CONSUMPTION SMOOTHING, ASSETS AND FAMILY LABOR SUPPLY

Richard Blundell  University College London & IFS

ECB October 2013
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- Focus on **labor market shocks** as the primitive source of uncertainty.

The link between the various types of inequality is mediated by multiple ‘insurance’ mechanisms, including:

- Labor supply: family labor supply (wages → earnings → joint earnings)
- Taxes and welfare: (earnings → income)
- Assets: Saving and borrowing (income → consumption)
- Informal contracts, gifts, etc.
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  - Taxes and welfare: (earnings $\rightarrow$ income)
  - Assets: Saving and borrowing (income $\rightarrow$ consumption)
  - Informal contracts, gifts, etc.
- But how important are each of these mechanisms?
Focus on how families deal with labor market shocks.
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Investigate how assets and labor market shocks combine to impact on consumption.
OVERVIEW

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- Point to the importance of constructing panel/administrative linked data that allow all three measures.
**Overview**

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- With Luigi and Itay we make use of the new PSID data on consumption, earnings and assets.

Finding: Once assets, family labor supply and taxes (and welfare) are properly accounted for, we can explain the link between these series and there is less evidence for additional insurance.

Some consumption inequality descriptives...
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- Some consumption inequality descriptives....
Consumption Inequality in the UK
By age and birth cohort

Variance of log nondurable consumption

Age
20 30 40 50 60 70

20 30 40 50 60 70

Var ianc e
.1
.15
.2
.25
.3
.35
.4
.45
.5

Var ianc e
.1
.15
.2
.25
.3
.35
.4
.45
.5

1940 1950 1960 1970
INCOME INEQUALITY IN THE UK
By age and birth cohort

![Graph showing variance of log net income by age and birth cohort]
CONSUMPTION INEQUALITY IN THE US

By age and birth cohort

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Diagram showing consumption inequality by age and birth cohort.
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A little background on the empirical strategy for income and consumption dynamics behind these results...
**INCOME DYNAMICS**

Consider consumer $i$ (of age $a$) in time period $t$, has log income $y_{it} (\equiv \ln Y_{i,a,t})$ written

$$y_{it} = Z_{it}' \varphi + f_{0i} + p_t f_{1i} + y_{it}^p + y_{it}^T$$
**Income Dynamics**

Consider consumer \( i \) (of age \( a \)) in time period \( t \), has log income \( y_{it} (\equiv \ln Y_{i,a,t}) \) written

\[
y_{it} = Z_{it}' \varphi + f_{0i} + pt f_{1i} + y_{it}^P + y_{it}^T
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where \( y_{it}^P \) is a persistent process of income shocks, say

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which adds to the individual-specific trend \( ptf_i \) and where \( y_{it}^T \) is a transitory shock represented by some low order MA process, say

\[
y_{it}^T = \varepsilon_{it} + \theta_1 \varepsilon_{i,t-1}
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- Find $ptf_{1i}$ to be less important and $\rho$ closer to unity, especially in the 30-59 age selection.
- Detailed work on Norwegian population register panel data....
**Life-cycle Income Dynamics**

Variance of permanent shocks over the life-cycle

![Graph showing variance of permanent shocks over the life-cycle](image)

Source: Blundell, Graber and Mogstad (2013), Norwegian Population Panel.
LIFE-CYCLE INCOME DYNAMICS
Norwegian population panel (low skilled)

Variance of Permanent Shocks : Average Cohort Effects LowSkilled

Source: Blundell, Graber and Mogstad (2013).
**Consumption Growth and Income "Shocks"**

To account for the impact of income shocks on consumption we introduce *transmission parameters*: $\kappa_{cvt}$ and $\kappa_{c\varepsilon t}$, writing consumption growth as:

$$\Delta \ln C_{it} \cong \Gamma_{it} + \Delta Z'_{it} \phi^c + \kappa_{cvt} v_{it} + \kappa_{c\varepsilon t} \varepsilon_{it} + \xi_{it}$$
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which provides the link between the consumption and income distributions.

For example, in Blundell, Low and Preston (QE, 2013) show, for any birth-cohort,

$$\Delta \ln C_{it} \equiv \Gamma_{it} + \Delta Z_{it}' \varphi^c + (1 - \pi_{it}) v_{it} + (1 - \pi_{it}) \gamma_{Lt} \epsilon_{it} + \zeta_{it}$$

where

$$\pi_{it} \approx \frac{\text{Assets}_{it}}{\text{Assets}_{it} + \text{Human Wealth}_{it}}$$

and $\gamma_{Lt}$ is the annuity value of a transitory shock for an individual aged $t$ retiring at age $L$. 
In this paper we look closer at four key mechanisms:

1. Self-insurance (i.e., savings) through a direct measure of $\pi_{it}$
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1. **Self-insurance** (i.e., savings) through a *direct* measure of $\pi_{it}$
2. Joint labor supply of each earner
3. Non-linear taxes and welfare

And test for "superior information".

Distinctive features of this paper:
- Allow for non-separability,
- Heterogeneous assets,
- Correlated shocks to individual wages.

- Use new data from the PSID 1999-2009
- More comprehensive consumption measure.
- Asset data collected in every wave.
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## NIPA-PSID Comparison

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>PSID Total</td>
<td>3,276</td>
<td>3,769</td>
<td>4,285</td>
<td>5,058</td>
<td>5,926</td>
<td>5,736</td>
</tr>
<tr>
<td>NIPA Total</td>
<td>5,139</td>
<td>5,915</td>
<td>6,447</td>
<td>7,224</td>
<td>8,190</td>
<td>9,021</td>
</tr>
<tr>
<td>ratio</td>
<td>0.64</td>
<td>0.64</td>
<td>0.66</td>
<td>0.70</td>
<td>0.72</td>
<td>0.64</td>
</tr>
<tr>
<td>PSID Nondurables</td>
<td>746</td>
<td>855</td>
<td>887</td>
<td>1,015</td>
<td>1,188</td>
<td>1,146</td>
</tr>
<tr>
<td>NIPA Nondurables</td>
<td>1,330</td>
<td>1,543</td>
<td>1,618</td>
<td>1,831</td>
<td>2,089</td>
<td>2,296</td>
</tr>
<tr>
<td>ratio</td>
<td>0.56</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.57</td>
<td>0.50</td>
</tr>
<tr>
<td>PSID Services</td>
<td>2,530</td>
<td>2,914</td>
<td>3,398</td>
<td>4,043</td>
<td>4,738</td>
<td>4,590</td>
</tr>
<tr>
<td>NIPA Services</td>
<td>3,809</td>
<td>4,371</td>
<td>4,829</td>
<td>5,393</td>
<td>6,101</td>
<td>6,725</td>
</tr>
<tr>
<td>ratio</td>
<td>0.66</td>
<td>0.67</td>
<td>0.70</td>
<td>0.75</td>
<td>0.78</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Note: PSID weights are applied for the non-sampled PSID data (47,206 observations for these years). Total consumption is defined as Nondurables + Services. PSID consumption categories include food, gasoline, utilities, health, rent (or rent equivalent), transportation, child care, education and other insurance. NIPA numbers are from NIPA table 2.3.5. All numbers are nonminal.
### Descriptive Statistics for Consumption

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>27,290</td>
<td>31,973</td>
<td>35,277</td>
<td>41,555</td>
<td>45,863</td>
<td>44,006</td>
</tr>
<tr>
<td>Nondurable Consumption</td>
<td>6,859</td>
<td>7,827</td>
<td>7,827</td>
<td>8,873</td>
<td>9,889</td>
<td>9,246</td>
</tr>
<tr>
<td>Food (at home)</td>
<td>5,471</td>
<td>5,785</td>
<td>5,911</td>
<td>6,272</td>
<td>6,588</td>
<td>6,635</td>
</tr>
<tr>
<td>Gasoline</td>
<td>1,387</td>
<td>2,041</td>
<td>1,916</td>
<td>2,601</td>
<td>3,301</td>
<td>2,611</td>
</tr>
<tr>
<td>Services</td>
<td>21,319</td>
<td>25,150</td>
<td>28,419</td>
<td>33,755</td>
<td>36,949</td>
<td>35,575</td>
</tr>
<tr>
<td>Food (out)</td>
<td>2,029</td>
<td>2,279</td>
<td>2,382</td>
<td>2,582</td>
<td>2,693</td>
<td>2,492</td>
</tr>
<tr>
<td>Health Insurance</td>
<td>1,056</td>
<td>1,268</td>
<td>1,461</td>
<td>1,750</td>
<td>1,916</td>
<td>2,188</td>
</tr>
<tr>
<td>Health Services</td>
<td>902</td>
<td>1,134</td>
<td>1,334</td>
<td>1,447</td>
<td>1,615</td>
<td>1,844</td>
</tr>
<tr>
<td>Utilities</td>
<td>2,282</td>
<td>2,651</td>
<td>2,702</td>
<td>4,655</td>
<td>5,038</td>
<td>5,600</td>
</tr>
<tr>
<td>Transportation</td>
<td>3,122</td>
<td>3,758</td>
<td>4,474</td>
<td>3,797</td>
<td>3,970</td>
<td>3,759</td>
</tr>
<tr>
<td>Education</td>
<td>1,946</td>
<td>2,283</td>
<td>2,390</td>
<td>2,557</td>
<td>2,728</td>
<td>2,584</td>
</tr>
<tr>
<td>Child Care</td>
<td>601</td>
<td>653</td>
<td>660</td>
<td>689</td>
<td>648</td>
<td>783</td>
</tr>
<tr>
<td>Home Insurance</td>
<td>430</td>
<td>480</td>
<td>552</td>
<td>629</td>
<td>717</td>
<td>729</td>
</tr>
<tr>
<td>Rent (or rent equivalent)</td>
<td>8,950</td>
<td>10,645</td>
<td>12,464</td>
<td>15,650</td>
<td>17,623</td>
<td>15,595</td>
</tr>
<tr>
<td>Observations</td>
<td>1,872</td>
<td>1,951</td>
<td>1,984</td>
<td>2,011</td>
<td>2,115</td>
<td>2,221</td>
</tr>
</tbody>
</table>

Notes: PSID data from 1999-2009 PSID waves. PSID means are given for the main sample of estimation: married couples with working males aged 30 to 65. SEO sample excluded. PSID rent is imputed as 6% of reported house value for homeowners. Missing values in consumption and assets sub-categories were treated as zeros.
# Descriptive Statistics for Assets and Earnings

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>Total assets</td>
<td>332,625</td>
<td>352,247</td>
<td>382,600</td>
<td>476,626</td>
<td>555,951</td>
<td>506,823</td>
</tr>
<tr>
<td>Housing and RE assets</td>
<td>159,856</td>
<td>187,969</td>
<td>227,224</td>
<td>283,913</td>
<td>327,719</td>
<td>292,910</td>
</tr>
<tr>
<td>Financial assets</td>
<td>173,026</td>
<td>164,567</td>
<td>155,605</td>
<td>192,995</td>
<td>228,805</td>
<td>214,441</td>
</tr>
<tr>
<td>Total debt</td>
<td>72,718</td>
<td>82,806</td>
<td>98,580</td>
<td>115,873</td>
<td>131,316</td>
<td>137,348</td>
</tr>
<tr>
<td>Mortgage</td>
<td>65,876</td>
<td>74,288</td>
<td>89,583</td>
<td>106,423</td>
<td>120,333</td>
<td>123,324</td>
</tr>
<tr>
<td>Other debt</td>
<td>7,021</td>
<td>8,687</td>
<td>9,217</td>
<td>9,744</td>
<td>11,584</td>
<td>14,561</td>
</tr>
<tr>
<td>First earner (head)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earnings</td>
<td>54,220</td>
<td>61,251</td>
<td>63,674</td>
<td>68,500</td>
<td>72,794</td>
<td>75,588</td>
</tr>
<tr>
<td>Hours worked</td>
<td>2,357</td>
<td>2,317</td>
<td>2,309</td>
<td>2,309</td>
<td>2,284</td>
<td>2,140</td>
</tr>
<tr>
<td>Second earner (wife)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Participation rate</td>
<td>0.81</td>
<td>0.8</td>
<td>0.81</td>
<td>0.81</td>
<td>0.81</td>
<td>0.8</td>
</tr>
<tr>
<td>Earnings (conditional on participation)</td>
<td>26,035</td>
<td>28,611</td>
<td>31,693</td>
<td>33,987</td>
<td>36,185</td>
<td>39,973</td>
</tr>
<tr>
<td>Hours worked (conditional on participation)</td>
<td>1,666</td>
<td>1,691</td>
<td>1,697</td>
<td>1,707</td>
<td>1,659</td>
<td>1,648</td>
</tr>
<tr>
<td>Observarions</td>
<td>1,872</td>
<td>1,951</td>
<td>1,984</td>
<td>2,011</td>
<td>2,115</td>
<td>2,221</td>
</tr>
</tbody>
</table>

Notes: PSID data from 1999-2009 PSID waves. PSID means are given for the main sample of estimation: married couples with working males aged 30 to 65. SEO sample excluded. PSID rent is imputed as 6% of reported house value for homeowners. Missing values in consumption and assets sub-categories were treated as zeros.
Household Decisions in a Unitary Framework

Household chooses \( \{ C_{i,t+j}, H_{i,1,t+j}, H_{i,2,t+j} \}_{j=0}^{T-t} \) to maximize

\[
\mathbb{E}_t \sum_{\tau=0}^{T-t} (1 + \delta)^{-\tau} v (C_{i,t+\tau}, H_{i,1,t+\tau}, H_{i,2,t+\tau}; Z_{i,t+\tau})
\]

subject to

\[
C_{i,t} + \frac{A_{i,t+1}}{1+r} = A_{i,t} + H_{i,1,t}W_{i,1,t} + H_{i,1,t}W_{i,2,t}
\]
Household chooses \( \{ C_{i,t+j}, H_{i,1,t+j}, H_{i,2,t+j} \}_{j=0}^{T-t} \) to maximize

\[
\mathbb{E}_t \sum_{\tau=0}^{T-t} (1 + \delta)^{-\tau} \nu \left( C_{i,t+\tau}, H_{i,1,t+\tau}, H_{i,2,t+\tau}; Z_{i,t+\tau} \right)
\]

subject to

\[
C_{i,t} + \frac{A_{i,t+1}}{1 + r} = A_{i,t} + H_{i,1,t} W_{i,1,t} + H_{i,1,t} W_{i,2,t}
\]

Our approach

- Extend previous work and express the distributional dynamics of consumption and earnings growth as functions of Frisch elasticities, ‘insurance parameters’ and wage shocks
Wage Process

For earner $j = \{1, 2\}$ in household $i$, period $t$, wage growth is:

$$\Delta \log W_{i,j,t} = \Delta X'_{i,j,t} \beta_j + \Delta u_{i,j,t} + v_{i,j,t}$$
**Wage Process**

For earner $j = \{1, 2\}$ in household $i$, period $t$, wage growth is:

$$\Delta \log W_{i,j,t} = \Delta X'_{i,j,t} \beta_j + \Delta u_{i,j,t} + \nu_{i,j,t}$$

\[
\begin{pmatrix}
  u_{i,1,t} \\
  u_{i,2,t} \\
  v_{i,1,t} \\
  v_{i,2,t}
\end{pmatrix}
\sim i.i.d.
\begin{pmatrix}
  0 \\
  0 \\
  0 \\
  0
\end{pmatrix},
\begin{pmatrix}
  \sigma_{u,1}^2 & \sigma_{u,1,u_2} & 0 & 0 \\
  \sigma_{u,1,u_2} & \sigma_{u,2}^2 & 0 & 0 \\
  0 & 0 & \sigma_{v,1}^2 & \sigma_{v,1,v_2} \\
  0 & 0 & \sigma_{v,1,v_2} & \sigma_{v,2}^2
\end{pmatrix}
\]
**Wage Process**

For earner $j = \{1, 2\}$ in household $i$, period $t$, wage growth is:

$$\Delta \log W_{i,j,t} = \Delta X'_{i,j,t} \beta_j + \Delta u_{i,j,t} + v_{i,j,t}$$

\[
\begin{pmatrix}
u_{i,1,t} \\
u_{i,2,t} \\
v_{i,1,t} \\
v_{i,2,t}
\end{pmatrix} \sim i.i.d. \begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix}, \begin{pmatrix}
\sigma^2_{u,1} & \sigma_{u,1,u_2} & 0 & 0 \\
\sigma_{u,1,u_2} & \sigma^2_{u,2} & 0 & 0 \\
0 & 0 & \sigma^2_{v,1} & \sigma_{v,1,v_2} \\
0 & 0 & \sigma_{v,1,v_2} & \sigma^2_{v,2}
\end{pmatrix}
\]

- Allow the variances to differ by gender and across the life-cycle.
## Wage Parameters Estimates

### Baseline

<table>
<thead>
<tr>
<th>Sample</th>
<th>Trans.</th>
<th>Perm.</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Males</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\sigma^2_{u_1})</td>
<td>(\sigma^2_{v_1})</td>
<td>0.033 (0.007)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Females</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\sigma^2_{u_2})</td>
<td>(\sigma^2_{v_2})</td>
<td>0.012 (0.006)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Correlation of shocks</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\rho_{u_1,u_2})</td>
<td>(\rho_{v_1,v_2})</td>
<td>0.244 (0.22)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The ‘Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\sim
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]
CONSUMPTION AND EARNINGS GROWTH

The ‘Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_j,u_j} = \left(1 + \eta_{h_j,w_j}\right) \rightarrow \text{[Frisch]}
\]
CONSUMPTION AND EARNINGS GROWTH

The ‘Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\sim
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_j,u_j} = \left(1 + \eta_{h_{j},v_j}\right) \rightarrow \text{[Frisch]} \quad \kappa_{y_j,v_j} \rightarrow \text{[Marshall]}
\]
CONSUMPTION AND EARNINGS GROWTH

The 'Simple' Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_{1,u_1}} & 0 & \kappa_{y_{1,v_1}} & \kappa_{y_{1,v_2}} \\
0 & \kappa_{y_{2,u_2}} & \kappa_{y_{2,v_1}} & \kappa_{y_{2,v_2}}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_{j,u_j}} = \left(1 + \eta_{h_{j,w_j}}\right) \quad \rightarrow \text{[Frisch]} \quad \kappa_{y_{j,v_j}} \quad \rightarrow \text{[Marshall]}
\]

\[
\kappa_{c,v_j} = (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_{j,w_j}}\right)}{\eta_{c,p} + \left(1 - \pi_{i,t}\right) \eta_{h,w}}
\]
CONSUMPTION AND EARNINGS GROWTH

The ‘Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_j,u_j} = \left(1 + \eta_{h_j,w_j}\right) \rightarrow \text{[Frisch]} \quad \kappa_{y_j,v_j} \rightarrow \text{[Marshall]}
\]

\[
\kappa_{c,v_j} = \left(1 - \pi_{i,t}\right) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_j,w_j}\right)}{\eta_{c,p} + (1 - \pi_{i,t}) \eta_{h,w}}
\]

\[
\pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}}
\]
CONSUMPTION AND EARNINGS GROWTH

The 'Simple' Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix} \sim
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_{1,u_1}} & 0 & \kappa_{y_{1,v_1}} & \kappa_{y_{1,v_2}} \\
0 & \kappa_{y_{2,u_2}} & \kappa_{y_{2,v_1}} & \kappa_{y_{2,v_2}}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_{j,u_j}} = \left(1 + \eta_{h_j,w_j}\right) \rightarrow \text{[Frisch]} \quad \kappa_{y_{j,v_j}} \rightarrow \text{[Marshall]}
\]

\[
\kappa_{c,v_j} = \left(1 - \pi_{i,t}\right) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_j,w_j}\right)}{\eta_{c,p} + (1 - \pi_{i,t}) \eta_{h,w}}
\]

\[
s_{i,j,t} \approx \frac{\text{Human Wealth}_{i,j,t}}{\text{Human Wealth}_{i,t}}
\]
CONSUMPTION AND EARNINGS GROWTH

The ‘Simple’ Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_{1,u_1}} & 0 & \kappa_{y_{1,v_1}} & \kappa_{y_{1,v_2}} \\
0 & \kappa_{y_{2,u_2}} & \kappa_{y_{2,v_1}} & \kappa_{y_{2,v_2}}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

where the key transmission parameters

\[
\kappa_{y_{j,u_j}} = \left(1 + \eta_{h_{j,w_j}}\right) \rightarrow \text{[Frisch]} \quad \kappa_{y_{j,v_j}} \rightarrow \text{[Marshall]}
\]

\[
\kappa_{c,v_j} = \left(1 - \pi_{i,t}\right) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h_{j,w_j}}\right)}{\eta_{c,p} + (1 - \pi_{i,t}) \bar{\eta}_{h,w}}
\]

\[
\bar{\eta}_{h,w} = s_{i,j,t} \eta_{h_{j,w_j}} + s_{i,-j,t} \eta_{h_{-j,w_{-j}}}
\]
CONSUMPTION AND EARNINGS GROWTH

The 'Simple' Separable Case

\[
\begin{pmatrix}
\Delta c_t \\
\Delta y_{1,t} \\
\Delta y_{2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
0 & 0 & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & 0 & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
0 & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{1,t} \\
\Delta u_{2,t} \\
v_{1,t} \\
v_{2,t}
\end{pmatrix}
\]

- Introduce now $\beta$, representing insurance over and above savings, taxes and labour supply $\rightarrow$ networks, etc.
- Key transmission parameter becomes:

\[
\kappa_{c,v_j} = (1 - \beta) (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left(1 + \eta_{h,j,w_j}\right)}{\eta_{c,p} + (1 - \beta) (1 - \pi_{i,t}) \eta_{h,w}}
\]
Identification with asset data

- Note that $\beta$ is not identified separately from $\pi$
- Back out $\pi$ from the data and estimate $\beta$

\[
\pi_{i,t} \approx \frac{\text{Observed in PSID}}{\text{Assets}_{i,t}} = \frac{\text{Human Wealth}_{i,t} + \text{Assets}_{i,t}}{\text{Projected lifetime earnings}}
\]

- Human wealth is projected using observables that evolve deterministically (e.g. age).
When preferences are non-separable, we have:

$$\begin{pmatrix} \Delta c_t \\ \Delta y_{1,t} \\ \Delta y_{2,t} \end{pmatrix} \sim \begin{pmatrix} \kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2} \\ \kappa_{y_1,u_1} & \kappa_{y_1,u_2} & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\ \kappa_{y_2,u_1} & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2} \end{pmatrix} \begin{pmatrix} \Delta u_{1,t} \\ \Delta u_{2,t} \\ v_{1,t} \\ v_{2,t} \end{pmatrix}$$

- $\kappa_{c,u_j} \rightarrow$ non-separability between consumption and leisure $j$
- $\kappa_{y_j,u_k} \rightarrow$ non-separability between spouses’ leisures
Distribution of $s$ by Age

$$s_{i,t} \approx \frac{\text{Human Wealth}_{\text{male},i,t}}{\text{Human Wealth}_{i,t}}$$

![Graph showing the distribution of $s$ by age](image)
Distribution of $s$ by Age

$$s_{i,t} \approx \frac{\text{Human Wealth}_{\text{male},i,t}}{\text{Human Wealth}_{i,t}}$$

The Distribution of $S$ by Age of Household Head
Distribution of $\pi$ by Age

$$\pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}}$$
**Distribution of $\pi$ by Age**

$$\pi_{i,t} \approx \frac{\text{Assets}_{i,t}}{\text{Assets}_{i,t} + \text{Human Wealth}_{i,t}} :$$

*The Distribution of Pi and Assets by Age of Household Head*

- **Pi (5th to 95th Percentiles) - Left axis**
- **Total Assets (Median, Thousands of Dollars) - Right axis**
**RESULTS: WITH AND WITHOUT SEPARABILITY**

<table>
<thead>
<tr>
<th></th>
<th>(1) Additive separ.</th>
<th>(2) Non-separab.</th>
<th>(3) Non-separab.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E (\pi)$</td>
<td>0.181 (0.008)</td>
<td>0.181 (0.008)</td>
<td>0.181 (0.008)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.741 (0.085)</td>
<td>-0.120 (0.098)</td>
<td>0</td>
</tr>
<tr>
<td>$\eta_{c,p}$</td>
<td>0.201 (0.077)</td>
<td>0.437 (0.124)</td>
<td>0.448 (0.126)</td>
</tr>
<tr>
<td>$\eta_{h_1,w_1}$</td>
<td>0.431 (0.097)</td>
<td>0.514 (0.150)</td>
<td>0.497 (0.150)</td>
</tr>
<tr>
<td>$\eta_{h_2,w_2}$</td>
<td>0.831 (0.133)</td>
<td>1.032 (0.265)</td>
<td>1.041 (0.275)</td>
</tr>
<tr>
<td>$\eta_{c,w_1}$</td>
<td>-- (0.051)</td>
<td>-0.141 (0.053)</td>
<td>-0.141 (0.053)</td>
</tr>
<tr>
<td>$\eta_{h_1,p}$</td>
<td>-- (0.030)</td>
<td>0.082 (0.031)</td>
<td>0.082 (0.031)</td>
</tr>
<tr>
<td>$\eta_{c,w_2}$</td>
<td>-- (0.139)</td>
<td>-0.138 (0.121)</td>
<td>-0.158 (0.121)</td>
</tr>
<tr>
<td>$\eta_{h_2,p}$</td>
<td>-- (0.166)</td>
<td>0.162 (0.145)</td>
<td>0.185 (0.145)</td>
</tr>
<tr>
<td>$\eta_{h_1,w_2}$</td>
<td>-- (0.052)</td>
<td>0.128 (0.064)</td>
<td>0.120 (0.064)</td>
</tr>
<tr>
<td>$\eta_{h_2,w_1}$</td>
<td>-- (0.103)</td>
<td>0.258 (0.119)</td>
<td>0.242 (0.119)</td>
</tr>
</tbody>
</table>
MARSHALLIAN ELASTICITIES: BY AGE

Marshallian Elasticities

Age of household head

Wife's Marshallian Elasticity
Head's Marshallian Elasticity

ECB October 2013
Marshallian Elasticities: By Age

Wife's Marshallian Elasticity ($\kappa_{12}$)

Age of household head

30-34 35-39 40-44 45-49 50-54 55-59 60-65

95th perc., 90th perc., 75th perc., 50th perc., 25th perc., 10th perc., 5th perc.
Marshallian Elasticities: By Age

Head's Marshallian Elasticity ($\kappa_7$)

Age of household head

30-34  35-39  40-44  45-49  50-54  55-59  60-65

95th perc.  90th perc.  75th perc.  50th perc.  25th perc.  10th perc.  5th perc.
INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)

The average response of total earnings \( (y = y_1 + y_2) \) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = s \cdot \frac{\partial \Delta y_1}{\partial v_1} + (1 - s) \cdot \frac{\partial \Delta y_2}{\partial v_1} = 0.44
\]

\( \tilde{s} = 0.69 \)

\( \tilde{\kappa}_{y_1,v_1} = 0.98 \)

\( 1 - \tilde{s} = 0.31 \)

\( \tilde{\kappa}_{y_2,v_1} = -0.81 \)
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)**

The average response of total earnings \( y = y_1 + y_2 \) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = s \cdot \frac{\partial \Delta y_1}{\partial v_1} + (1 - s) \cdot \frac{\partial \Delta y_2}{\partial v_1}
\]

\[
\begin{align*}
\hat{s} &= 0.69 \\
\hat{k}_{y_1,v_1} &= 0.98 \\
\hat{k}_{y_2,v_1} &= -0.81
\end{align*}
\]

Response of consumption to a 10% permanent decrease in the male’s wage rate \( v_1 = -0.1 \):

- one earner, fixed labor supply and no insurance: -10%
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)**

The average response of total earnings \((y = y_1 + y_2)\) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = s * \frac{\partial \Delta y_1}{\partial v_1} + (1 - s) * \frac{\partial \Delta y_2}{\partial v_1} = 0.44
\]

Response of consumption to a 10% permanent decrease in the male’s wage rate \((v_1 = -0.1)\):

- one earner, fixed labor supply and no insurance  
  -10%
- two earners, fixed labor supply and no insurance  
  -6.9%
The average response of total earnings ($y = y_1 + y_2$) to a permanent shock to the male’s wages:

$$\frac{\partial \Delta y}{\partial v_1} = s \cdot \frac{\partial \Delta y_1}{\partial v_1} + (1-s) \cdot \frac{\partial \Delta y_2}{\partial v_1} = 0.44$$

Response of consumption to a 10% permanent decrease in the male’s wage rate ($v_1 = -0.1$):

- one earner, fixed labor supply and no insurance -10%
- two earners, fixed labor supply and no insurance -6.9%
- with husband labor supply adjustment -6.8%
- with family labor supply adjustment and other insurance -3.8%
**INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES)**

The average response of total earnings \( y = y_1 + y_2 \) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = s \left( \frac{\partial \Delta y_1}{\partial v_1} \right) + (1 - s) \left( \frac{\partial \Delta y_2}{\partial v_1} \right) = 0.44
\]

\( \hat{s} = 0.69 \), \( \hat{\kappa}_{y_1,v_1} = 0.98 \), \( 1 - \hat{s} = 0.31 \), \( \hat{\kappa}_{y_2,v_1} = -0.81 \)

Response of **consumption** to a 10% permanent decrease in the male’s wage rate \( v_1 = -0.1 \):

- one earner, fixed labor supply and no insurance: -10%
- two earners, fixed labor supply and no insurance: -6.9%
- with husband labor supply adjustment: -6.8%
- with family labor supply adjustment: -4.4%
INTERPRETATION: INSURANCE VIA LABOR SUPPLY
(SHOCK TO MALE WAGES)

The average response of total earnings \((y = y_1 + y_2)\) to a permanent shock to the male’s wages:

\[
\frac{\partial \Delta y}{\partial v_1} = s \cdot \frac{\partial \Delta y_1}{\partial v_1} + (1 - s) \cdot \frac{\partial \Delta y_2}{\partial v_1} = 0.44
\]

Response of consumption to a 10% permanent decrease in the male’s wage rate \((v_1 = -0.1)\):

- one earner, fixed labor supply and no insurance: -10%
- two earners, fixed labor supply and no insurance: -6.9%
- with husband labor supply adjustment: -6.8%
- with family labor supply adjustment: -4.4%
- with family labor supply adjustment and other insurance: -3.8%
INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES): BY AGE

Response of Consumption to a 10% Permanent Decrease in the Male’s Wage Rate

Age of household head

30-34 35-39 40-44 45-49 50-54 55-59 60-65

fixed labor supply and no insurance
with family labor supply adjustment
with family labor supply adjustment and other insurance
INSURANCE VIA LABOR SUPPLY (SHOCK TO MALE WAGES): BY AGE

Consumption Response to a -10% Permanent Shock to Head's Wages ($\kappa_3$)

Age of household head

30-34 35-39 40-44 45-49 50-54 55-59 60-65

5th perc. 10th perc. 25th perc. 50th perc. 75th perc. 90th perc. 95th perc.
The average response of total earnings to a permanent shock to the female’s wages:

\[
\frac{\partial \Delta y}{\partial v_2} = s \cdot \frac{\partial \Delta y_1}{\partial v_2} + (1 - s) \cdot \frac{\partial \Delta y_2}{\partial v_2} = 0.25
\]

Response of consumption to a 10% permanent decrease in the female’s wage rate \((v_2 = -0.1)\):

- two earners, fixed labor supply and no insurance -3.1%
INTERPRETATION: INSURANCE VIA LABOR SUPPLY (SHOCK TO FEMALE WAGES)

The average response of total earnings to a permanent shock to the female’s wages:

\[
\frac{\partial \Delta y}{\partial v_2} = s \cdot \frac{\partial \Delta y_1}{\partial v_2} + (1 - s) \cdot \frac{\partial \Delta y_2}{\partial v_2} = 0.25
\]

\[\kappa_{y_1,v_2} = -0.23 \quad \kappa_{y_2,v_2} = 1.32\]

Response of consumption to a 10% permanent decrease in the female’s wage rate \(v_2 = -0.1\):

- two earners, fixed labor supply and no insurance: -3.1%
- with family labor supply adjustment: -2.5%
**Interpretation: Insurance Via Labor Supply (Shock to Female Wages)**

The average response of total earnings to a permanent shock to the female’s wages:

\[
\frac{\partial \Delta y}{\partial v_2} = s \cdot \frac{\partial \Delta y_1}{\partial v_2} + (1 - s) \cdot \frac{\partial \Delta y_2}{\partial v_2} = 0.25
\]

\[\kappa_{y_1, v_2} = -0.23\]
\[\kappa_{y_2, v_2} = 1.32\]

Response of consumption to a 10% permanent decrease in the female’s wage rate \((v_2 = -0.1)\):

- two earners, fixed labor supply and no insurance \(-3.1\%\)
- with family labor supply adjustment \(-2.5\%\)
- with family labor supply adjustment and other insurance \(-2.1\%\)
Focus on understanding the transmission of inequality over the working life.
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Found that family labor supply is a key mechanism for smoothing consumption.
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- especially for those with limited access to assets,
- and non-separability between consumption and labour supply is essential.
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Once family labor supply, assets and taxes (and benefits) are properly accounted for, there is less evidence for additional insurance.
Summary and Conclusions...

- Focus on understanding the transmission of inequality over the working life.

- Found that family labor supply is a key mechanism for smoothing consumption
  - especially for those with limited access to assets,
  - and non-separability between consumption and labour supply is essential.

- Once family labor supply, assets and taxes (and benefits) are properly accounted for, there is less evidence for additional insurance
  - lots to be done to dig deeper into these, and other, mechanisms.
  - consider detailed consumption components and form of non-separability.
## Results by Age, Education and Asset Selections

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Age 30-55</th>
<th>Some college+</th>
<th>Top 2 asset terc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E (\pi)$</td>
<td>0.181</td>
<td>0.142</td>
<td>0.202</td>
<td>0.245</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.120</td>
<td>-0.177</td>
<td>0.117</td>
<td>-0.046</td>
</tr>
<tr>
<td></td>
<td>(0.098)</td>
<td>(0.089)</td>
<td>(0.072)</td>
<td>(0.084)</td>
</tr>
<tr>
<td>$\eta_{c,p}$</td>
<td>0.437</td>
<td>0.465</td>
<td>0.368</td>
<td>0.343</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.044)</td>
<td>(0.05)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>$\eta_{h_1,w_1}$</td>
<td>0.514</td>
<td>0.467</td>
<td>0.542</td>
<td>0.388</td>
</tr>
<tr>
<td></td>
<td>(0.150)</td>
<td>(0.036)</td>
<td>(0.045)</td>
<td>(0.037)</td>
</tr>
<tr>
<td>$\eta_{h_2,w_2}$</td>
<td>1.032</td>
<td>1.039</td>
<td>0.858</td>
<td>0.986</td>
</tr>
<tr>
<td></td>
<td>(0.265)</td>
<td>(0.099)</td>
<td>(0.097)</td>
<td>(0.105)</td>
</tr>
<tr>
<td>$\eta_{c,w_1}$</td>
<td>-0.141</td>
<td>-0.113</td>
<td>-0.162</td>
<td>-0.127</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.018)</td>
<td>(0.022)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$\eta_{h_1,p}$</td>
<td>0.082</td>
<td>0.065</td>
<td>0.087</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.01)</td>
<td>(0.012)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\eta_{c,w_2}$</td>
<td>-0.138</td>
<td>-0.083</td>
<td>-0.142</td>
<td>-0.129</td>
</tr>
<tr>
<td></td>
<td>(0.139)</td>
<td>(0.029)</td>
<td>(0.032)</td>
<td>(0.154)</td>
</tr>
<tr>
<td>$\eta_{h_2,p}$</td>
<td>0.162</td>
<td>0.097</td>
<td>0.169</td>
<td>0.154</td>
</tr>
<tr>
<td></td>
<td>(0.166)</td>
<td>(0.034)</td>
<td>(0.038)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>$\eta_{h_1,w_2}$</td>
<td>0.128</td>
<td>0.101</td>
<td>0.115</td>
<td>0.079</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.011)</td>
<td>(0.012)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>$\eta_{h_2,w_1}$</td>
<td>0.258</td>
<td>0.205</td>
<td>0.255</td>
<td>0.172</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.022)</td>
<td>(0.027)</td>
<td>(0.021)</td>
</tr>
</tbody>
</table>

Note: Specifications (2) to (4) - Non-bootstrap s.e.’s
Concavity and Advance Information

- Concavity of preferences. Use the fact that:

\[
\begin{pmatrix}
\eta_p \frac{c}{p} & \eta_cw_1 \frac{c}{w_1} & \eta_cw_2 \frac{c}{w_2} \\
-\eta_{h_1} \frac{h_1}{p} & -\eta_{h_1} \frac{h_1}{w_1} & -\eta_{h_1} \frac{h_1}{w_2} \\
-\eta_{h_2} \frac{h_2}{p} & -\eta_{h_2} \frac{h_2}{w_1} & -\eta_{h_2} \frac{h_2}{w_2}
\end{pmatrix}
= \lambda
\begin{pmatrix}
\frac{d^2u}{dc^2} & \frac{d^2u}{dcdl_1} & \frac{d^2u}{dcdl_2} \\
\frac{d^2u}{dl_1dc} & \frac{d^2u}{dl_1^2} & \frac{d^2u}{dl_1dl_2} \\
\frac{d^2u}{dl_2dc} & \frac{d^2u}{dl_2dl_1} & \frac{d^2u}{dl_2^2}
\end{pmatrix}^{-1}
\]

- Appendix shows concavity cannot be rejected, and is numerically satisfied at average values of wages, hours, consumption.
Concavity and Advance Information

- **Concavity of preferences.** Use the fact that:

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\begin{pmatrix}
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-\eta_{h_1} \frac{h_1}{p} & -\eta_{h_1} \frac{h_1}{w_1} & -\eta_{h_1} \frac{h_1}{w_2} \\
-\eta_{h_2} \frac{h_2}{p} & -\eta_{h_2} \frac{h_2}{w_1} & -\eta_{h_2} \frac{h_2}{w_2}
\end{pmatrix}
= \lambda
\begin{pmatrix}
\frac{d^2u}{dc^2} & \frac{d^2u}{dcl_1} & \frac{d^2u}{dcl_2} \\
\frac{d^2u}{dl_1 dc} & \frac{d^2u}{dl_1^2} & \frac{d^2u}{dl_1 dl_2} \\
\frac{d^2u}{dl_2 dc} & \frac{d^2u}{dl_2 dl_1} & \frac{d^2u}{dl_2^2}
\end{pmatrix}^{-1}
\]

- Appendix shows concavity cannot rejected, and is numerically satisfied at average values of wages, hours, consumption.

- **Advance Information.** Consumption growth should be correlated with future wage growth (Cunha et al., 2008, and BPP 2008).
  - Test has p-value 13%
**RESULTS: EXTENSIVE MARGIN**

- Estimate a "conditional" Euler equation, controlling for changes in hours (intensive margin) and changes in participation (extensive margin)

<table>
<thead>
<tr>
<th></th>
<th>EMP</th>
<th>∆EMP</th>
<th>∆h</th>
<th>∆EMP</th>
<th>∆h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>1</td>
<td>0.144</td>
<td>0.269</td>
<td>23.4</td>
<td>0.013</td>
</tr>
<tr>
<td>Female</td>
<td>1</td>
<td>0.356</td>
<td>0.169</td>
<td>98.4</td>
<td>0.171</td>
</tr>
</tbody>
</table>

Sample All EMP

Instruments 2nd, 4th lags 2nd, 4th lags

Note: ∆xₜ is defined as (xₜ - xₜ₋₁) / [0.5(2xₜ + xₜ₋₁)]
### RESULTS: EXTENSIVE MARGIN

- Estimate a "conditional" Euler equation, controlling for changes in hours (intensive margin) and changes in participation (extensive margin)

<table>
<thead>
<tr>
<th></th>
<th>Regression results</th>
<th>First stage F-stats</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>$\Delta EMP_t(Male)$</td>
<td>0.144 (0.269)</td>
<td></td>
</tr>
<tr>
<td>$\Delta h_t(Male)   $</td>
<td>$-0.073 (0.075)$</td>
<td>$-0.013 (0.021)$</td>
</tr>
<tr>
<td>$\Delta EMP_t(Female)$</td>
<td>0.356 (0.169)</td>
<td>0.362 (0.176)</td>
</tr>
<tr>
<td>$\Delta h_t(Female)$</td>
<td>$-0.220 (0.100)$</td>
<td>$-0.171 (0.094)$</td>
</tr>
</tbody>
</table>

- Sample: All $EMP_t(Male)=1$
- Instruments: $2nd, 4th$ lags

Note: $\Delta x_t$ is defined as $(x_t - x_{t-1}) / [0.5 (x_t + x_{t-1})]$
## Wage Parameters by Assets and Age

<table>
<thead>
<tr>
<th>Sample</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>All</td>
<td>1(^{st}) asset tercile</td>
<td>2(^{nd}), 3(^{rd}) asset terciles</td>
<td>age&lt;40</td>
<td>age&gt;=40</td>
</tr>
<tr>
<td>Males</td>
<td>Trans.</td>
<td>$\sigma^2_{u1}$</td>
<td>0.033</td>
<td>0.03</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>Perm.</td>
<td>$\sigma^2_{v1}$</td>
<td>0.035</td>
<td>0.027</td>
<td>0.039</td>
</tr>
<tr>
<td>Females</td>
<td>Trans.</td>
<td>$\sigma^2_{u2}$</td>
<td>0.012</td>
<td>0.023</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>Perm.</td>
<td>$\sigma^2_{v2}$</td>
<td>0.046</td>
<td>0.036</td>
<td>0.05</td>
</tr>
<tr>
<td>Correlations of Shocks</td>
<td>Trans.</td>
<td>$\sigma_{u1,u2}$</td>
<td>0.202</td>
<td>-0.264</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>Perm.</td>
<td>$\sigma_{v1,v2}$</td>
<td>0.153</td>
<td>0.366</td>
<td>0.096</td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td></td>
<td>8,191</td>
<td>2,626</td>
<td>5,565</td>
</tr>
</tbody>
</table>
**Transmission Parameters:**

Consumption response to $j$’s permanent wage shock:

$$
\kappa_{c,v_j} = (1 - \beta) (1 - \pi_{i,t}) s_{i,j,t} \frac{\eta_{c,p} \left( 1 + \eta_{h_j,w_j} \right)}{\eta_{c,p} + (1 - \beta) (1 - \pi_{i,t}) \eta_{h,w}}
$$
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- declines with \( \pi_{i,t} \) (accumulated assets allow better insurance of shocks)
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$$

- declines with $\pi_{i,t}$ (accumulated assets allow better insurance of shocks)
- declines with $\beta$ (outside insurance allows more smoothing)
**TRANSMISSION PARAMETERS:**

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- declines with \( \eta_{h_j,w_j} \) ("added worker" effect)
- declines with \( \eta_{h_j,w_j} \) only if \( j \)'s labor supply responds negatively to own permanent shock. In one-earner case, true if

\[
(1 - \beta) (1 - \pi_{i,t}) - \eta_{c,p} > 0
\]
DATA AND SAMPLE SELECTION

- PSID biennial 1999-2009:
  - PSID consumption went through a major revision in 1999
    - ~70% of consumption expenditures. Good match with NIPA
    - The sum of food at home, food away from home, gasoline, health, transportation, utilities, etc.
    - Main items that are missing: clothing (now included), recreation, alcohol and tobacco
  - Earning and hours for each earner
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- **To begin with focus on:**
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  - Working males (93% in this age group)
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  - Working males (93% in this age group)
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- **Methodology:** Use structural restrictions that ‘theory’ imposes on the variance covariance structure of $\Delta c_{i,t}$, $\Delta y_{i,1,t}$ and $\Delta y_{i,2,t}$
Some Econometrics Issues

- Measurement error
  - For consumption, use martingale assumption and mean-reversion
  - For wages, use external estimates from Bound et al. (1994)
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  - Selection adjusted second moments
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  - Selection adjusted second moments

- **Inference**
  - Multi-step procedure
  - Block bootstrap standard errors
Inference

- Multi-step estimation procedure:
  - Regress $c_{i,t}, y_{i,j,t}, w_{i,j,t}$ on observable characteristics, and construct the residuals $\Delta c_{i,t}, \Delta y_{i,j,t}$ and $\Delta w_{i,j,t}$
  - Estimate the wage parameters using the conditional second order moments for $\Delta w_{i,1,t}$ and $\Delta w_{i,2,t}$
  - Estimate $\pi_{i,t}$ and $s_{i,t}$ using asset and (current and projected) earnings data
  - Estimate preference parameters using restrictions on the joint behavior of $\Delta c_{i,t}, \Delta y_{i,j,t}$ and $\Delta w_{i,j,t}$

- GMM with standard errors corrected by the block bootstrap.
\[
\begin{pmatrix}
\Delta w_{i,1,t} \\
\Delta w_{i,2,t} \\
\Delta c_{i,t} \\
\Delta y_{i,1,t} \\
\Delta y_{i,2,t}
\end{pmatrix}
\overset{\sim}{\sim}
\begin{pmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
\kappa_{c,u_1} & \kappa_{c,u_2} & \kappa_{c,v_1} & \kappa_{c,v_2} \\
\kappa_{y_1,u_1} & \kappa_{y_1,u_2} & \kappa_{y_1,v_1} & \kappa_{y_1,v_2} \\
\kappa_{y_2,u_1} & \kappa_{y_2,u_2} & \kappa_{y_2,v_1} & \kappa_{y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{i,1,t} \\
\Delta u_{i,2,t} \\
\nu_{i,1,t} \\
\nu_{i,2,t}
\end{pmatrix}
+ \begin{pmatrix}
\Delta \xi_{i,1,t}^w \\
\Delta \xi_{i,2,t}^w \\
\Delta \xi_{i,t}^c \\
\Delta \xi_{i,1,t}^y \\
\Delta \xi_{i,2,t}^y
\end{pmatrix}
\]

where \(\xi_{i,j,t}^w, \xi_{i,t}^c\) and \(\xi_{i,j,t}^y\) are measurement errors in log wages of earner \(j\), log consumption, and log earnings of earner \(j\).
Our focus here is on non-stationarity, heterogenous profiles, and shocks of varying persistence.
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Individual $i$ of age $a$ in time period $t$, has log income $y_{i,a}(\equiv \ln Y_{i,a,t})$

$$y_{i,a} = Z_{i,a}^T \phi_a + f_{0i} + f_{1i} \beta_p + y_{i,a}^p + \epsilon_{i,a}$$

where $\beta_i \beta_p$ is an individual-specific trend, allow non-zero covariance between $f_0$ and $f_1$. 

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Individual $i$ of age $a$ in time period $t$, has log income $y_{i,a}(\equiv \ln Y_{i,a,t})$

$$y_{i,a} = Z_{i,a}^T \varphi_a + f_{0i} + f_{1i}p_a + y_{i,a}^P + \epsilon_{i,a}$$

where $\beta_{i}p_a$ is an individual-specific trend, allow non-zero covariance between $f_0$ and $f_1$.

$y_{i,a}^T$ is the persistent process with variance $\sigma_a^2$

$$y_{i,a}^T = \rho y_{i,a-1}^T + \nu_{i,a}$$

and $\epsilon_{i,a}$ is a transitory process (can be low order MA) with variance $\omega_a^2$ (can be low order MA).
Our focus here is on non-stationarity, heterogenous profiles, and shocks of varying persistence.

Individual $i$ of age $a$ in time period $t$, has log income $y_{i,a}(\equiv \ln Y_{i,a,t})$

$$y_{i,a} = Z_{i,a}^T \varphi_a + f_0 + f_1 p_a + y_{i,a}^p + \varepsilon_{i,a}$$

where $\beta_i p_a$ is an individual-specific trend, allow non-zero covariance between $f_0$ and $f_1$.

$y_{i,a}^T$ is the persistent process with variance $\sigma_a^2$

$$y_{i,a}^T = \rho y_{i,a-1}^T + \nu_{i,a}$$

and $\varepsilon_{i,a}$ is a transitory process (can be low order MA) with variance $\omega_a^2$ (can be low order MA).

Allow variances (or factor loadings) of $\nu_{i,a}$ and $\varepsilon_{i,a}$ to vary with age/time for each birth cohort and education group.
**Idiosyncratic Trends**

- The idiosyncratic trend term $p_{tf_{1i}}$ could take a number of forms. Two alternatives are worth highlighting:
  - (a) Deterministic idiosyncratic trend:
    $$p_{tf_{1i}} = r(t)f_{1i}$$
    where $r$ is a known function of $t$, e.g. $r(t) = t$,
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- (b) Stochastic trend in ‘ability prices’:

  \[ p_t = p_{t-1} + \zeta_t \]

  with $E_{t-1} \zeta_t = 0$. 

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**Blundell, UCL & IFS ( )**

**Consumption and Family Labor Supply**

**ECB October 2013**
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Evidence points to some periods of time where each of these is of key importance. Deterministic trends as in (a), appear most prominently early in the working life. Formally, this is a life-cycle effect.
Idiosyncratic Trends

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- Evidence points to some periods of time where each of these is of key importance. Deterministic trends as in (a), appear most prominently early in the working life. Formally, this is a life-cycle effect.

- Alternatively, stochastic trends (b) are most likely to occur during periods of technical change when skill prices are changing across the unobserved ability distribution. Formally, this is a calendar time effect.
**Idiosyncratic Trends**

- For each cohort we consider several alternative models for the heterogenous profile $\beta_i p_a$:

  1. **Baseline Specification:**
     \[ f_1^i = 0 \]
  2. **Linear Specification:**
     \[ p_a = \gamma_1 a + \gamma_0, \text{ so that } \Delta \rho p_a = (1 - \rho_1) \gamma_0 \xi_0 + \gamma_1 \xi_0 \]
  3. **Quadratic Specification:**
     \[ p_a = \gamma_0 + \gamma_1 a + \gamma_2 a^2 \]
  4. **Piecewise-Linear Specification:**
     \[ p_a = \begin{cases} \kappa_1 a + 35 \gamma_1 a \kappa_2 a + 52 \gamma_2 a & \text{if } a \leq 35 \\ & \text{otherwise} \end{cases} \]
     with knots at age 35 and age 52.
  5. **Polynomials up to degree 4.**
**Idiosyncratic Trends**

- For each cohort we consider several alternative models for the heterogenous profile $\beta_i p_a$:

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Idiosyncratic Trends

For each cohort we consider several alternative models for the heterogeneous profile $\beta_i p_a$:

1. Baseline Specification: $f_{1i} = 0$

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$$\Delta^\rho p_a = (1 - \rho) \gamma_0 \iota + \gamma_1 \xi_0$$

where $\xi_0 \equiv [a - \rho (a - 1)]$. 
**Idiosyncratic Trends**

- For each cohort we consider several alternative models for the heterogenous profile $\beta_ip_a$:
  
  1. **Baseline Specification:** $f_{1i} = 0$
  2. **Linear Specification:** $p_a = \gamma_1a + \gamma_0$, so that
     
     $$
     \Delta^\rho p_a = (1 - \rho)\gamma_0i + \gamma_1\xi_0
     $$
     
     where $\xi_0 \equiv [a - \rho (a - 1)]$.
  3. **Quadratic Specification:** $p_a = \gamma_0 + \gamma_1a + \gamma_2a^2$
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   \[
   p_a = \begin{cases} 
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   a & \text{otherwise} \\
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   \end{cases}
   \]
   with knots at age 35 and age 52.

---

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   with knots at age 35 and age 52.
5. Polynomials up to degree 4.
Covariance structure

Suppose we observe individual $i$ at age $a = 1, \ldots, T$, we then have $T - 1$ equations $\Delta^\rho y_{ia} (\equiv y_{i,a} - \rho y_{i,a-1})$. In vector form

$$
\Delta^\rho y_i = ((1 - \rho) \iota, \Delta^\rho p_a) \begin{pmatrix} f_{0i} \\ f_{1i} \end{pmatrix} + v_i + \Delta^\rho \varepsilon_i.
$$
**Covariance Structure**

- Suppose we observe individual $i$ at age $a = 1, ..., T$, we then have $T - 1$ equations $\Delta^\rho y_{ia} (\equiv y_{i,a} - \rho y_{i,a-1})$. In vector form

$$\Delta^\rho \mathbf{y}_i = ((1 - \rho) \mathbf{v}, \Delta^\rho \mathbf{p}_a) \begin{pmatrix} f_{0i} \\ f_{1i} \end{pmatrix} + \mathbf{v}_i + \Delta^\rho \mathbf{e}_i.$$ 

- The Variance-Covariance matrix in general has the form: 

$$\text{Var}(\Delta^\rho \mathbf{y}_i) = \mathbf{\Omega} + \mathbf{W}$$ 

where $\mathbf{W} =$

$$
\begin{pmatrix}
\sigma_2^2 + \omega_2^2 + \rho^2 \omega_1^2 & -\rho \omega_2^2 & 0 & 0 \\
-\rho \omega_2^2 & \sigma_3^2 + \omega_3^2 + \rho^2 \omega_2^2 & -\rho \omega_3^2 & 0 \\
0 & -\rho \omega_3^2 & \ddots & -\rho \omega_{T-1}^2 \\
0 & 0 & -\rho \omega_{T-1}^2 & \sigma_T^2 + \omega_T^2 + \rho^2 \omega_{T-1}^2
\end{pmatrix}
$$
Covariance structure

Suppose we observe individual $i$ at age $a = 1, ..., T$, we then have $T - 1$ equations $\Delta^\rho y_{ia} (\equiv y_{i,a} - \rho y_{i,a-1})$. In vector form

$$\Delta^\rho y_i = ((1 - \rho) \iota, \Delta^\rho p_a) \left( \begin{array}{c} f_{0i} \\ f_{1i} \end{array} \right) + \nu_i + \Delta^\rho \varepsilon_i.$$ 

The Variance-Covariance matrix in general has the form:

$$\text{Var}(\Delta^\rho y_i) = \Omega + W$$

where $W =$

$$
\begin{pmatrix}
\sigma_2^2 + \omega_2^2 + \rho^2 \omega_1^2 & -\rho \omega_2^2 & 0 & 0 \\
-\rho \omega_2^2 & \sigma_3^2 + \omega_3^2 + \rho^2 \omega_2^2 & -\rho \omega_3^2 & 0 \\
0 & -\rho \omega_3^2 & \ddots & -\rho \omega_{T-1}^2 \\
0 & 0 & -\rho \omega_{T-1}^2 & \sigma_T^2 + \omega_T^2 + \rho^2 \omega_{T-1}^2 \\
\end{pmatrix}
$$

For the linear heterogeneous profiles case:

$$\Omega = [(1 - \rho) \iota, \xi_0] \left( \begin{array}{cc} \sigma_0^2 & \rho_{01} \sigma_0 \sigma_1 \\ \rho_{01} \sigma_0 \sigma_1 & \sigma_1^2 \end{array} \right) [(1 - \rho) \iota, \xi_0]^T.$$
Removing Additive Separability: Theory

Approximating the first order conditions (intensive margin):

\[ \Delta c_{i,t} \approx \left( \eta_{c,w_1} + \eta_{c,w_2} - \eta_{c,p} \right) \Delta \ln \lambda_{i,t} \]
\[ + \eta_{c,w_1} \Delta w_{i,1t+1} + \eta_{c,w_2} \Delta w_{i,2t+1} \]
Approximating the first order conditions (intensive margin):

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Interpretation:

- C and H substitutes \((\eta_{c,w_j} < 0)\) ⇒ Excess smoothing
- C and H complements \((\eta_{c,w_j} > 0)\) ⇒ Excess sensitivity
Approximating the first order conditions (intensive margin):

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\]

Interpretation:
- C and H substitutes \((\eta_{c,w_j} < 0)\) \(\Rightarrow\) Excess smoothing
- C and H complements \((\eta_{c,w_j} > 0)\) \(\Rightarrow\) Excess sensitivity

\[
\begin{pmatrix}
\Delta c_{i,t} \\
\Delta y_{i,1,t} \\
\Delta y_{i,2,t}
\end{pmatrix}
\approx
\begin{pmatrix}
\kappa_{i,c,u_1} & \kappa_{i,c,u_2} & \kappa_{i,c,v_1} & \kappa_{i,c,v_2} \\
\kappa_{i,y_1,u_1} & \kappa_{i,y_1,u_2} & \kappa_{i,y_1,v_1} & \kappa_{i,y_1,v_2} \\
\kappa_{i,y_2,u_1} & \kappa_{i,y_2,u_2} & \kappa_{i,y_2,v_1} & \kappa_{i,y_2,v_2}
\end{pmatrix}
\begin{pmatrix}
\Delta u_{i,1,t} \\
\Delta u_{i,2,t} \\
v_{i,1,t} \\
v_{i,2,t}
\end{pmatrix}
\]

where (for \(j = 1, 2\))

\[
\kappa_{i,c,u_j} = \eta_{c,w_j}; \quad \kappa_{i,y_j,u_j} = 1 + \eta_{h_j,w_j}; \quad \kappa_{i,y_j,v_{-j}} = \eta_{h_j,w_{-j}}
\]
Non-linear Taxes

\[ \tilde{Y}_{it} = (1 - \chi_t) (H_{1,t} W_{1,t} + H_{2,t} W_{2,t})^{1-\mu_t} \]
\[ \tilde{Y}_{it} = (1 - \chi_t) (H_{1,t} W_{1,t} + H_{2,t} W_{2,t})^{1-\mu_t} \]

- Implications for underlying structural preference parameters, e.g.

\[ \tilde{\eta}_{h_j, w_j} = \frac{\eta_{h_j, w_j} (1 - \mu)}{1 + \mu \eta_{h_j, w_j}} \text{ (with } \tilde{\eta}_{h_j, w_j} \leq \eta_{h_j, w_j} \text{ for } 0 \leq \mu \leq 1) \]
Non-linear Taxes

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- Labor supply elasticities (w.r.t. $W$) are dampened: Return to work decreases as people cross tax brackets
## Loading Factor Matrix: Estimates

<table>
<thead>
<tr>
<th>Response to</th>
<th>Separable case</th>
<th></th>
<th></th>
<th>Non-separable case</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Consump.</td>
<td>Husband’s earnings</td>
<td>Wife’s earnings</td>
<td>Consump.</td>
<td>Husband’s earnings</td>
<td>Wife’s earnings</td>
</tr>
<tr>
<td>v₁</td>
<td>0.13</td>
<td>1.15</td>
<td>-0.54</td>
<td>0.38</td>
<td>0.98</td>
<td>-0.81</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.067)</td>
<td>(0.206)</td>
<td>(0.057)</td>
<td>(0.131)</td>
<td>(0.180)</td>
</tr>
<tr>
<td>v₂</td>
<td>0.07</td>
<td>-0.16</td>
<td>1.53</td>
<td>0.21</td>
<td>-0.23</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.057)</td>
<td>(0.101)</td>
<td>(0.037)</td>
<td>(0.048)</td>
<td>(0.087)</td>
</tr>
<tr>
<td>Δu₁</td>
<td>0</td>
<td>1.43</td>
<td>0</td>
<td>-0.14</td>
<td>1.51</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.097)</td>
<td></td>
<td>(0.051)</td>
<td>(0.150)</td>
<td>(0.103)</td>
</tr>
<tr>
<td>Δu₂</td>
<td>0</td>
<td>0</td>
<td>1.83</td>
<td>-0.14</td>
<td>0.13</td>
<td>2.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.133)</td>
<td>(0.139)</td>
<td>(0.051)</td>
<td>(0.265)</td>
</tr>
</tbody>
</table>
### Heterogeneity:

<table>
<thead>
<tr>
<th></th>
<th>(1) Baseline</th>
<th>(2) Age 30-55</th>
<th>(3) Some college+</th>
<th>(4) Top 2 asset terc.</th>
<th>(5) Age variance</th>
<th>(6) Sel.correct.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\pi)$</td>
<td>0.181</td>
<td>0.142</td>
<td>0.202</td>
<td>0.245</td>
<td>0.181</td>
<td>0.176</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.120</td>
<td>-0.177</td>
<td>0.117</td>
<td>-0.046</td>
<td>-0.109</td>
<td>-0.129</td>
</tr>
<tr>
<td>$\eta_{c,p}$</td>
<td>0.437</td>
<td>0.465</td>
<td>0.368</td>
<td>0.343</td>
<td>0.42</td>
<td>0.473</td>
</tr>
<tr>
<td>$\eta_{h1,w1}$</td>
<td>0.514</td>
<td>0.467</td>
<td>0.542</td>
<td>0.388</td>
<td>0.575</td>
<td>0.509</td>
</tr>
<tr>
<td>$\eta_{h2,w2}$</td>
<td>1.032</td>
<td>1.039</td>
<td>0.858</td>
<td>0.986</td>
<td>1.005</td>
<td>1.095</td>
</tr>
<tr>
<td>$\eta_{c,w1}$</td>
<td>-0.141</td>
<td>-0.113</td>
<td>-0.162</td>
<td>-0.127</td>
<td>-0.15</td>
<td>-0.150</td>
</tr>
<tr>
<td>$\eta_{h1,p}$</td>
<td>0.082</td>
<td>0.065</td>
<td>0.087</td>
<td>0.07</td>
<td>0.087</td>
<td>0.088</td>
</tr>
<tr>
<td>$\eta_{c,w2}$</td>
<td>-0.138</td>
<td>-0.083</td>
<td>-0.142</td>
<td>-0.129</td>
<td>-0.11</td>
<td>-0.122</td>
</tr>
<tr>
<td>$\eta_{h2,p}$</td>
<td>0.162</td>
<td>0.097</td>
<td>0.169</td>
<td>0.154</td>
<td>0.129</td>
<td>0.143</td>
</tr>
<tr>
<td>$\eta_{h1,w2}$</td>
<td>0.128</td>
<td>0.101</td>
<td>0.115</td>
<td>0.079</td>
<td>0.141</td>
<td>0.125</td>
</tr>
<tr>
<td>$\eta_{h2,w1}$</td>
<td>0.258</td>
<td>0.205</td>
<td>0.255</td>
<td>0.172</td>
<td>0.285</td>
<td>0.253</td>
</tr>
</tbody>
</table>

Note: Specifications (2) to (6) - Non-bootstrap s.e.’s
Approximation of the Euler Equation (1)

- From $\lambda_{i,t} = \frac{1+\delta}{1+r} E_t \lambda_{i,t+1}$, use a second order Taylor approximation (with $r = \delta$) to yield:

$$\Delta \ln \lambda_{i,t+1} \approx \omega_t + \varepsilon_{i,t+1}$$

- where

$$\omega_t = -\frac{1}{2} E_t (\Delta \ln \lambda_{i,t+1})^2$$

$$\varepsilon_{i,t+1} = \Delta \ln \lambda_{i,t+1} - E_t (\Delta \ln \lambda_{i,t+1})$$

- Then use the fact that

$$\Delta \ln U_{C_{i,t+1}} = \Delta \ln \lambda_{i,t+1}$$

$$\Delta \ln U_{H_{i,j,t+1}} = -\Delta \ln \lambda_{i,t+1} - \Delta \ln W_{i,j,t+1}$$
APPORXIMATION OF THE EULER EQUATION (2)

Consider now Taylor expansion of $U_{C_{i,t+1}}(= \lambda_{i,t+1})$:

$$
\frac{U_{C_{i,t+1}} - U_{C_{i,t}}}{U_{C_{i,t}}} \approx U_{C_{i,t}} + (C_{i,t+1} - C_{i,t}) \frac{U_{C_{i,t}} C_{i,t}}{C_{i,t}}
$$

$$
\Delta \ln U_{C_{i,t+1}} \approx - \frac{1}{\eta_{c,p}} \Delta \ln C_{i,t+1}
$$

and therefore, from

$$
\Delta \ln \lambda_{i,t+1} \approx \omega_{t+1} + \epsilon_{i,t+1}
$$

get

$$
\Delta \ln C_{i,t+1} = - \eta_{c,p} (\omega_{t+1} + \epsilon_{i,t+1})
$$
Approximation of the Life Time Budget Constraint

- Use the fact that

\[
\mathbb{E}_I \left[ \ln \sum_{i=0}^{T-t} X_{t+i} \right] = \ln \sum_{i=0}^{T-t} \exp \mathbb{E}_{t-1} \ln X_{t+i} \\
+ \sum_{i=0}^{T-t} \frac{\exp \mathbb{E}_{t-1} \ln X_{t+i}}{\sum_{j=0}^{T-t} \exp \mathbb{E}_{t-1} \ln X_{t+j}} \left( \mathbb{E}_I - \mathbb{E}_{t-1} \right) \ln X_{t+i} \\
+ O \left( \mathbb{E}_I \left\| \xi_t \right\|^2 \right)
\]

for \( X = C, WH \) and appropriate choice of \( \mathbb{E}_I \).

- Goal: obtain a mapping from wage innovations to innovations in consumption (marginal utility of wealth)