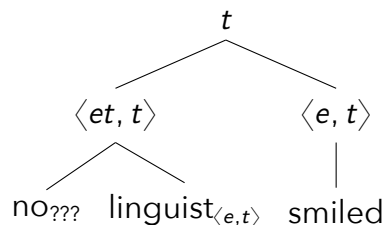


1 Determiners

From the above analysis of quantificational DPs as generalized quantifiers, we can deduce the denotations of determiners. Recall that the denotations of nouns like *linguist* are of type $\langle e, t \rangle$, so we have the following semantic types.



What is the semantic type of 'no' here? There are two options:

- We can apply Functional Application, if $\llbracket \text{no} \rrbracket^M$ is an entity, so is of type e , but the output won't be of type $\langle et, t \rangle$. so this is not a viable option.
- The other possibility is that $\llbracket \text{no} \rrbracket^M$ is a function that takes $\llbracket \text{linguist} \rrbracket^M$ and returns the generalized quantifier $\llbracket \text{no linguist} \rrbracket^M$, i.e.

$$(1) \quad \llbracket \text{no linguist} \rrbracket^M = \llbracket \text{no} \rrbracket^M(\llbracket \text{linguist} \rrbracket^M)$$

This means that $\llbracket \text{no} \rrbracket^M$ is of type $\langle et, \langle et, t \rangle \rangle$.

Now, let's figure out which function of type $\langle et, \langle et, t \rangle \rangle$ it is. We already know $\llbracket \text{no linguist} \rrbracket^M$ and $\llbracket \text{linguist} \rrbracket^M$. Substituting these in the above equation (1), we get:

$$\begin{aligned}
 [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff for no linguist } x \text{ in } M, f(x) = 1] &= \llbracket \text{no} \rrbracket^M(\llbracket \text{linguist} \rrbracket^M) \\
 &= \llbracket \text{no} \rrbracket^M([\lambda x \in D_e. 1 \text{ iff } x \text{ is a linguist in } M])
 \end{aligned}$$

In words, $\llbracket \text{no} \rrbracket^M$ takes the type- $\langle e, t \rangle$ function $[\lambda x \in D_e. 1 \text{ iff } x \text{ is a linguist in } M]$ and returns the generalized quantifier on the left of $=$. Abstracting over this particular type- $\langle e, t \rangle$ function, we get the following as the denotation of 'no'.

$$(2) \quad \text{For any model } M, \\ \llbracket \text{no} \rrbracket^M = [\lambda g \in D_{\langle e, t \rangle}. [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff for no individual } x \text{ such that } g(x) = 1, f(x) = 1]]$$

Here g is the NP-denotation and f is the VP-denotation. g determines which individuals the generalized quantifier will be about, namely the individuals that g maps to 1. Call these individuals g -individuals. The determiner says how many of the g -individuals f needs to map to 1 to make the sentence true. In the case of 'no', when f maps zero g -individuals to 1, the sentence will be true.

It is easy to generalize this analysis to other quantificational determiners.

$$(3) \quad \text{For any model } M, \\ \begin{array}{l} \text{a. } \llbracket \text{every} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff for every individual } x \text{ such that } \right. \right. \\ \left. \left. g(x) = 1, f(x) = 1 \right] \right] \\ \text{b. } \llbracket \text{some} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff for some individual } x \text{ such that } \right. \right. \\ \left. \left. g(x) = 1, f(x) = 1 \right] \right] \end{array}$$

$$c. \quad \llbracket \text{most} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for most individuals } x \text{ such that} \\ g(x) = 1, f(x) = 1 \end{array} \right] \right]$$

All of these determiners say how many g -individuals f needs to map to 1 to make the sentence true. Specifically, (3a) says the sentence will be true if f maps all g -individuals to 1; (3b) says the sentence will be true if f maps (at least) some g -individuals to 1; (3c) says the sentence will be true if f maps most g -individuals to 1.

Note that for 'every', the first argument g will be a singular noun, while for 'most', it is a plural noun, and for 'some', it can be either.

- (4) a. Every linguist is tall.
 b. Some linguist is tall.
 c. *Most linguist is tall.
- (5) a. *Every linguists are tall.
 b. Some linguists are tall.
 c. Most linguists are tall.

We are disregarding the difference between singular and plural nouns for now, because we haven't really discussed the semantics of plural nouns. We will deal with this issue in the second half of the course, so for now, let's make the following (unrealistic) assumption:

$$(6) \quad \llbracket \text{linguist} \rrbracket^M = \llbracket \text{linguists} \rrbracket^M = [\lambda x \in D_e. x \text{ is a linguist in } M]$$

We need to eventually say something about nominal number, because it clearly has semantic consequences. Simply put, the singular noun 'linguist' is about one individual, while the plural noun 'linguists' is (typically) about more than one individual.

2 Sets and Their Characteristic Functions

When talking about generalized quantifiers, the notion of *characteristic functions* becomes handy. Take a function of type $\langle e, t \rangle$, say $\llbracket \text{smokes} \rrbracket^M$.

$$(7) \quad \llbracket \text{smokes} \rrbracket^M = [\lambda x \in D_e. 1 \text{ iff } x \text{ smokes in } M]$$

This function maps anybody in M to 1 if he or she smokes in M , and to 0 if not. So it divides the inhabitants of M into two groups, smokers and non-smokers. We can represent them as the following sets:

- (8) a. $\{x \mid x \in D_e \text{ and } x \text{ smokes in } M\}$
 b. $\{x \mid x \in D_e \text{ and } x \text{ doesn't smoke in } M\}$

$\llbracket \text{smokes} \rrbracket^M$ maps the individuals in (8a) to 1 and the individuals in (8b) to 0.

In this situation, we say $\llbracket \text{smokes} \rrbracket^M$ *characterizes* the set in (8a). Or equivalently, we say $\llbracket \text{smokes} \rrbracket^M$ is the *characteristic function* of the set in (8a).

Generally, any function of type $\langle e, t \rangle$ characterizes some set. More precisely, any function f of type $\langle e, t \rangle$ characterizes the set $\{x \mid x \in D_e \text{ and } f(x) = 1\}$. Generalizing this further, we can say that each function f of type $\langle \sigma, t \rangle$ characterizes the set $\{x \mid x \in D_\sigma \text{ and } f(x) = 1\}$ for

any semantic type σ . Keep in mind that only functions of type $\langle \sigma, t \rangle$ characterize sets. So, for instance, functions of type $\langle e, \langle e, t \rangle \rangle$, like $\llbracket \text{loves} \rrbracket^M$, do not characterize sets. The output type needs to be t , i.e. the function needs to return a truth-value.

Correspondingly, when a set whose members are all of the same type is given, e.g. a set of individuals, we can tell which function characterizes the set. For instance, take a set of individuals $\{a, b, c\}$. This is (by assumption) a set whose members are all of type e . This set is characterized by the following type- $\langle e, t \rangle$ function: $[\lambda x \in D_e. 1 \text{ iff } x = a \text{ or } x = b \text{ or } x = c]$. More generally, any set of individuals S is characterized by a function of type $\langle e, t \rangle$, namely $[\lambda x \in D_e. 1 \text{ iff } x \in S]$. Generalizing over semantic types, any set S whose members are all of type σ is characterized by the function of type $\langle \sigma, t \rangle$, $[\lambda x \in D_\sigma. 1 \text{ iff } x \in S]$.¹

It is important to notice that when any function of type $\langle \sigma, t \rangle$ is given (for any semantic type σ), we can uniquely determine the set it characterizes. If f is a function of type $\langle \sigma, t \rangle$, the set it characterizes is $\{x \mid x \in D_\sigma \text{ and } f(x) = 1\}$. It doesn't characterize any other set. Furthermore, for any set whose members are of the same type, there is a unique function that characterizes it. Specifically, for a set S whose members are all of type σ , i.e. $S \subseteq D_\sigma$, its characteristic function is $[\lambda x \in D_\sigma. 1 \text{ iff } x \in S]$, and no other function of type $\langle \sigma, t \rangle$ characterizes S . Notice also that for any function f of type $\langle \sigma, t \rangle$, the characteristic function of the set f characterizes is f itself.

This means that there is a *one-to-one correspondence* between functions of type $\langle \sigma, t \rangle$ and the sets they characterize. If a function f of type $\langle \sigma, t \rangle$ is given, it can be uniquely determined which set f characterizes and from that set, the function f can be reconstructed, using the above recipe. This is important, because it means that these two kinds of objects—functions and sets—carry the same amount of information. That is to say, although formally distinct, they are indistinguishable in some sense, and we can treat them as the 'same thing' for certain purposes.

More concretely, we can regard the denotation of 'smokes', $\llbracket \text{smokes} \rrbracket^M$, as either a function of type $\langle e, t \rangle$ or alternatively, as the set in (8a). These things are formally distinct, but one can be recovered from the other. Likewise, recall that we defined $\llbracket \text{boy} \rrbracket^M$ and $\llbracket \text{blond} \rrbracket^M$ as functions of type $\langle e, t \rangle$, but alternatively we can treat them as the set of boys in M , and the set of blond people in M , respectively.

The one-to-one correspondence between sets and their characteristic functions guarantees that such 'set talk' is harmless, because we are not losing any information by converting type- $\langle e, t \rangle$ functions into sets and vice versa. But why do we do this? Because it allows us to look at the same thing from a different angle and it can be quite informative, as we will see below.

Before moving on, let us introduce some notations. For any function f of type $\langle \sigma, t \rangle$, we denote the set it characterizes by $\text{set}(f)$. Similarly, for any set S such that $S \subseteq D_\sigma$, we denote its characteristic function by $\text{func}(S)$. To stress the main point, we can regard f and $\text{set}(f)$ as the 'same thing', because $f = \text{func}(\text{set}(f))$, and similarly, S and $\text{func}(S)$ as the 'same thing' because $S = \text{set}(\text{func}(S))$.

¹We will not talk about sets whose members are not of a uniform type. Technically it is possible to define characteristic functions of such sets, but such sets and their characteristic functions do not play a role in our semantic theory.

3 Determiners as Relations between Sets

Recall the denotations of quantificational determiners.

- (9) For any model M ,
- a. $\llbracket \text{every} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for every individual } x \text{ such that} \\ g(x) = 1, f(x) = 1 \end{array} \right] \right]$
 - b. $\llbracket \text{no} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for no individual } x \text{ such that} \\ g(x) = 1, f(x) = 1 \end{array} \right] \right]$
 - c. $\llbracket \text{some} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for some individual } x \text{ such that} \\ g(x) = 1, f(x) = 1 \end{array} \right] \right]$
 - d. $\llbracket \text{most} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for most individuals } x \text{ such that} \\ g(x) = 1, f(x) = 1 \end{array} \right] \right]$

Let's re-state these in terms of sets. The determiners themselves are of type $\langle et, \langle et, t \rangle \rangle$, so they don't characterize sets. But their arguments are of type $\langle e, t \rangle$, so they characterize sets of individuals.

Take the denotation of 'every' in (9a). We can rewrite it using sets as follows.

- (10) For any model M ,
- $$\llbracket \text{every} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for every individual } x \text{ such that} \\ x \in \text{set}(g), x \in \text{set}(f) \end{array} \right] \right]$$

This is not so different from the representation in (9a). But now notice that this is essentially saying that $\text{set}(g)$ is a subset of $\text{set}(f)$ (Recall: set A is a subset of set B iff every member of A is a member of B). So we can write the denotation more economically as follows.

- (11) For any model M ,
- $$\llbracket \text{every} \rrbracket^M = [\lambda g \in D_{\langle e, t \rangle}. [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \text{set}(g) \subseteq \text{set}(f)]]$$

That is to say, 'every' expresses a subset relation between two sets. This might sound a bit surprising, but it intuitively makes sense. If you take a concrete example like 'Every linguist smiled', its truth-condition can be paraphrased as 'The set of linguists is a subset of the set of people who smiled'.

Let's now reanalyze 'some' using sets. First, take the 'function-talk' denotation in (3b) and convert the functions into sets as follows.

- (12) For any model M ,
- $$\llbracket \text{some} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for some individual } x \text{ such that} \\ x \in \text{set}(g), x \in \text{set}(f) \end{array} \right] \right]$$

This means that $\text{set}(g)$ and $\text{set}(f)$ have some member in common. So using symbols from Set Theory, we can re-state it as:

- (13) For any model M ,
- $$\llbracket \text{some} \rrbracket^M = [\lambda g \in D_{\langle e, t \rangle}. [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \text{set}(g) \cap \text{set}(f) \neq \emptyset]]$$

Take a concrete example, say, 'Some boy is blond'. This is indeed paraphrased by 'The set of boys and the set of blond people have a non-empty intersection.'

'No' is essentially the converse of 'some'. It says that the intersection is empty.

$$(14) \text{ For any model } M, \\ \llbracket \text{no} \rrbracket^M = [\lambda g \in D_{\langle e,t \rangle}. [\lambda f \in D_{\langle e,t \rangle}. 1 \text{ iff } \text{set}(g) \cap \text{set}(f) = \emptyset]]$$

Let us take a concrete example, say, 'No semanticist is left-handed'. This is the same as 'The intersection of the set of semanticists and the set of left-handed people is empty'.

The meaning of 'most' is more complex, but it is possible to express it in terms of sets as well. Let us first simply re-write the denotation in (3c) using sets:

$$(15) \text{ For any model } M, \\ \llbracket \text{most} \rrbracket^M = \left[\lambda g \in D_{\langle e,t \rangle}. \left[\lambda f \in D_{\langle e,t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for most individuals } x \text{ such that} \\ x \in \text{set}(g), x \in \text{set}(f) \end{array} \right] \right]$$

Now take a concrete example, 'Most dogs are brown'. When is this true? In terms of sets, it means: 'The majority of members of the set of dogs are also members of the set of brown things.' That is, more members of the set of dogs are in the set of brown things than not. We can express this using symbols from Set Theory as follows. Recall that $|A|$ is the cardinality of the set A , which is the number of distinct members of A , and $A - B$ is the complement of A relative to B , defined as $\{x \mid x \in A \text{ and } x \notin B\}$.

$$|\{x \mid x \text{ is a dog}\} \cap \{x \mid x \text{ is brown}\}| > |\{x \mid x \text{ is a dog}\} - \{x \mid x \text{ is brown}\}|$$

The left-hand side of $>$ is the cardinality of the set of brown dogs, and the right-hand side is the cardinality of the set of non-brown dogs. Now using this, we can re-write (15) as follows.

$$(16) \text{ For any model } M, \\ \llbracket \text{most} \rrbracket^M = [\lambda g \in D_{\langle e,t \rangle}. [\lambda f \in D_{\langle e,t \rangle}. 1 \text{ iff } |\text{set}(g) \cap \text{set}(f)| > |\text{set}(f) - \text{set}(g)|]]$$

In words, $|\text{set}(g) \cap \text{set}(f)|$ is the number of common members of g and f . In our example, this is the number of brown dogs. $|\text{set}(f) - \text{set}(g)|$ is the number of members of f that are not in g . In our example, this is the number of dogs that are not brown. And the inequality says that the former number is greater than the latter.

We can write the same relationship in different ways. For instance, the following means the same thing as above.

$$(17) \text{ For any model } M, \\ \llbracket \text{most} \rrbracket^M = \left[\lambda g \in D_{\langle e,t \rangle}. \left[\lambda f \in D_{\langle e,t \rangle}. 1 \text{ iff } |\text{set}(g) \cap \text{set}(f)| > \frac{|\text{set}(f)|}{2} \right] \right]$$

If the number of brown dogs is more than half the number of all dogs, the majority of the dogs must be brown. So this is actually the same statement as before. You can further transform it to:

$$(18) \text{ For any model } M, \text{ and any assignment } a, \\ \llbracket \text{most} \rrbracket^M = \left[\lambda g \in D_{\langle e,t \rangle}. \left[\lambda f \in D_{\langle e,t \rangle}. 1 \text{ iff } \frac{|\text{set}(g) \cap \text{set}(f)|}{|\text{set}(f)|} > \frac{1}{2} \right] \right]$$

These are all equivalent.

A note on the meaning of ‘most’: According to the present analysis, it means the same thing as ‘more than half’, but you might have a quibble about that. In fact, if 53 out of 100 dogs are brown, it’s a bit strange to say ‘Most of the dogs are brown’, if not outright false, while ‘More than half of the dogs are brown’ sounds true true. So ‘most’ and ‘more than half’ should not mean the exact same thing. Intuitively, ‘most’ seems to require the fraction $\frac{|\text{set}(g) \cap \text{set}(f)|}{|\text{set}(f)|}$ to be much larger than $\frac{1}{2}$. But at the same time, we do not have clear intuitions about exactly which fraction it needs to be larger than. We just can’t really name such a threshold fraction for ‘most’. Rather, it seems to be inherently vague somehow. Our semantic system so far has no resources to deal with vague expressions like this, because in our semantics, every statement is clearly either true or false. To deal with this and related issues of *vagueness*, we need to enrich our model. Several such ideas have been proposed, but this is outside of the scope of this course (but would be a good thesis/Long Essay project!).

4 Summary

We analyzed quantificational DPs as Generalized Quantifiers, which are functions of type $\langle et, \langle et, t \rangle \rangle$.

- (19) For any model M ,
- $\llbracket \text{every linguist} \rrbracket^M = [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff for every linguist } x \text{ in } M, f(x) = 1]$
 - $\llbracket \text{no linguist} \rrbracket^M = [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff for no linguist } x \text{ in } M, f(x) = 1]$
 - $\llbracket \text{some linguist} \rrbracket^M = [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff for some linguist } x \text{ in } M, f(x) = 1]$
 - $\llbracket \text{most linguists} \rrbracket^M = [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff for most linguists } x \text{ in } M, f(x) = 1]$

This analysis is general enough to cover all sorts of quantificational DPs.

- (20) Let QP be a quantificational DP. Then, for any model M ,
- $$\llbracket \text{QP} \rrbracket^M = [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff for QP } x \text{ in } M, f(x) = 1]$$

Based on this, we arrived at the denotations of quantificational determiners.

- (21) For any model M ,
- $\llbracket \text{every} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for every individual } x \text{ such that} \\ g(x) = 1, f(x) = 1 \end{array} \right] \right]$
 - $\llbracket \text{no} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for no individual } x \text{ such that} \\ g(x) = 1, f(x) = 1 \end{array} \right] \right]$
 - $\llbracket \text{some} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for some individual } x \text{ such that} \\ g(x) = 1, f(x) = 1 \end{array} \right] \right]$
 - $\llbracket \text{most} \rrbracket^M = \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \begin{array}{l} \text{for most individuals } x \text{ such that} \\ g(x) = 1, f(x) = 1 \end{array} \right] \right]$

These can be re-written as follows, using sets.

- (22) For any model M ,
- $\llbracket \text{every} \rrbracket^M = [\lambda g \in D_{\langle e, t \rangle}. [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \text{set}(g) \subseteq \text{set}(f)]]$
 - $\llbracket \text{no} \rrbracket^M = [\lambda g \in D_{\langle e, t \rangle}. [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \text{set}(g) \cap \text{set}(f) = \emptyset]]$
 - $\llbracket \text{some} \rrbracket^M = [\lambda g \in D_{\langle e, t \rangle}. [\lambda f \in D_{\langle e, t \rangle}. 1 \text{ iff } \text{set}(g) \cap \text{set}(f) \neq \emptyset]]$

$$d. \llbracket \text{most} \rrbracket^M = \left[\lambda g \in D_{\langle e,t \rangle}. \left[\lambda f \in D_{\langle e,t \rangle}. 1 \text{ iff } \frac{|\text{set}(g) \cap \text{set}(f)|}{|\text{set}(f)|} > \frac{1}{2} \right] \right]$$

In the next lecture we'll discuss formal properties of these determiners, and how they might be linguistically relevant.

4.1 One Word Quantificational DPs

English has several words that function as quantificational DPs. They include:

- (23) a. somebody, everybody, nobody, anybody
 b. something, everything, nothing, anything
 c. somewhere, everywhere, nowhere, anywhere
 d. somehow, anyhow

We can analyze the ones in (23a) and (23b) as follows. Take 'somebody' as an example. It seems to be made up of the determiner 'some' and another morpheme '-body'. Essentially, 'somebody' means the same thing as 'some person'. Thus, we can keep the analysis of 'some' we came up with above, and simply analyze '-body' as meaning the same thing as person.

- (24) For any model M ,
- $$\llbracket \text{somebody} \rrbracket^M = \llbracket \text{some person} \rrbracket^M = \left[\lambda f \in D_{\langle e,t \rangle}. \begin{array}{l} 1 \text{ iff for some person } x \text{ in } M \\ f(x) = 1 \end{array} \right]$$

Similarly for other words in (23a) and (23c).

For the words in (23c) and (23d), as they (can) function as adverbs, we need a theory of how to analyze adverbs, which requires a non-trivial extension of the semantics we've been developing here, and this might be a good essay topic.

It is also interesting to notice that (23c) and (23d) involve *wh*-words like 'where' and 'how'. Cross-linguistically, it is very common to use *wh*-words to form quantificational words like these. And there must be some deep semantic reason for this. This is another intriguing topic that we need to leave open here.

4.2 Indefinite Article 'a(n)'

Here's one more open issue. Recall that in PLIN2001 Semantic Theory, we analyzed the indefinite article 'a' as a semantically vacuous item. Recall also that semantically vacuous items denote identity functions. More concretely, we gave it the following denotation.

- (25) For any model M ,
- $$\llbracket a \rrbracket^M = [\lambda f \in D_{\langle e,t \rangle}. f]$$

However, in sentences like (26), we need a different lexical entry for 'a'.

- (26) A linguist smiled.

In fact, in (26), 'a' means something very similar to 'some'.

- (27) Some linguist smiled.

Although you might feel that (27) has an extra connotation that the identity of the linguist is unknown (which is itself a very interesting phenomenon), the two sentences have very similar truth-conditions, namely, they are true if there is at least one linguist who smiled, and false otherwise. Then, we should analyze 'a' in (26) as a quantificational determiner, as in (28).

$$\begin{aligned}
 (28) \quad & \text{For any model } M, \\
 \llbracket a_{\text{det}} \rrbracket^M &= \left[\lambda g \in D_{\langle e, t \rangle}. \left[\lambda f \in D_{\langle e, t \rangle}. \begin{array}{l} 1 \text{ iff for an individual } x \in D_e \\ \text{such that } g(x) = 1, f(x) = 1 \end{array} \right] \right] \\
 &= [\lambda g \in D_{\langle e, t \rangle}. [\lambda f \in D_{\langle e, t \rangle}. \text{set}(g) \cap \text{set}(f) \neq \emptyset]]
 \end{aligned}$$

Having two lexical entries for 'a' is certainly theoretically undesirable, so we should try to explain away one of them using the other. We will not attempt to do it here, but you could maybe try it in your essay. It is also noticeable in this connection that in many languages (though not in English), 'a' in a predicative NP like (29) is either optional or completely absent.

(29) Alice is a linguist.