When we discussed Classical Generalised Quantifier Theory in the first half of the term, we did not distinguish singular and plural NPs, but this is clearly an oversimplification. For example, the following contrasts suggest that singular and plural noun phrases have different meanings.

(1) a. John is a boy.
b. *John and Bill are a boy.

(2) a. *John is boys.
b. John and Bill are boys.

One might think that this might be a syntactic phenomenon where the two DPs in predicational sentences need to agree in number. However, number agreement is not always required, as shown by (3).

(3) a. John and Bill are a couple.
b. These assignments are a nightmare.

Another reason to believe that singular and plural nouns have different semantics comes from the fact that plural noun phrases mean 'plural' after all (at least in canonical cases; we'll discuss exceptions later). For instance, (4b) sounds false if John only read one book, unlike (4a).

(4) a. John read a book.
b. John read books.

What exactly is the semantics of plural marking? The simplest hypothesis is that plural nouns like *books* only have plural entities in their denotation, and singular nouns like *book* only has singular entities in their denotation, as in (5).

(5) a. $[\![\text{book}]\!]^M = \lambda x \in SG.\ 1$ iff x is a book in M
b. $[\![\text{books}]\!]^M = \lambda x \in PL.\ 1$ iff each singular part y of x is a book in M

More concretely, if there are three books b_1, b_2 and b_3 in the model M:

(6) a. $\text{set}\left([\![\text{book}]\!]^M\right) = \{\, b_1, b_2, b_3 \,\}$
b. $\text{set}\left([\![\text{books}]\!]^M\right) = \left\{ \begin{array}{ccc} b_1 \oplus b_2, & b_1 \oplus b_3, & b_2 \oplus b_3, \\ & b_1 \oplus b_2 \oplus b_3 & \end{array} \right\}$

However, it turns out that the semantics of plural nouns is not that straightforward.

1 Unmarked Plural

1.1 Singular Nouns

We do not need to revise our analysis of singular nouns. That is, they denote functions of type $\langle e, t\rangle$ that are true of singular entities. This analysis gives a (partial) explanation for why singular nouns cannot be true of plural entities in sentences like (7).

(7) *Andrew and Bill are a boy.

This is because a plural entity like Andrew$\oplus$Bill is never in the extension of a singular noun *boy*.

However, this is not a complete explanation, given that we have postulated the distributivity operator Δ. To remind you, its denotation is as in (8).

(8) For any model M,
$[\![\Delta]\!]^M = \lambda P_{\langle e,t \rangle}.\lambda x \in PL.\ 1$ iff for each $y \in SG$ such that $y \sqsubseteq x$, $P(y) = 1$

We used this operator to account for the compatibility of a plural subject and a distributive predicate, as in (9).

(9) Andrew and Bill Δ(speak a Romance language)

If this operator can appear in the predicational copula construction, we do indeed predict (7) to be fine. That is, (10) is true iff Andrew is a boy and Bill is a boy.

(10) *Andrew and Bill Δ(are a boy).

We could stipulate a constraint that prohibits Δ in predicational sentences, but why such a constraint exists needs to be explained.

1.2 Plural Nouns

Let us discuss two possible analyses for plural nouns.

1. Plural means more than one (> 1)

   (11) For any model M,
   $[\![\text{books}]\!]^M = \left[\lambda x \in D_e.\ \begin{array}{l} 1 \text{ iff } x \text{ is a plural individual} \\ \text{each of whose singular part is a book in } M \end{array}\right]$

2. Plural means one or more (> 0)

   (12) For for any model M,
   $[\![\text{books}]\!]^M = \left[\lambda x \in D_e.\ \begin{array}{l} 1 \text{ iff } x \text{ is a book or a plural individual} \\ \text{each of whose singular part is a book in } M \end{array}\right]$

At first sight, the first option seems to be better. However, there are arguments for the second analysis. That is, there are some cases where the plural means one or more (> 0), rather than more than one (> 1).

- Plural indefinites in questions

  (13) Do you have children?
  a. Yes, I have one.
  b. #No, I (only) have one.

  This question is neutral with respect to the number. Compare this to (14):

  (14) Do you have more than one child?
  a. #Yes, I have one.

b. No, I (only) have one.

These two questions clearly have different meanings.

- Plural indefinites in negative sentences

(15) John doesn't have children.

This sentence entails that John does not have a child. Again, compare this to (16), which does not entail it.

(16) John doesn't have more than one child.

- Plural indefinites in conditionals

(17) If you have coins in your pocket, put them in a tray.

This conditional sentence is number neutral in the sense that it says something about situations where you only have one coin in your pocket. Compare this to (18), which does exclude such situations.

(18) If you have more than one coin in your pocket, put them in a tray.

- Plural definites in ignorance situations

Consider the following scenario (this example is taken from Sauerland, Anderssen & Yatsushiro 2005):

(19) You are inviting an old friend who you have not seen in years. you heard that he has a family now, but you have no idea how many children he has.

In this scenario, it is more natural to use a plural:

(20) a. You are welcome to bring your children.
b. #You are welcome to bring your child.

(19a) means something closer to *child or children*.

(21) You are welcome to bring your child or children.

These observations lead us to assume that the denotation of a plural noun includes singular entities as well. For instance, if there are three books in the model M:

(22) a. $\text{set}\left(\llbracket\text{book}\rrbracket^M\right) = \{\, b_1, b_2, b_3 \,\}$

b. $$\text{set}\left(\llbracket\text{books}\rrbracket^M\right) = \left\{ \begin{array}{c} b_1, \quad b_2, \quad b_3, \\ b_1 \oplus b_2, \quad b_1 \oplus b_3, \quad b_2 \oplus b_3, \\ b_1 \oplus b_2 \oplus b_3 \end{array} \right\}$$

According to this analysis, the plural does not mean more than one (> 1), but rather, it is number neutral!

But the plural does mean more than one in other cases like the following examples.

(23) a. John has children.
b. I like Paul's books on semantics.

(23a) entails that John has more than one child, and the possessive DP in (23b) refers to more than one book on semantics by Paul. Let us call these inferences **plurality inferences** (alt.: multiplicity inferences). The key question is, when do we get tha plurality inference and when do we not? And why?

Let us explore one specific account of the distribution of the plurality inference. Consider the following rule (see Sauerland et al. 2005 for a more precise formulation of this; see also Pearson, Khan & Snedeker 2010)

(24) **Unmarked Plural Rule (UPR)**
If you mean 'exactly one' you cannot use the plural.

To be concrete, let us apply this rule to the following example.

(25) John has children.

According to our number neutral semantics for plural nouns, (25) means 'John has one or more children'. The UPR demands that if you want to mean 'John has exactly one child', you cannot use this sentence. Conversely, you can use (25) if you do not mean 'John has exactly one child'. Thus, together with the meaning of the sentence (John has at least one child), it follows that John has two or more children.

Incidentally, the UPR says nothing about situations where you do *not* want to mean 'exactly one'. So in such contexts, you can use (25) or (26).

(26) John has a child.

(You might think (26) sounds like John only has one child, but this 'exactly one' inference is likely to be a scalar implicature; it's an interesting question how this scalar implicature is derived.)

Let us now go through cases without plurality inferences.

- Plural indefinites in questions

  (27) Does John have children?

  The UPR demands that if you want to mean 'exactly one', i.e. 'Does John have exactly one child'?, you cannot use (27). In all other contexts, you can use (27), including when you want to ask a number neutral question.
  Recall that if you do not mean 'exactly one', the UPR has nothing to say. In particular, it does not prevent you from using (28) to mean the same thing as (27).

  (28) Does John have a child?

- The same reasoning applies to plural indefinites in negative sentences and conditionals. Let us take conditionals (the negative example can be analyzed in the same way).

  (29) If you have coins in your pocket, put them in a tray.

  The UPR says, if you want to mean 'If you have exactly one coin in your pocket, put it in a tray', you cannot use (29), but you can use (29) to mean 'If you have any coin in your pocket, put it in a tray'. Again, there are no restrictions on the singular counterpart, so nothing prevents (30) from meaning the same thing as (29).

  (30) If you have a coin in your pocket, put it in a tray.

- Plural definites in ignorance contexts

  (31) You are welcome to bring your children.

  If you know that your old friend has exactly one child, the UPR says you cannot use (31). But in all other contexts, including contexts where you do not know how many children your friend has, you can use (31) to mean 'You are welcome to bring your child or children'. Again, the UPR says nothing about (32).

  (32) You are welcome to bring your child.

But (32) is only fine in contexts where your friend has exactly one child, due to the meaning of the singular noun and the definiteness (we'll come back to definiteness below).

To sum up, in some cases the plural is number neutral but in other cases it gives rise to a plurality inference ('more than one'). We assume a number neutral meaning of the plural and derive the plurality inference via the Unmarked Plural Rule (UPR).

(33) a. $[\![\text{book}]\!]^M = \lambda x \in SG.\ 1$ iff x is a book in M
b. $[\![\text{books}]\!]^M = \lambda x \in D_e.\ 1$ iff for each $y \in SG$, if $y \sqsubseteq x$, y is a book in M

(34) **Unmarked Plural Rule (UPR)**
If you mean 'exactly one' you cannot use the plural.

Given the above analysis, we can assign the following meaning to the plural morpheme. It takes the denotation of a singular noun and 'closes it with $\oplus$':

(35) $[\![\text{-s}]\!]^M = \lambda P \in D_{\langle e,t\rangle}.\lambda x \in D_e.\ 1$ iff for each $y \in SG$ such that $y \sqsubseteq x$, $P(x) = 1$

2 Definites

Suppose that there are three books, b_1, b_2 and b_3 (call this model M_1). Then, the phrase *the book* is infelicitous, while *the books* denotes the plural individual consisting of these three books.

(36) a. $\text{set}\left([\![\text{books}]\!]^{M_1}\right) = \left\{ \begin{array}{c} b_1, \quad b_2, \quad b_3, \\ b_1 \oplus b_2, \quad b_1 \oplus b_3, \quad b_2 \oplus b_3, \\ b_1 \oplus b_2 \oplus b_3 \end{array} \right\}$
b. $[\![\text{the books}]\!]^{M_1} = b_1 \oplus b_2 \oplus b_3$

Suppose now that there is only one book b_1 (call this situation M_2). Then, the phrase *the book* denotes this unique book, while *the books* is infelicitous (because of the UPR).

(37) a. $\text{set}\left([\![\text{book}]\!]^{M_2}\right) = [\![\text{books}]\!]^{M_2} = \{ b_1 \}$
b. $[\![\text{the book}]\!]^{M_2} = b_1$

Generally, 'the NP(s)' denotes the **unique maximal entity** satisfying the NP, if any. In M_1, $[\![\text{books}]\!]^{a,M_1}$ has a unique maximal entity, $b_1 \oplus b_2 \oplus b_3$. It's maximal in the sense that everything else in $[\![\text{books}]\!]^M$ is a part of it. On the other hand, $[\![\text{book}]\!]^{M_1}$ does not have a maximal entity in M_1, because there are three independent books and hence three maximal elements in $[\![\text{book}]\!]^{M_1}$. Consequently *the book* has nothing to denote (we call such a case a presupposition failure; see discussion in Heim & Kratzer 1998:§4.4 for more on this). In M_2, $[\![\text{book}]\!]^{a,M_2}$ does have a unique maximal entity, namely b_1, so *the book* refers to it. As noted above, *the books* is infelicitous in M_2 because you would mean 'the exactly one book', and violate the UPR.

3 Generalised Quantifiers with Plurality

Some quantificational determiners combine with a singular noun:

(38) a. every book

b. *every books

Others select for a plural noun:

(39) a. *most book
b. most books

Others are neutral:

(40) a. some book
b. some books

Let us modify Classical Generalise Quantifier Theory to incorporate the semantics of plural noun phrases developed above.

3.1 Indefinites

According to the generalised quantifier analysis of indefinites, a singular indefinite like *some book* is an existential quantifier:

(41) For any model M,
$[\![\text{some book}]\!]^M = \lambda P \in D_{\langle e,t\rangle}.\ 1$ iff there is a book x in M such that $P(x) = 1$

(42) For any model M,
$$\begin{aligned}[\![\text{some}]\!]^M &= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1 \text{ iff there is } x \in D_e \text{ such that } Q(x) = P(x) = 1\\ &= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1 \text{ iff set}(Q) \cap \text{set}(P) \neq \varnothing\end{aligned}$$

This semantics can be used with a plural noun as is, as shown in (43).

(43)
$$\begin{aligned}[\![\text{some books}]\!]^M &= \lambda P \in D_{\langle e,t\rangle}.\ 1 \text{ iff there is } x \in D_e \text{ such that } [\![\text{books}]\!]^M(x) = P(x) = 1\\ &= \lambda P \in D_{\langle e,t\rangle}.\left[\begin{array}{l}1 \text{ iff there is } x \in D_e \text{ such that } x \text{ is a book in } M\\ \text{or each singular part of } x \text{ is a book in } M\\ \text{and } P(x) = 1\end{array}\right]\end{aligned}$$

This analysis accounts for the following sentences (recall that the UPR prohibits the 'exactly one' meaning).

(44) a. Some children Δ cried. (Distributive)
b. Some children gathered in the park. (Collective)

Notice in particular, since $[\![\text{children}]\!]^M$ includes plural entities, collective predication is made possible, as in (44b).

When the noun is singular, only singular entities can be true of the type-$\langle e, t\rangle$ function that the singular noun denotes. Consequently, collective predication is incompatible with a singular indefinite.

(45) *Some child gathered in the park.

Since $[\![\text{child}]\!]^M$ does not include a plural entity in its extension, the sentence cannot be true (as discussed in the lecture notes from last week, this can be though of as a purely semantic anomaly or syntactic anomaly, or both).

Let us now consider *a*. Its semantics is very similar to that of *some*, but one crucial difference is that it only combines with a singular noun.

(46) a. a child
b. *a children

This could be seen as a morphosyntactic restriction. Or alternatively, it could be analyzed as a presupposition. Or, perhaps both of these are true. We leave it open here.

English does not have a plural version of *a*, but bare plurals can be seen as existential quantifiers in sentences like (47).

(47) a. Children Δ cried.
b. Children gathered in the park.

One way to analyze these sentences is by a null determiner $\exists$, which by assumption has the same meaning as *some* (and is realised overtly in other languages like French and Spanish).

(48) $$\begin{aligned}[\![\exists]\!]^M &= [\![\text{some}]\!]^M \\ &= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1 \text{ iff there is } x \in D_e \text{ such that } Q(x) = P(x) = 1 \\ &= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1 \text{ iff set}(Q) \cap \text{set}(P) \neq \varnothing\end{aligned}$$

However, there are several differences between *some NPs* and bare plurals. Firstly, bare plurals always take narrow scope, while *some NPs* is a so-called Positive Polarity Item and cannot take scope below a clause-mate negation. So the following two sentences are not synonymous.

(49) a. John didn't read some books.
b. John didn't read books.

Secondly, bare plurals have kind readings, while *some NPs* only have sub-kind readings.

(50) a. Some dogs are mammals.
b. Dogs are mammals.

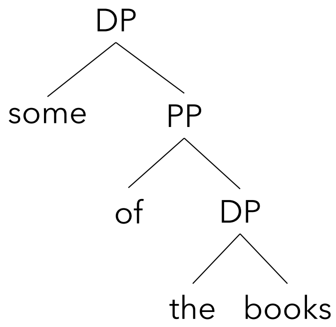
Thirdly, only *some* can appear in partitive DPs.

(51) a. Some of the books are interesting.
b. *Of the books are interesting.

Some argue that bare plurals do not actually involve determiners (Carlson 1977, Chierchia 1998). See Carlson (1977), Chierchia (1998), Chung & Ladusaw (2003), Diesing (1992), Van Geenhoven (1998), Delfitto (2005), Dayal (2011) for more on bare plurals, other bare NPs, and cross-linguistic facts.

Some of the books is also an existential quantifier, but it contains a definite DP. Let's assume the following LF for this DP.

(52)



Recall from the previous section that *the books* denotes the unique maximal entity. Since *some* requires a predicate of type $\langle e, t\rangle$ as its argument, let's assume that *of* turns this plural entity into a predicate in the following manner:

(53) For any model M,
$[\![\text{of}]\!]^M = \lambda x \in D_e.\ \lambda y \in D_e.\ 1$ iff $y \sqsubseteq x$

Then we have:

(54) a. $[\![\text{of the books}]\!]^M = [\![\text{books}]\!]^M$
b. $[\![\text{some of the books}]\!]^M = [\![\text{some books}]\!]^M$

3.2 *No*

<u>*No*</u> is another number neural determiner.

(55) a. no book
b. no books

In the first half of the course, we gave the following analysis, which is the negation of *some*:

(56) For any model M,
$[\![\text{no}]\!]^M = \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1$ iff there is no $x \in D_e$ such that $Q(x) = P(x) = 1$
$= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1$ iff $Q \cap P = \varnothing$

This will work as is with plural noun phrases.

(57) a. No students Δ were asleep during my lecture.
b. No students gathered.

(Notice also that this is another case where the plural should not have a plurality inference).

3.3 Universal Quantifiers

According to Classical Generalized Quantifier Theory, universal quantifiers express the subset relation. We analyzed *each*, $[\![\text{every}]\!]^M$, and *all* to be synonymous:

(58) For any model M,
$[\![\text{each}]\!]^M = [\![\text{every}]\!]^M = [\![\text{all}]\!]^M$
$= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ \left[\begin{array}{l} 1 \text{ iff for each } x \in D_e \\ \text{such that } Q(x) = 1,\ P(x) = 1 \end{array}\right]$

$= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1 \text{ iff } Q \subseteq P$

However, these three universal quantifiers are not completely identical. One notable difference is number marking.

(59) a. each book
b. *each books

(60) a. every book
b. *every books

(61) a. *all book
b. all books

Also, *every* cannot occur in partitives.

(62) a. each of the books
b. *every of the books
c. all of the books

Their difference goes beyond their morphosyntactic properties. *All* is compatible with collective predication, while *each* is not.

(63) a. *Each student gathered in the courtyard.
b. All (of) the students gathered in the courtyard.

Every seems to be fine here (although in the literature it is sometimes remarked that this is not the case).

(64) Every student gathered in the courtyard.

The classical generalized quantifier analysis above works well for *every* and *each*. But for *all*, it predicts too strong meaning. For instance, (63b) is predicted to be true iff $[\![\text{of the students}]\!]^M$ is a subset of $[\![\text{gathered}]\!]^M$. Suppose that there are four students, $s_1, s_2, \ldots, s_4$ and there is only one gathering by $s_1 \oplus s_2 \oplus s_3 \oplus s_4$. Then, we have:

(65) a. $$\text{set}\left([\![\text{students}]\!]^M\right) = \left\{ \begin{array}{c} s_1, \quad s_2, \quad s_3, \quad s_4, \\ s_1 \oplus s_2, \quad s_1 \oplus s_3, \quad s_1 \oplus s_4, \quad s_2 \oplus s_3, \quad s_2 \oplus s_4, \quad s_3 \oplus s_4, \\ s_1 \oplus s_2 \oplus s_3, \quad s_1 \oplus s_2 \oplus s_4, \quad s_1 \oplus s_3 \oplus s_4, \quad s_2 \oplus s_3 \oplus s_4, \\ s_1 \oplus s_2 \oplus s_3 \oplus s_4 \end{array} \right\}$$

b. $$\text{set}\left([\![\text{gathered}]\!]^M\right) = \{ s_1 \oplus s_2 \oplus s_3 \oplus s_4 \}$$

Intuitively, the sentence (63b) is true in this situation, but (65a) is clearly not a subset of (65b)! In fact, it is predicted that (64b) can never be true, because a collective predicate is never true of singular entities, while (65a) contains singular entities.

How do we account for *all*? One analytical possibility is that *all* is not a quantifier after all. Such an analysis is proposed by Brisson (2003). Her idea is that *all (of) the students* denotes the maximal individual, just like *the students*. Then, the sentence in (63b) is correctly predicted to be true in the scenario depicted in (65).

But of course there is a difference between *all of the students* and *the students*. One such difference is the (in)tolerance of exceptions. It is often remarked (e.g. see Lasersohn 1999) that definite descriptions can be used somewhat loosely. For instance, (66) sounds true, even when, say, 3 of the 200 students are not happy with the program.

(66) The students are happy with the program.

But with *all* the sentence does sound false, unless literally all of them are happy.

(67) All the students are happy with the program.

Brisson (2003) proposes that the elimination of this looseness is the semantic contribution of *all*. We will not try to formalize this idea here.

This analysis, however, is not without problems. For instance, observe the following contrast:

(68) a. The students are numerous.
b. *All the students are numerous.

If we analyze *all the NP* as the same thing as *the NP*, we cannot account for this contrast. Notice that (68) implies that there are two types of collective predicates, those that are compatible with quantifiers like *gather* and those that are not like *are numerous*. For more on this issue and its theoretical consequences, see Dowty (1987) and Winter (2001, 2002)

3.4 Numerals

Classical Generalized Quantifier Theory analyzes numerals as quantificational determiners:

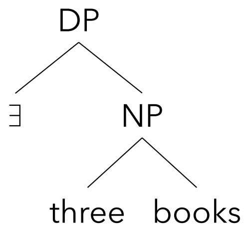
(69) For any model M,

a. $[\![\text{three}]\!]^M = \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\begin{bmatrix} 1 \text{ iff there are three individuals } x \\ \text{such that Q(x)=P(x)=1} \end{bmatrix}$
$= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1 \text{ iff } |P \cap Q| = 3$

b. $[\![\text{at least three}]\!]^M$
$= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\begin{bmatrix} 1 \text{ iff there are at least three individuals } x \\ \text{such that Q(x)=P(x)=1} \end{bmatrix}$
$= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1 \text{ iff } |P \cap Q| \geqslant 3$

c. $[\![\text{at most three}]\!]^M$
$= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\begin{bmatrix} 1 \text{ iff there are at most three individuals } x \\ \text{such that Q(x)=P(x)=1} \end{bmatrix}$
$= \lambda Q \in D_{\langle e,t\rangle}.\lambda P \in D_{\langle e,t\rangle}.\ 1 \text{ iff } |P \cap Q| \leqslant 3$

But numerals are morphosyntactically not determiners, since they can co-occur with *the*, e.g. *the three books I read*, etc. It seems to make more sense to analyze numerals as nominal modifiers of type $\langle e, t\rangle$, which combines with $[\![\text{books}]\!]^M$ via Predicate Modification. For instance, we assume the following LF for *three books*. ($\exists$ is the null existential determiner, synonymous with *some*, mentioned above.)

(70) 

Recall that *books* is true of any entity whose singular parts are books (including singular books). We assume that *three* sieves out those entities that are not comprised of three books:

(71) For any model M,
  a. $[\![\text{three books}]\!]^M = \lambda x \in D_e.\, 1$ iff x has three singular parts and $[\![\text{books}]\!]^M(x) = 1$
  b. $[\![\text{three}]\!]^M = \lambda x \in D_e.\, 1$ iff x has three singular parts

For instance, suppose that there are four books in M_4. Then,

(72) a. $\text{set}\left([\![\text{books}]\!]^{M_4}\right)$

$$= \left\{ \begin{array}{c} b_1, \quad b_2, \quad b_3, \quad b_4, \\ b_1 \oplus b_2, \quad b_1 \oplus b_3, \quad b_1 \oplus b_4, \quad b_2 \oplus b_3, \quad b_2 \oplus b_4, \quad b_3 \oplus b_4, \\ b_1 \oplus b_2 \oplus b_3, \quad b_1 \oplus b_2 \oplus b_4, \quad b_1 \oplus b_3 \oplus b_4, \quad b_2 \oplus b_3 \oplus b_4, \\ b_1 \oplus b_2 \oplus b_3 \oplus b_4 \end{array} \right\}$$

  b. $\text{set}\left([\![\text{three books}]\!]^{M_4}\right)$

$$= \{\, b_1 \oplus b_2 \oplus b_3, \quad b_1 \oplus b_2 \oplus b_4, \quad b_1 \oplus b_3 \oplus b_4, \quad b_2 \oplus b_3 \oplus b_4 \,\}$$

Then,

(73) $[\![\exists \text{ three books}]\!]^{M_4} = \lambda P \in D_{\langle e,t \rangle}.\, 1$ iff there is a member of (72b) for which P is true.

Notice that the definite *the three books* will be infelicitous in M_4, because *three books* has no unique maximal element. In fact, *the three books* is only felicitous in contexts where there are exactly three books. This is a good result.

It should also be noticed that what we derive for sentences like (74) is an 'at-least' reading.

(74) $\exists$ three students Δ were late for class.

According to our semantics, (74) is true iff there is a plural entity consisting of three students and each of these students was late for class. This is going to be true in a situation where there are four or more students who were late, because in such a situation you can just take three of them to make the sentence true!

This is not a problem because it's possible that the 'exact'-reading is a scalar implicature. That is, when somebody asserts (74), you compare it to the versions of sentences with different numerals:

(75) a. $\exists$ four students Δ were late for class.
  b. $\exists$ five students Δ were late for class.
  c. $\exists$ six students Δ were late for class.

...

Since these sentences are stronger, i.e. they asymmetrically entail (74), you conclude that they are not true. Therefore, exactly three students were late for class.

Notice that you do not want to derive scalar implicatures for collective sentences. In order to see this, consider (76), where *combined* is collective with respect to its object.

(76) John combined $\exists$ three PDFs (into a single PDF).

This sentence does not implicate that he did not make another PDF with four or more PDFs, which you would infer by negating the alternatives in (77).

(77) a. John combined $\exists$ four PDFs.
b. John combined $\exists$ five PDFs.
...

Importantly, with collective predicates like *combine*, these sentences do not stand in an entailment relation. That is, (77a) does *not* entail (76). If you believe that scalar implicatures are only generated based on alternatives that asymmetrically entail what is uttered, the incorrect inference is blocked.

Notice furthermore, that this means that we do not have an 'at least' reading for (76), because in a situation where John combined four PDFs, (76) is simply not true. It is only true in a situation where John combined a plural entity that has three parts, each of which is a PDF file.

Let us now consider modified numerals. Let's try to give similar predicative analyses:

(78) For any model M,
a. $[\![\text{at least three}]\!]^M = \lambda x \in D_e.\, 1$ iff x has at least three singular parts
b. $[\![\text{exactly three}]\!]^M = \lambda x \in D_e.\, 1$ iff x has exactly three singular parts
c. $[\![\text{at most three}]\!]^M = \lambda x \in D_e.\, 1$ iff x has at most three singular parts

(78a) actually is not a bad analysis, but there will be an open question regarding the scalar implicatures of distributive sentences. In order to see this, consider (79).

(79) $\exists$ at least three students Δ were late for class.

This is true iff there is a plural entity consisting of three or more singular parts that are all students, and each of them was late for class. Notice that this has exactly the same truth-conditions as (80).

(80) $\exists$ three students Δ were late for class.

But there is a notable difference, i.e. (79) does not have an 'exact'-reading, unlike (80). Or to put it differently, (79) has no scalar implicature. Why this is the case is left unsolved here. See Krifka (1999), Büring (2009), Fox & Hackl (2006), Mayr (2013), Schwarz (2016) for ideas.

For *exactly three* and *at most three*, the above analysis predicts truth-conditions that are too weak (**Van Bentham's trap**). Let us illustrate the problem with *at most three* (the problem for *exactly three* is left for an exercise). The following sentence is true iff there is a plural entity consisting of at most three students such that each of them was late for class.

(81) $\exists$ at most three students Δ were late for class.

But notice that if four (or more) students were late for class, you can find a plural entity consisting of three or fewer students who were late. Then it is predicted that the sentence is true in such a situation!

The above semantics is problematic with collective predicates too.

(82) $\exists$ at most three students gathered.

This sentence is predicted to be true in the following scenario: there are two gatherings, one by $s_1 \oplus s_2$ and another by $s_3 \oplus s_4 \oplus s_5 \oplus s_6$. In this context, there is a plural entity consisting of three or fewer students , namely $s_1 \oplus s_2$, that gathered. But intuitively the sentence is false. However, this might not be a problem after all. Recent work by Marty, Chemla & Spector (2015) found that in certain experimental settings, this seemingly problematic reading is indeed detected. See Spector (2014) for a pragmatic explanation of this, and related discussion.

4 Further Readings

For the unmarkedness of plural noun phrases, see Sauerland (2003), Sauerland et al. (2005), Pearson et al. (2010). Farkas & de Swart (2010) pursue a different analysis where the plural is not unmarked. There are also some works on the meanings of plural indefinites in particular: Spector (2007), Zweig (2009), and Ivlieva (2013). See Heim (2008) and Sauerland (2008) for related discussion. Among these papers, you should find Sauerland et al. (2005) and Sauerland (2008) particularly accessible.

For the semantics of quantifiers, see Scha (1981), Van der Does (1993), and Winter (2001, 2002). These tend to be a little complicated. In addition, I find van den Berg (1996) very useful in understanding the meanings of plural quantifiers, but it is highly technical (in part because it deals with dynamic semantics).

For bare plurals, Carlson (1977), Chierchia (1998), Chung & Ladusaw (2003), Diesing (1992) and Van Geenhoven (1998) are major works, as mentioned above. There are also some overview articles: Delfitto (2005) and Dayal (2011).

There is a lot of recent studies on numerals, especially modified numerals, starting from Krifka (1999). See for instance, Hackl (2000), Takahashi (2006), Nouwen (2010), Geurts & Nouwen (2007), Mayr (2013), Schwarz (2016), and the works cited in these papers. There is also a very useful overview article, Spector (2014). Also see Marty et al. (2015) for an experimental work on the 'weak' reading of certain modified numerals mentioned above.

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