APPENDIX

A computer model of the Land Mondrian Retinex experiment

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In the Land Mondrian retinex experiment (Land, 1974) a multicoloured abstract scene consisting of rectangular patches is illuminated using three projectors. Each projector is fitted with a coloured filter which passes either predominantly long-wave, medium-wave or short-wave light. The intensity of light from each projector may be independently varied. Constructing a computerised version of this experiment requires two steps; firstly, the colour of each patch must be calculated given the intensities of the three projectors and secondly that colour must be emulated on the graphics output device of the computer (cathode ray tube (CRT) or liquid crystal display (LCD) projector).

XYZ values for each of the three illuminators (long, medium and short-wave) can be calculated separately and then added together to give a combined XYZ for the patch. Using this method the XYZ tristimulus values for any patch can be calculated for any combination of intensities of the three illuminators.

Measuring the full reflectance and radiant spectra as described above has the advantage that XYZ tristimulus values can be generated for any combination of patch and illuminator. However, it is sufficient to measure just the XYZ tristimulus values for each patch in turn when each is lit in turn by each illuminator at maximum intensity.

Emulation of a particular colour on a computer graphics output device

The computer graphics output devices we used employ three variable intensity light sources to generate colours. In the case of a CRT monitor there are three coloured phosphors, red, green and blue (the RGB elements). In the case of LCD projectors there are red, green and blue elements in the LCD panel. In both cases the intensities of the RGB elements may be controlled directly by the computer.

For the purposes of this study two assumptions were made about the RGB elements. Firstly, that the colour of each element does not change for different intensities (i.e. the xyz chromaticity co-ordinates remain constant at different intensities). Second, that the elements are independent so that the output of a particular red element for example is constant regardless of the output of surrounding elements.

Given a reflectance spectrum for a patch and a radiant spectrum for an illuminator, it is a simple matter to calculate the spectrum that would be reflected. Using tables it is possible to convert this spectrum into 1931 CIE XYZ tristimulus values (X, Y and Z). These contain information about both intensity and colour, and can be converted into the CIE xyz chromaticity co-ordinates, which are independent of the intensity, as follows:

\[ x = \frac{X}{X+Y+Z}, \quad y = \frac{Y}{X+Y+Z}, \quad z = \frac{Z}{X+Y+Z}. \]

The required colour matching functions are tabulated in Wyszecki & Stiles (1982), Table I (3.3.1).

The XYZ values thus obtained are for the respective illuminator at maximum intensity. If the illuminator were at 50% brightness the resulting XYZ values would be at 50% of these values. Thus the XYZ values of the patch can be calculated in simple proportion for any required brightness of that illuminator. Moreover, the XYZ values are additive so that the

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The XYZ tristimulus values of each of the individual RGB elements at peak intensity were measured. Let these be:

- $X_r, Y_r, Z_r$ for the red element
- $X_g, Y_g, Z_g$ for the green element
- $X_b, Y_b, Z_b$ for the blue element

The RGB elements can have different intensities denoted by $I_r$, $I_g$, $I_b$, where $I$ varies linearly from 0 to 1. The resulting tristimulus values for the RGB elements combined would be:

- $X = X_r I + X_g I_g + X_b I_b$
- $Y = Y_r I + Y_g I_g + Y_b I_b$
- $Z = Z_r I + Z_g I_g + Z_b I_b$

Equation E1 - relationship between RGB intensity and output XYZ

The tristimulus values of each of the individual RGB elements are known and so the three simultaneous linear equations above can be solved for any required value of XYZ, giving the required intensities for the three RGB elements $I_r$, $I_g$, $I_b$.

The next step was to produce these required intensities under program control. Usually there is a graphics command on the computer to set the individual colour levels for the RGB elements. On the Macintosh they can be set on a scale from 0 to 65535. The relationship between colour level (e.g. 0 to 65535 for the Macintosh) and intensity on the graphics output device is usually non-linear and is commonly known as a gamma function (not related to statistical gamma distributions). A typical gamma function for a monitor is shown in Appendix Figure 4. The gamma functions for the three RGB elements may be obtained using a light meter (although we used the PR650 SpectraColorimeter). It is not usually necessary to take more than about 64 calibration points along the curve; spline interpolation between these points should be accurate enough for the intervening areas.

Problems arising from the model

Two problems may arise from the solution of equation E1 above:

1/ One or more of the intensities $I_r$, $I_g$ or $I_b$ is negative.

This means that the graphics output device is unable to generate the required colour. The device can only generate colours that lie within the triangle formed by the CIE 1931 chromaticity co-ordinates of the RGB elements of the device; the XYZ tristimulus values have chromaticity co-ordinates which lie outside that triangle.

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2/ One or more of the intensities $I_r$, $I_g$ or $I_b$ is greater than one.

This means that the graphics output device is unable to generate the required colour at the required intensity; the maximum brightness of the graphics device RGB elements in the required ratio is still not enough to produce the required intensity.

Problem 1 above is difficult to deal with. One approach is to generate a colour close to requirements. This could be done by resetting the negative intensity to zero, by taking the absolute value of the minimum negative intensity and adding it to all the intensities (effectively adding white and desaturating the colour), or by calculating the closest colour on the 1931 CIE colour space which lies within the RGB element triangle. All these methods will produce an approximation to the required colour. Another approach (the one we use) is to modify the
experiment so that less saturated colours are required, colours which can be generated by the graphics device. In the case of our Land Mondrian retinex experiment we do this by limiting the minimum intensity of the three illuminators to say 20% of full intensity rather than allowing them to switch off completely. This effectively reduces the saturation of the colours that are reflected from the Mondrian patches.

Problem 2 above may be dealt with in the following way. First of all we calculate the maximum intensity $I_r$, $I_g$ or $I_b$ which may be required under any circumstances. This will be for one of the patches when lit by all three illuminators at maximum intensity. Let this value be $I_{\text{Max}}$. The inverse of this can be used as a normalisation factor. If every required intensity is always multiplied by this normalisation factor then all the resulting $I_r$, $I_g$ or $I_b$ values will be less than or equal to one and therefore they will all be able to be generated on the graphics device. All the required colours will be the same as before and all the relative intensities will also be correct, but the absolute intensities will all be dimmer by the normalisation factor.

Discrepancies between predicted and measured XYZ

The method described above produces a good emulation of the Land Mondrian retinex experiment but there are still some discrepancies between predicted and measured XYZ values, typically between 5% and 10%. This is due to failure of the two assumptions of the model: inconstancy of the hue of the RGB elements with intensity and non-independence of the RGB elements. Other factors may also have an effect; for example, the colour and intensity of a very small patch may be different from that of a very large patch even though the computer RGB levels may be identical. Also, different areas on the graphics device may have a slight colour cast or different intensity, particularly when comparing the periphery to the centre of the screen. There may also be a temporal effect; transient changes in colour may not be delivered accurately by the graphics device. This has been noted when using LCD projectors.

REFERENCES
