ON THE SIMPLEX AND INTERIOR POINT APPROACHES TO MULTI-OBJECTIVE LINEAR PROGRAMMING

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Abstract

In the last four to five decades a number of algorithms have been suggested for Multiple Objective Linear Programming (MOLP). Most are based on the simplex algorithm and interior-point methods for Linear Programming. However, objective space based methods are becoming more and more prominent. This paper investigates three algorithms namely the Multiobjective Simplex Algorithm (MSA), Arbel's Affine Scaling Interior-point (ASIMOLP) algorithm and Benson's objective space Outer Approximation (BOA) algorithm. Numerical results on non-trivial MOLP problems show that MSA works only on small scale problems. BOA, on the other hand, is superior in terms of the quality of the solutions it returns to both MSA and ASIMOLP, while ASIMOLP more than holds its own in terms of computing efficiency. Matlab implementations of these algorithms and the experimental results will be explained and discussed.

Keywords: Multiple Objective Linear Programming, Arbel's affine scaling interior MOLP algorithm, Benson's outer approximation algorithm.

Table 1: Computational results for individual problem

Algorithms				MSA		ASIMOLP		BOA	
Prob.	m	n	k	Most preferred Solutions	CPU time (s)	Most preferred Solutions	CPU time (s)	Most preferred Solutions	CPU time (s)
1	4	2	2	f1 = 12 f2 = -9	0.028	f1 = 4.86 f2 = -6.60	0.035	f1 = 12 f2 = -9	0.127
2	6	3	3	f1 = -2 f2 = 10 f3 = -5	0.046	f1 = -1.75 f2 = 5.56 f3 = -2.76	0.037	f1 = -2 f2 = 10 f3 = -5	0.143
3	8	4	3	f1 = -42.50 f2 = -42.50 f3 = -10	0.244	f1 = -28.64 f2 = -26.45 f3 = -48.81	0.076	f1 = -42.50 f2 = -42.50 f3 = -10	0.190
4	15	10	3	f1 = -51.37 f2 = -357.40 f3 = -313.84	0.388	f1 = 33.65 f2 = -285.72 f3 = -383.71	0.187	f1 = -51.37 f2 = -357.40 f3 = -313.84	0.239
5	25	15	3	f1 = -160.86 f2 = -114.00 f3 = -188.83	91.450	f1 = -107.15 f2 = -169.94 f3 = -166.26	0.211	f1 = -363.82 f2 = -33.70 f3 = -136.71	0.586
6	30	15	3	f1 = -360.62 f2 = -33.70 f3 = -136.71	17.225	f1 = -150.44 f2 = -203.65 f3 = -295.21	0.357	f1 = -362.78 f2 = -32.75 f3 = -138.88	0.619
7	35	15	3		forcefully terminated after 7days	f1 = -354.93 f2 = -220.10 f3 = -269.35	0.896	f1 = -363.82 f2 = -47.30 f3 = -136.71	1.051
8	40	20	3		forcefully terminated after 7days	f1 = -52.86 f2 = -62.94 f3 = -40.83	0.257	f1 = -101.54 f2 = -61.59 f3 = -7.99	0.727
9	50	30	3		forcefully terminated after 7days	f1 = -93.29 f2 = -114.10 f3 = -49.79	0.305	f1 = -134.00 f2 = -89.28 f3 = -14.01	1.001
10	60	40	3		forcefully terminated after 7days	f1 = -159.59 f2 = -114.21 f3 = -101.03	0.339	f1 = -248.59 f2 = -98.70 f3 = -37.65	1.557
11	65	45	3		forcefully terminated after 7days	f1 = 135.45 f2 = -118.84 f3 = -75.78	0.546	f1 = -169.34 f2 = -128.37 f3 = -70.97	2.000
12	70	50	3		forcefully terminated after 7days	f1 = -265.42 f2 = -192.32 f3 = -181.14	0.763	f1 = -414.48 f2 = -206.73 f3 = -85.49	2.825
13	75	55	3		forcefully terminated after 7days	f1 = -232.17 f2 = -241.12 f3 = -412.42	2.450	f1 = -274.86 f2 = -226.87 f3 = -515.13	12.631
14	80	60	3		forcefully terminated after 7days	f1 = -217.55 f2 = -375.40 f3 = -624.05	2.616	f1 = -45.47 f2 = -290.74 f3 = -757.53	7.540

As we can see from Table 1, the CPU times for all algorithms increase from problem 1 down to problem 14 as the problem size increases. We notice that from problem 7 to 14, the MSA could not produce result even after running for 7 days, it was forcefully terminated. Looking at the quality of solutions returned for all problems, and considering that we are solving minimization problems, we observed that BOA is superior to both ASIMOLP and MSA in terms of the quality of solution it returns while ASIMOLP outperforms the simplex and objective space algorithms in terms of computing efficiency.

Some relevant references

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