E_t \pi_{t+1}$. Eliminating x_t from the pair of equations, this implies

$$\pi_t = \left(\frac{\theta}{1 + \frac{1}{\beta\gamma^2}} \right) E_t \pi_{t+1}$$

where we define the term in brackets as τ ; it lies between zero and one. Since the *MR* and *NKPC* hold each period, $E_t \pi_{t+i} = \tau E_t \pi_{t+i+1}$ for all i . Hence if we rule out the possibility that $E_t \pi_{t+i} \rightarrow \infty$ with i , we have the result $E_t \pi_{t+i} = 0$ (or in the case where the inflation target is not zero, $E_t \pi_{t+i} = \pi^T$).

Deriving $E_t\pi_{t+1}$ in the *NKPC* with autocorrelated cost-push added

Clarida et al. (1999) attempt to build more realistic results into their model by introducing into the *NKPC* an exogenous “cost-push” factor, c , in effect an inflation shock, the effect of which is assumed to diminish over time. Because this development is both important and difficult to follow in their article it is explained here. As far as we can see, it lacks microfoundations.

The *NKPC* now becomes

$$\pi_t = \frac{\alpha\delta}{1-\delta}x_t + \theta E_t\pi_{t+1} + c_t$$

where $c_t = \rho c_{t-1} + \varepsilon_t$, with ρ a positive constant less than one and ε_t , a random variable with mean zero that is not autocorrelated. The assumed form of the cost-push factor, c_t , is crucial in generating inflation dynamics in the *NKPC* model: the cost-push this period is a fraction of cost-push last period (plus a random component).

The introduction of the cost-push factor makes evaluating $E_t\pi_{t+1}$ slightly more difficult. Here is how it is done. With rational expectations and dropping the expectation operator to simplify the notation, we have the Phillips curve:

$$\pi_{t+1} = \alpha \left(\frac{\delta}{1-\delta} \right) x_{t+1} + \theta\pi_{t+2} + \rho c_t$$

and the monetary rule:

$$\pi_{t+1} = -\gamma x_{t+1},$$

which together imply

$$\pi_{t+1} = \theta b\pi_{t+2} + \rho b c_t$$

where $b \equiv 1 / \left(1 + (\beta\gamma^2)^{-1} \right)$; and working out π_{t+2} the same way:

$$\pi_{t+2} = \theta b\pi_{t+3} + \rho^2 b c_t.$$

Note that the impact of c_t declines by ρ each period. Successively substituting, we get

$$\pi_{t+1} = b\rho c_t [1 + \theta b\rho + \theta^2 b^2 \rho^2 \dots] = \frac{b}{1 - \theta b\rho} \rho c_t.$$

For simplicity and restoring the expectation operator, we write this as:

$$E_t\pi_{t+1} = \lambda\rho c_t,$$

where $0 < \lambda < 1$. This means that the ‘new’ *NKPC* is given by

$$\pi_t = \frac{\alpha\delta}{1-\delta}x_t + \theta E_t\pi_{t+1} + c_t = \frac{\alpha\delta}{1-\delta}x_t + (1 + \theta\lambda\rho)c_t.$$

The *NKPC* is shown in Fig.14 and the term $(1 + \theta\lambda\rho)c_t$ is measured by the rate of inflation when output is at equilibrium.

The geometry of the model looks similar to that of the standard *IS-PC-MR* model. The key difference centres on what determines the position of the Phillips curve and on what shifts it. In

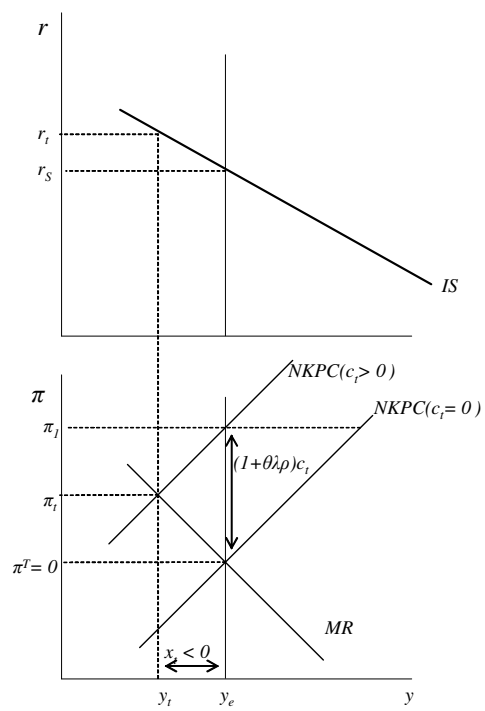


Figure 14: *NKPC* with an exogenous cost push term

the *IS-PC-MR* model the Phillips curve is shifted by inertial inflation, with the consequence that the Phillips curve shifts endogenously as inflation adjusts. With the *NKPC*, the exogenous cost push term, c_t , fixes the position of the Phillips curve and the assumption that this becomes weaker at the rate ρ leads the *NKPC* to shift. To see how this works, we need to assume that the cost push effect persists, i.e. $\rho > 0$ since if $\rho = 0$, the expected future inflation term disappears. It is simplest to assume that $c_t = c^*$ and that $\varepsilon_{t+i} = 0$ for all $i > 0$. In this case the *NKPC* at t is $\pi_t = \frac{\alpha\delta}{1-\delta}x_t + (1 + \theta\lambda\rho)c^*$, where $E_t\pi_{t+1} = \lambda\rho c^*$. So again determining π_t by the intersection of *MR* and *NKPC* we get

$$\pi_t = \frac{1 + \theta\lambda\rho}{[1 + (1/\beta\gamma^2)]}c^*$$

Bearing in mind that the influence of c^* diminishes exponentially by ρ each period, the *NKPC* in period $t + 1$ is $\pi_{t+1} = \frac{\alpha\delta}{1-\delta}x_{t+1} + (1 + \theta\lambda\rho)\rho c^*$ and hence

$$\pi_{t+1} = \frac{1 + \theta\lambda\rho}{[1 + (1/\beta\gamma^2)]}\rho c^* = \rho\pi_t.$$

So with $\rho > 0$ there is a gradual move of the *NKPC* down the *MR* curve and the economy moves back to equilibrium along the path from B to C and back to A as shown in Fig.15 with the central bank adjusting the interest rate appropriately as shown in the upper panel. But note that any one period inflation shock has no effect on next period's inflation: in so far as inflation persists in this ad hoc model it is driven purely by the autocorrelation of past shocks.

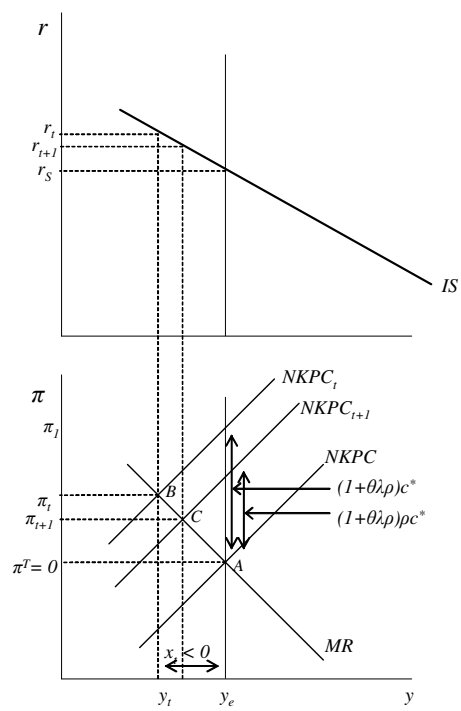


Figure 15: *NKPC* with an auto-correlated cost-push term

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