MATH6502 Example Sheet 8. Not for credit.

1. Using the cofactor method, find the inverse of the matrix

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 1 & -2 \end{pmatrix}.$$

2. Use the Gauss-Jordan method to find the inverse of both these matrices:

$$\underline{\underline{B}} = \begin{pmatrix} 3 & 2 & 2 \\ 6 & 5 & 4 \\ 2 & 1 & 5 \end{pmatrix} \quad \text{and} \quad \underline{\underline{A}} = \begin{pmatrix} 5 & 4 & 5 \\ 5 & 4 & 4 \\ 2 & 3 & 4 \end{pmatrix}.$$

   Now find the product $\underline{\underline{A}}\,\underline{\underline{B}}$. Find its inverse using the cofactor method, and verify that your results satisfy $(\underline{\underline{A}}\,\underline{\underline{B}})^{-1} = \underline{\underline{B}}^{-1}\underline{\underline{A}}^{-1}$.

3. Consider the matrix

$$\underline{\underline{D}} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & x \end{pmatrix}$$

   (i) Find any values of x for which $(\underline{\underline{I}} - \underline{\underline{D}})^{-1}$ does not exist.

   (ii) Given that $x = 4$, write down $(\underline{\underline{I}} - \underline{\underline{D}})^{-1}$.

   (iii) If $x = 4$ and $\underline{\underline{D}}(\underline{\underline{I}} + \underline{\underline{C}}) = \underline{\underline{C}}$ then find $\underline{\underline{C}}$.

4. Use the definition of the matrix inverse to prove that:

   (i) $\det(\underline{\underline{A}}^{-1}) = 1/\det(\underline{\underline{A}})$

   (ii) $(\underline{\underline{A}}^{-1})^{-1} = \underline{\underline{A}}$

   (iii) $(\underline{\underline{A}}\,\underline{\underline{B}})^{-1} = \underline{\underline{B}}^{-1}\underline{\underline{A}}^{-1}$

5. Solve the following set of equations by first finding the inverse of the matrix of coefficients (even though we would usually just use Gaussian elimination):

(i)	(ii)	(iii)	(iv)
$3x + 2y + z = 1$	$x + y + z = 10$	$2x - y + 5z = 2$	$5x - 14y + 2z = -15$
$4x + 3y - 4z = \frac{3}{2}$	$x + 2y + z = 13$	$x + 7y - 10z = 1$	$-10x - 5y - 10z = -15$
$x + 2y - 3z = 1$	$x + y + 2z = 15$	$x + y + z = 2$	$10x + 2y - 11z = -30$

6. Using the Gauss-Jordan method, find the inverse of these matrices:

$$\underline{\underline{A}} = \begin{pmatrix} -1 & -1 & 2 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix} \qquad \underline{\underline{B}} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 3 & 3 & 2 \end{pmatrix}$$