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Assessing Answer Patterns in Questionnaire / Item Response Data Using Mixtures of Rasch Models

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Outline

- Latent traits
- Rasch model
- Mixture models
- Rasch mixture models
- Application: CRAN motivation survey
- Summary

Latent traits

- Aim: Measure latent traits.
- Examples:
  - Intelligence, abilities (e.g., knowledge, teamwork).
  - Attitudes (e.g., towards strangers, the EU).
  - Responsiveness to stimuli (e.g., advertising).
- Measurement tool: Sets of items, e.g., problem solving for measuring ability, agreement with statements for measuring attitudes.
- Here: Binary items. Solve a problem yes / no, agree with a statement yes / no.
- State-of-the-art model for binary items in item response theory: Rasch model.

Rasch Model

Probability for person *i* to solve item *j*:

$$P(Y_{ij} = y_{ij} | \theta_i, \beta_j) \; = \; \frac{\exp\{y_{ij}(\theta_i - \beta_j)\}}{1 + \exp\{\theta_i - \beta_j\}}.$$

- y_{ij}: Response by person *i* to item *j*.
- θ_i: Ability of person *i*.
- β_j: Difficulty of item *j*.

By construction:

- No covariates, all information is captured by ability and difficulty.
- Both parameters θ and β are on the same scale: If $\beta_1 > \beta_2$, then item 1 is more difficult than item 2 for *all* subjects.

Central assumption of measurement invariance needs to be checked for both manifest and latent subject groups.

Rasch Model: Estimation

- Joint estimation of θ and β is inconsistent.
- Conditional ML (CML) estimation: Use factorization of the full likelihood on basis of the scores $r_i = \sum_{j=1}^{m} y_{ij}$:

$$\begin{aligned} L(\theta, \beta) &= f(y|\theta, \beta) \\ &= h(y|r, \theta, \beta) g(r|\theta, \beta) \\ &= h(y|r, \beta) g(r|\theta, \beta). \end{aligned}$$

  Estimate β from maximization of $h(y|r, \beta)$.
- Also maximizes $L(\theta, \beta)$ if $g(r|\cdot)$ is assumed to be independent of θ and β; but potentially depending on auxiliary parameters δ: $g(r|\delta)$.

Mixture Model

- Assumption: Data stems from different classes but class membership is unknown.
- Modeling tool: Mixture models.
- Mixture model = $\sum$ weight $\times$ component.
- Components represent the latent classes. They are densities or (regression) models.
- Weights are a priori probabilities for the components / classes, treated either as parameters or modeled through concomitant variables.

Rasch Mixture Model: Framework

Full mixture:

- Weights: Either (non-parametric) prior probabilities π_k or weights $\pi(k|x, \alpha)$ based on concomitant variables x, e.g., a multinomial logit model.
- Components: Conditional likelihood for item parameters and specification of score probabilities

$$f(y|\alpha, \beta, \delta) = \prod_{i=1}^{n} \sum_{k=1}^{K} \pi(k|x_i, \alpha)\; h(y_i|r_i, \beta_k)\; g(r_i|\delta_k).$$

- Estimation of all parameters via ML through the EM algorithm.

Rasch Mixture Model: Score Probabilities

- Original proposition by Rost (1990): Saturated model. Discrete distribution with parameters (probabilities) $g(r) = \Psi_r$.
- Number of parameters necessary is potentially very high: (number of items $-$ 1) $\times$ (number of components).
- More parsimonious: Assume parametric model on score probabilities, e.g., using mean and variance parameters.
- General approach: Conditional logit model encompassing the original saturated parameterization and a mean / variance parameterization (with only two parameters per component) as special cases

$$g(r|\delta) \;=\; \frac{\exp\{z_r^\top \delta\}}{\sum_{j=1}^{m-1} \exp\{z_j^\top \delta\}}.$$

Software

- Available in R in package **psychomix** at `http://CRAN.R-project.org/package=psychomix`
- Based on package **flexmix** (Grün and Leisch, 2008) for flexible estimation of mixture models.
- Based on package **psychotools** for estimation of Rasch models.
- Frick et al. (2012), provides implementation details and hands-on practical guidance. See also `vignette("raschmix", package = "psychomix")`.

CRAN Motivation: Data

Survey data from 2010 among 658 developers of R packages. Assesses psychological traits like motivation, values, and work design as well as research and R related activities.

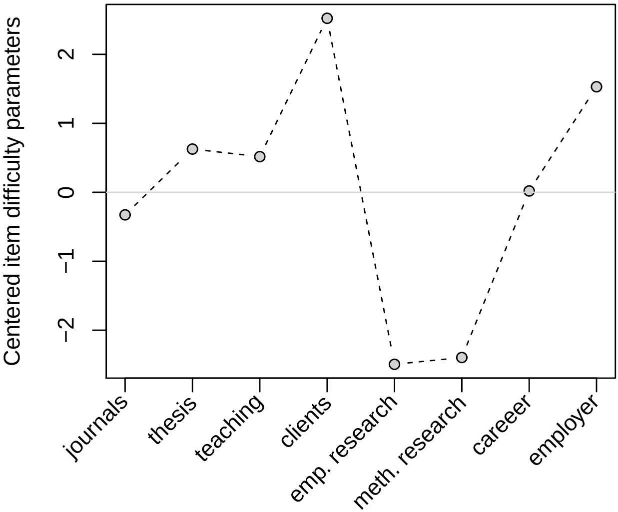
Here: Subset of 8 items measuring external regulation of motivation:

- "I can publish the packages in scientific **journals**."
- "They are part of my master / PhD **thesis**."
- "I need them for **teaching** courses."
- "I develop them for **clients** who pay me."
- "They are a byproduct of my **empirical research**. If I cannot find suitable existing software to analyze my data, I develop software components myself."
- "They are a byproduct of my **methodological research**. If I develop / extend methods, I develop accompanying software, e.g., for illustrations and simulations."
- "I expect an enhancement of my **career** from it."
- "My **employer** pays me to do so."

CRAN Motivation: Analysis

- First approach: analyze full sample with single Rasch model employing a mean-variance specification of the scores.
  Question: Is this appropriate?
- Check assumptions via mixture of Rasch models:
  Is there more than one latent class? Select number of components via BIC.
- Inspect item profiles / answer patterns in latent class(es).
- Covariates: Occupational status, PhD, job in academia.
  Can they explain class membership? Employ either ex-post or in a concomitant variable model.

CRAN Motivation: Single Rasch Model



CRAN Motivation: Mixtures of Rasch Models

Fit model:

```
R> mix <- raschmix(exReg ~ 1, data = CRAN, k = 1:4, nrep = 5,
+    scores = "meanvar")
R> mixC <- raschmix(exReg ~ academic + occupation + phd,
+    data = CRAN, k = 1:4, nrep = 5, scores = "meanvar")
```

Select model:

```
R> rbind(mix = BIC(mix), mixC = BIC(mixC))
```

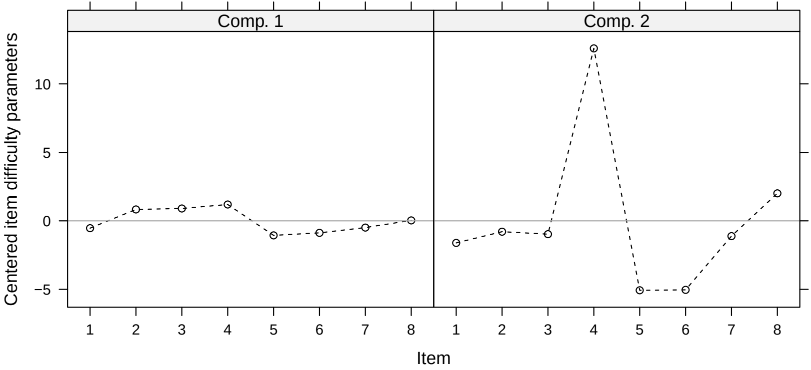
```
            1        2        3        4
mix  5222.706 5170.803 5191.648 5206.042
mixC 5222.706 5103.511 5095.773 5111.783
```

```
R> mix2 <- getModel(mix, which = "2")
R> mixC3 <- getModel(mixC, which = "3")
```

Plot item profiles and effects of concomitant variables:

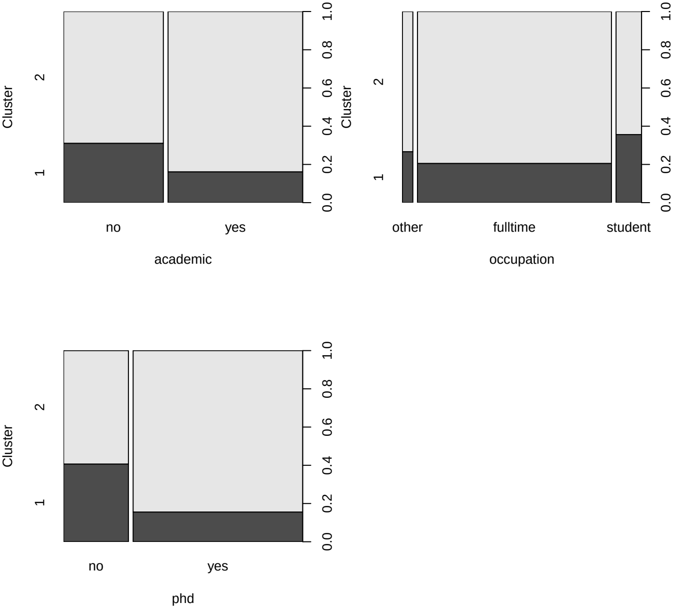
```
R> xyplot(mix2)
R> xyplot(mixC3)
R> effectsplot(mixC3)
```

CRAN Motivation: Item Profiles

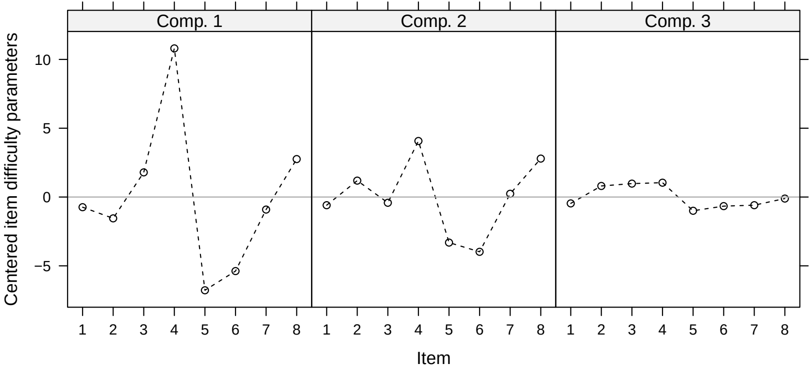


1 = journals	4 = clients	7 = career
2 = thesis	5 = emp. research	8 = employer
3 = teaching	6 = meth. research	

CRAN Motivation: Covariates



CRAN Motivation: Item Profiles



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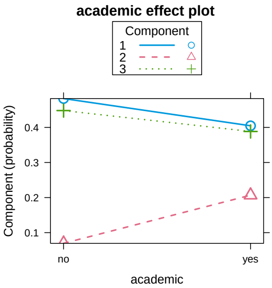
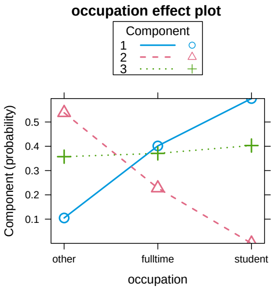
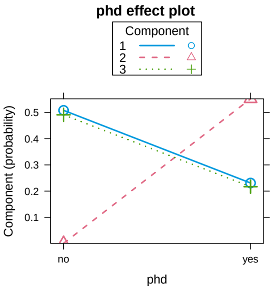
CRAN Motivation: Effects Displays







Summary

- The Rasch model is a useful tool to analyze binary questionnaire / item response data.
- Mixtures of Rasch models are a flexible means to check a necessary assumption to provide fair comparisons, i.e., assess if different answer patterns are present.
- General framework incorporates concomitant variable models for mixture weights along with various score models.
- Concomitant variable models are a convenient extension to the otherwise covariate-free Rasch model.
- Implementation of all flavors in R package **psychomix** at `http://CRAN.R-project.org/package=psychomix`

References

Frick H, Strobl C, Zeileis A (2013). “Rasch Mixture Models for DIF Detection: A Comparison of Old and New Score Specifications.” *Working Paper 2013-36*, Working Papers in Economics and Statistics, Research Platform Empirical and Experimental Economics, Universität Innsbruck.

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