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1 Conservativity

Determiners denote functions of type $\langle et, \langle et, t\rangle\rangle$. We can talk about a number of different formal/mathematical properties of such functions, but it is not immediately clear which of them are of interest for analyses of linguistic phenomena. In this lecture, we will focus on one formal property, *conservativity*.

Conservativity is defined as (1).

(1) A function $Q \in D_{\langle et,\langle et,t\rangle\rangle}$ is *conservative* iff for any functions $f, g \in D_{\langle e,t\rangle}$, $Q(f)(g) = Q(f)([\lambda x \in D_e.\ f(x) = 1 \text{ and } g(x) = 1])$.

This states that with a conservative determiner Q, replacing g with $[\lambda x \in D_e.\ f(x) = g(x) = 1]$ does not matter for the overall truth-conditions. It is perhaps easier to understand the idea behind conservativity, when we state it in terms of sets. In particular, notice that:

$$\text{set}([\lambda x \in D_e.\ f(x) = 1 \text{ and } g(x) = 1]) = \text{set}(f) \cap \text{set}(g)$$

So, if Q is conservative, you can replace set(g) with set(f) $\cap$ set(g), without affecting the overall denotation. This means that in order to determine whether $Q(f)(g)$ is true or false, you need not look at those entities that are not in set(f). Or in other words, all that matters is the entities in set(f).

Let us take a couple of concrete examples. 'Every' happens to be conservative. Recall its set denotation:

(2) $[\![\text{every}]\!]^M = [\lambda f \in D_{\langle e,t\rangle}.[\lambda g \in D_{\langle e,t\rangle}.\ \text{set}(f) \subseteq \text{set}(g)]]$

Every expresses the subset relation. Replacing set(g) with set(f) $\cap$ set(g) will not change anything because of the following fact (I omit a proof here, but try to see why this is the case):

$$\text{set}(f) \subseteq \text{set}(g) \quad \text{iff} \quad \text{set}(f) \subseteq (\text{set}(f) \cap \text{set}(g))$$

So we can actually state the lexical entry for *every* as follows:

(3) $[\![\text{every}]\!]^M = [\lambda f \in D_{\langle e,t\rangle}.[\lambda g \in D_{\langle e,t\rangle}.\ \text{set}(f) \subseteq \text{set}(g) \cap \text{set}(f)]]$

Consequently, in evaluating the truth of '$[\![\text{every}]\!]^M(f)(g)$', all you need to look at is the entities in set(f). If all of them are also in set(g), the sentence is true; if not, the sentence is false. The entities that are not in set(f) simply don't matter.

More concretely, in order to determine whether (4) is true or false, all you need to look at is the linguists. Non-linguists do not matter for the truth/falsity of this sentence.

(4) Every linguist smokes.

Let us next consider 'no'. $[\![\text{no}]\!]^M$ is also conservative. Recall its set denotation:

(5) $[\![\text{no}]\!]^M = [\lambda f \in D_{\langle e,t\rangle}.[\lambda g \in D_{\langle e,t\rangle}.\ \text{set}(f) \cap \text{set}(g) = \varnothing]]$

We can replace set(g) with set(f) $\cap$ set(g), because of the following equivalence:

$$\text{set}(f) \cap \text{set}(g) = \varnothing \text{ iff } \text{set}(f) \cap (\text{set}(f) \cap \text{set}(g)) = \varnothing$$

Again, this means that for the truth of $[\![\text{no}]\!]^M(f)(g)$, those entities outside of set(f) are irrelevant. More concretely, in order to evaluate the truth of (6), information about non-linguists is unnecessary. You can just zoom in on the linguists and check if any of them smoke.

(6) No linguist smokes.

Thus, the idea behind conservativity is this: if a determiner is conservative, the first argument–i.e. the NP denotation–determines the 'domain' that the sentence is about. The entities outside of this domain do not matter for the truth or falsity of the sentence.

1.1 Conservativity Universal

Conservativity is of interest for linguistics, because it seems that all determiners in natural languages are conservative. This hypothesis is called the *Conservativity Universal*.

(7) *Conservativity Universal*:
All determiners in natural languages denote conservative functions of type $\langle et, \langle et, t\rangle\rangle$.

Indeed, $[\![\text{some}]\!]^M$, $[\![\text{most}]\!]^M$, $[\![\text{exactly three,}]\!]^M$ etc. are also conservative (in order to see this, you should ask yourself "Do I need to check entities outside of set($[\![\text{NP}]\!]^M$)?").

It should be stressed that the Conservativity Universal is only about determiners. In fact, there are non-determiners that denote non-conservative functions. The most famous among such cases is 'only' as in (8).

(8) Only linguists are smokers.

In order to evaluate the truth of this sentence, you clearly need to look at non-linguists. If there are non-linguists who smoke, the sentence is false. This does not mean, however, that the Conservativity Universal is false, because the word 'only' is arguably not a determiner. Although it looks like one in (8), 'only' has a much wider distribution than determiners, as illustrated by the following examples.

(9) a. Only John and Mary are dating.
b. John is only 20 years-old.
c. Mary will come to the party, only if John doesn't come.

Real determiners cannot appear in these positions.

It is also instructive to think about hypothetical non-conservative determiners that are conceivable but do not seem to exist. For example, it seems to be natural to have a determiner denotation that says that the size of set(f) is smaller than the size of set(g).

(10) $[\lambda f \in D_{\langle e,t\rangle}.[\lambda g \in D_{\langle e,t\rangle}.\ |\text{set}(f)| < |\text{set}(g)|]]$

This function is not conservative, because when set(g) is replaced with set(f) $\cap$ set(g), it will mean something else (Exercise: show that the two versions of the determiner mean different things). Or to put it differently, to evaluate the truth of this sentence, you need to check whether there are entities outside of set(f) that belong to set(g). This is arguably an intuitively natural meaning to express—namely, comparison of numerosities—but there seems to be no determiner that denotes it in any language (although you of course cannot prove the non-existence).

1.2 Potential Counter-example: ***Many*** and ***Few***

That said, there are potential counter-examples, namely 'many' and 'few'. These two determiners are quite peculiar and have three readings, and crucially, one of the readings seems to be non-conservative. Let us go through the readings one by one.

1. The most prominent reading of 'many' and 'few' is the *cardinality reading*, which is about the number of individuals. For instance, consider (11).

   (11) a. Many linguists smoke.
   b. Few linguists smoke.

   Very roughly, (11a) means the number of linguists who smoke is large, where what counts as large is inherently vague and context dependent (just like the meanings of adjectives like 'rich' and 'tall' are). (11b) says the opposite: the number of linguists who smoke is small. Again, what counts as a small number is vague and contextually determined. Putting vagueness/contextual dependency aside, these cardinal readings are conservative: If you know what counts as large/small, you only need to look at linguists to determine the truth/falsity of the sentences in (11).[1]

2. 'Many' and 'few' also have a reading that concerns proportions, the *proportional reading*. This reading is facilitated when the partitive structure is used, as in (12).

   (12) a. Many of the Lichtenschteiners are car-owners.
   b. Few of the Chinese are car-owners.

   Here are some facts relevant for the truths of these sentences. Roughly, about 80% of the Lichtenschteiners own cars, while only 10% of the Chinese do. However, since the population of Lichtenschtein is only 37,000, there are only about 30,000 car-owners in Lichtenstein. On the other hand, the population of China being huge, the number of car-owners is staggering 155,000,000!
   Thus, the cardinality readings of the sentences in (12) are false (although (12a) might be true in some contexts where 30,000 is large enough). Nonetheless, the sentences are judged true according to these numbers. These sentences have readings that are about the proportions, rather than the cardinality. Specifically, (12a) means that the proportion of car-owners among the Lichtensteiners, (13a), is large, and (12b) means that the proportion of car-owners among the Chinese, (13b), is small.

   (13) a. $\dfrac{|\{\, x \mid x \text{ is a Lichtenschteiner car-owner} \,\}|}{|\{\, x \mid x \text{ is a Lichtenschteiner} \,\}|}$

   b. $\dfrac{|\{\, x \mid x \text{ is a Chinese car-owner} \,\}|}{|\{\, x \mid x \text{ is a Chinese} \,\}|}$

   The proportional readings of 'many' and 'few' are also conservative (but see the caveat

[1] A complication here is that in order to determine what counts as large/small, you might have to look at non-linguists. It is, however, not entirely clear how exactly the contextual standard is determined, and it is even less clear whether the procedure to determine the contextual standard is semantically encoded. Conservativity being a property of the lexical semantic representation of the determiner meanings, one could insist that the context-sensitivity does not make 'many' and 'few' non-conservative under the cardinality reading, assuming how to determine the contextual standard is not part of the lexical semantics of the determiners.

in the footnote). That is to say, if one knows the contextual standard for 'large' and 'small', one need not look at car-owners in other countries to evaluate the truths of these sentences.

3. The non-conservative reading that 'many' and 'few' give rise to also has to do with proportions as well, but is distinct from the propositional reading. To illustrate, consider the sentences in (14).

    (14) a. Many Suedes are Nobel laureates.
    b. Few Japanese applied to UCL.

    There is a reading of (14a) that means "The proportion of Nobel laureates among the Suedes is high". Similarly (14b) can mean "The proportion of Japanese applicants among all the UCL applicants is low". So these are also proportional readings. (13a) says that the proportion in (15a) is large, while (14a) is says the proportion in (15b) is small.

    (15) a. $$\frac{|\{\, x \mid x \text{ is a Swedish Nobel laureate} \,\}|}{|\{\, x \mid x \text{ is a Nobel laureate} \,\}|}$$
    b. $$\frac{|\{\, x \mid x \text{ is a Japanese UCL applicant} \,\}|}{|\{\, x \mid x \text{ is a UCL applicant} \,\}|}$$

    These readings have one crucial difference from the proportional readings represented in (13) above. In (13), the denominator is the NP-denotation, while in (15), the denominator is the VP-denotation. Schematically, (16a) is the fraction that the proportional reading illustrated by (12) is about, and (16b) is the fraction that the proportional reading illustrated by (14) is about.

    (16) a. $$\frac{|\text{set}([\![\text{NP}]\!]^M) \cap \text{set}([\![\text{VP}]\!]^M)|}{|\text{set}([\![\text{NP}]\!]^M)|}$$
    b. $$\frac{|\text{set}([\![\text{NP}]\!]^M) \cap \text{set}([\![\text{VP}]\!]^M)|}{|\text{set}([\![\text{VP}]\!]^M)|}$$

    Notice furthermore that one can replace $\text{set}([\![\text{VP}]\!]^M)$ in (16a) with $\text{set}([\![\text{NP}]\!]^M) \cap \text{set}([\![\text{VP}]\!]^M)$ without affecting the truth-conditions, because the following equivalence holds:

    $$\text{set}([\![\text{NP}]\!]^M) \cap \text{set}([\![\text{VP}]\!]^M) = \text{set}([\![\text{NP}]\!]^M) \cap (\text{set}([\![\text{NP}]\!]^M) \cap \text{set}([\![\text{VP}]\!]^M))$$

    This means that (16a) is conservative.

    By contrast, replacing the two occurrences of $\text{set}([\![\text{VP}]\!]^M)$ with $\text{set}([\![\text{NP}]\!]^M) \cap \text{set}([\![\text{VP}]\!]^M)$ in (16b) would result in a different reading. Specifically, since the denominator will be also $\text{set}([\![\text{NP}]\!]^M) \cap \text{set}([\![\text{VP}]\!]^M)$, it will always be 1!! Therefore this reading is not conservative. Or to put it differently, in order to evaluate the truth of the Swedish Nobel prize example in (14a), one clearly needs to look at non-Suedes, because one needs to know the number of all Nobel laureates to compute the denominator. Similarly for the example in (14b), one needs to look at the non-Japanese applicants to UCL.

So, the third reading, which is sometimes called the *reverse proportional reading* of 'many' and 'few' seem to be problematic for the Conservativity Universal. However, researchers have noticed that reverse proportional readings of 'many' and 'few' have some peculiar properties, e.g. they seem to require a particular type of intonation. Based on this, it has been claimed that these determiners actually always have conservative denotations, but due to the interactions with other factors such as intonation (and its semantic correlates like focus-topic), the resulting meaning looks as if it is non-conservative. Since we cannot discuss the details of such analyses in this course, we will leave this issue open here (but you are encouraged to look into this topic for your essay).