1 Problem Class, 31 Oct

1.1 Question 2

(There was a slight mistake at the problem class, see correct phrasing here)
Show that an irrotational flow in an annulus with solid boundaries must be zero everywhere (Use methods from this course), if the circulation around the cylinder is zero. The annulus is given by a region between two concentric circles with radius $a < b$, i.e. $a^2 < x^2 + y^2 < b^2$.

Solution:

The streamfunction must take the form (see lectures)

$$\psi = A_0\theta + B_0 \log r + \sum_{n=1}^{\infty} \left((A_n r^n + B_n r^{-n}) \cos n\theta + (C_n r^n + D_n r^{-n}) \sin n\theta \right). \quad (1)$$

Solid body at $r = a$ means that ψ is constant at $r = a$, i.e. $A_n a^n + B_n a^{-n} = 0$, $C_n a^n + D_n a^{-n} = 0$, $A_0 = 0$ and $B_0 \log a$ is constant. Similarly, for the boundary at $r = b$, $A_n b^n + B_n b^{-n} = 0$, $C_n b^n + D_n b^{-n} = 0$, $A_0 = 0$ and $B_0 \log b$ is constant. Since $b > a$, $A_n = B_n = C_n = D_n = 0$ for all $n > 0$. The only remaining term $B_0 \log r$ vanishes because the circulation around the cylinder is zero.