1 Problem Class, 21 Nov

1.1 Question 1

Let $\boldsymbol{u}$ be an incompressible flow and $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$.

Show that

$$\nabla \times (\boldsymbol{\omega} \times \boldsymbol{u}) = (\boldsymbol{u} \cdot \nabla)\boldsymbol{\omega} - (\boldsymbol{\omega} \cdot \nabla)\boldsymbol{u}. \tag{1}$$

1.2 Question 2

In a cylinder with z-axis as geometric axis, a flow satisfies $u = 4\alpha y(x^2 + y^2)$, $v = -4\alpha x(x^2 + y^2)$.

(a) Show that streamlines are circles.

(b) Assume steady, homogeneous flow (constant density). By using Euler's equations with gravity as the sole external forcing, find the pressure in Cartesians.

(c) The height of the water surface at $r = 0$ is h. Determine the surface height as a function of r.