1 Problem Class, 14 Nov

1.1 Question 1

Two line vortices are located in the upper half-plane with a wall at $y = 0$. The strength of the vortices are $2\pi\kappa_1$ and $2\pi\kappa_2$ and are located at $z = -d + bi, d + bi$, respectively.

(a) Find the complex velocity potential of the system by using the method of images.

(b) Suppose the vortices are free to move under the influence of the other vortex and the wall. Find the velocity of the two vortices. Deduce that they approach/get away from the wall with the same speed if and only if $\kappa_1 + \kappa_2 = 0$

(c) Suppose $\kappa_1 + \kappa_2 = 0$. Find an evolution equation for d and b.

(d) Deduce that $\dot{d} = -\partial_b H, \dot{b} = \partial_d H$, where $H = \frac{\kappa_1}{2} \log\left(\frac{db}{\sqrt{d^2+b^2}}\right)$, i.e. the system is Hamiltonian. Deduce that $r \sin 2\theta$ is a conserved quantity of the motion, where r, θ are the distance of vortex κ_2 from the origin and θ is its orientation. Sketch the trajectory of the vortices.