Dynamical Elliptical Diagnostics of the Antarctic Polar Vortex

Márton Mester, Gavin Esler
University College London

INTRODUCTION

Elliptical diagnostics[5,8] are commonly used to construct an ellipse that approximates the shape of the polar vortex on a given isentropic level.

The diagnostics are determined by calculating zeroth-, first- and second-order vortex integral moments of the vortex, and then defining an ellipse to match these.

The resulting time series of the vortex centroid, area, aspect ratio and orientation provide a detailed picture of the climatology, interannual variability, seasonal cycle and vertical structure of the vortices in each hemisphere.

The elliptical diagnostics have been argued to be a useful measure of vortex variability, capturing stratospheric sudden warmings (SSWs) as extreme events in the diagnostics[7]. Undisturbed, displaced and split vortex states naturally arise from the diagnostics via a clustering method[2].

Here, we go further by addressing the *dynamical* evolution of the elliptical diagnostics of the Antarctic polar vortex, by making use of a simple model, which is Kida's elliptical two-dimensional vortex in a linear background flow[3].

METHODS

An elliptical patch of uniform vorticity ω stays elliptical in a linear background flow[3] $\boldsymbol{u} = \mathcal{A}\boldsymbol{x}$, $\mathcal{A} = \begin{pmatrix} \Gamma_1 & \Gamma_2 - \Gamma_3 \\ \Gamma_2 + \Gamma_3 & -\Gamma_1 \end{pmatrix}$. The aspect ratio λ and orientation θ are converted to ellipticity variables[6] $X \equiv \begin{pmatrix} X \\ Y \end{pmatrix} = (\lambda^{1/2} - \lambda^{-1/2}) \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix}$ that evolve according to Hamilton's equations $\dot{X} = \nabla^{\perp} H$, where the Hamiltonian is given by

$$H(\boldsymbol{X}) = \left(|\boldsymbol{X}|^2 + 4\right)^{1/2} \boldsymbol{\Gamma} \cdot (\boldsymbol{k} \times \boldsymbol{X}) + |\boldsymbol{X}|^2 \, \boldsymbol{\Gamma} \cdot \boldsymbol{k} + \omega \log\left(\frac{|\boldsymbol{X}|^2 + 4}{4}\right),$$

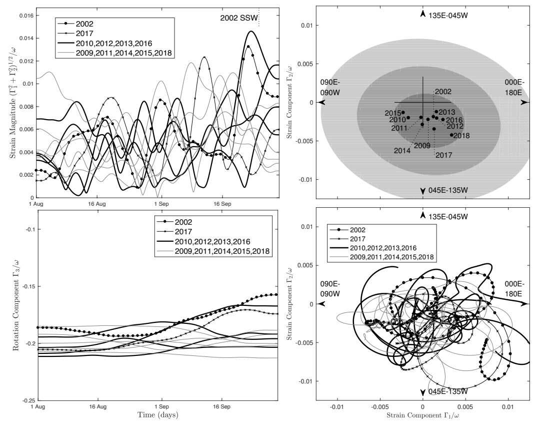
where $\Gamma = (\Gamma_1, \Gamma_2, \Gamma_3)^T$. The events $H(\boldsymbol{X}) = H_c(\boldsymbol{\Gamma})$ can be associated to SSWs[1].

The dynamical elliptical diagnostics $\{X_a(t), \Gamma(t)\}$ are minimisers of the functional

$$J(\boldsymbol{\Gamma}, \boldsymbol{X}, \boldsymbol{\mu}) = \int_0^T \left(|\dot{\boldsymbol{\Gamma}}|^2 + \epsilon^2 |\boldsymbol{X} - \boldsymbol{X}_{\boldsymbol{obs}}(t)|^2 + \boldsymbol{\mu} \cdot (\dot{\boldsymbol{X}} - \nabla^{\perp} H) \right) \mathrm{d}t,$$

and they represent a Kida-vortex in a background flow that is the most consistent with observations*. The minimisation is carried out with standard techniques of variational calculus.

RESULTS



CONCLUSIONS

The assimilation technique constructs a representative Kida-vortex that captures the dynamical evolution of the polar vortex.

The obtained time-series for the rotational component reveal a slowly evolving stratospheric jet each year, with no rapid fluctuations.

A Gaussian fit for the strain components show the presence of a weak stationary forcing (stationary topographic wave) that is dominated by a stochastic component (planetary-scale waves generated by the interaction of baroclinic eddies at tropopause level). This result suggests that

1) vortex splits on the Southern Hemisphere have no strongly preferred direction
2) SSWs may occur purely as a result of random fluctuations in accordance with [4].

References

1) Esler, J. G., and M. Mester, 2019: Noise-induced vortex splitting stratospheric sudden warmings. Q. *J. R. Meteorol. Soc.*, **145**, 476–494.

2) Hannachi, A. et al., 2011: On the use of geometric moments to examine the continuum of sudden stratospheric warmings. *J. Atmos. Sci.*, **68**, 657–674.

3) Kida S., 1981: Motion of an Elliptic Vortex in a Uniform Shear Flow. *J. Phys. Soc. Jap.*, **50**, 3517–3520.

4) Kushner, P. J., and L. M. Polvani, 2005: A very large, spontaneous stratospheric sudden warming in a simple GCM: A prototype for the Southern Hemisphere warming of 2002? J. Atmos. Sci., 62, 890–897.

5) Matthewman N. J. et al., 2009: A new look at stratospheric sudden warmings. Part III. Polar vortex evolution and vertical structure. *J. Clim.*, **22**, 1566–1585.

6) Melander, M. V. et al., 1986: A moment model for vortex interactions of the two- dimensional Euler equations. Part 1. Computational validation of a Hamiltonian elliptical representation. *J. Fluid Mech.*, **167**, 95–115.

7) Mitchell D. M. et al., 2011: Characterizing the variability and extremes of the stratospheric polar vortices using 2D moment analysis. *J. Atmos. Sci.*, **68**, 1194–1213.

8) Waugh D. W., and W. J. Randel, 1999: Climatology of Arctic and Antarctic polar vortices using elliptic diagnostics. *J. Atmos. Sci.*, **56**, 1594–1613.

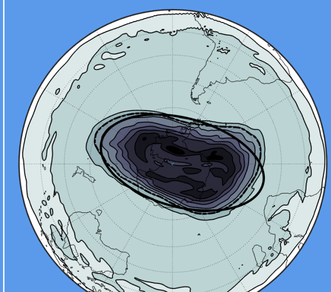
This work is funded by EPSRC award 1777889 (Márton Mester) and UK NERC grant NE/S00985X/1 (Gavin Esler).
*Our observational data is derived from the ERA-Interim dataset, provided by the British Atmospheric Data Centre.

EPSRC Engineering and Physical Sciences Research Council

ASSIMILATION

Potential vorticity distribution on a given isentropic level is used to define the vortex boundary and apply elliptical diagnostics.

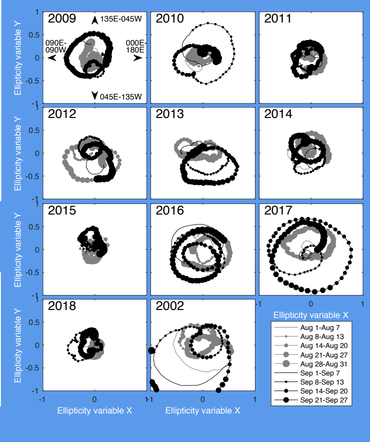
The constructed ellipse provides compact information about the vortex shape and the structure of the vorticity field.



The aspect ratio and orientation of the ellipse are converted to ellipticity variables.

$$\begin{pmatrix} X \\ Y \end{pmatrix} = (\lambda^{1/2} - \lambda^{-1/2}) \begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix}$$

$X - Y$ phase-portraits visualise polar vortex evolutions in a given time window.



A variational data assimilation technique determines the assimilated ellipticity variables $(X, Y)_a$.

$(X, Y)_a$ satisfy Kida's equations in an unsteady linear background flow, determined by the assimilation.