MATH6502 Example Sheet 5. Hand in all questions from section A.
Cover sheet with DEPARTMENT/TUTOR/YOUR NAME & signed.
Due into Maths room 6.10 by 2pm on Wednesday 19 November.

Section A

1. Given these two matrices, calculate the products $\underline{\underline{A}}\,\underline{\underline{B}}$ and $\underline{\underline{B}}\,\underline{\underline{A}}$:

$$\underline{\underline{A}} = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix} \qquad \underline{\underline{B}} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix}.$$

2. Calculate <u>every</u> possible product of any two of the following matrices (including the square of a matrix where possible):

$$\underline{\underline{A}} = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} \quad \underline{\underline{B}} = \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix} \quad \underline{\underline{C}} = \begin{pmatrix} 2 & 5 & -3 \\ -8 & 1 & 0 \\ 6 & 5 & -1 \end{pmatrix} \quad \underline{\underline{D}} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad \begin{array}{l} \underline{\underline{E}} = \begin{pmatrix} 1 & 3 & 8 \end{pmatrix} \\ \underline{\underline{F}} = \begin{pmatrix} 1 & -3 \end{pmatrix} \end{array}$$

Section B

1. Solve the simultaneous equations $3x + 2y = 7$ and $x + y = 3$ (you can do this with techniques from school). Now write the system as a matrix-vector multiplication and verify that the vector $\begin{pmatrix} x \\ y \end{pmatrix}$, with the values of x and y you have just found, is a solution of the matrix problem.

2. If $\underline{\underline{A}}\,\underline{\underline{B}} + \underline{\underline{B}}\,\underline{\underline{A}} = \underline{\underline{B}}$, show that $\underline{\underline{B}}^2\underline{\underline{A}} = \underline{\underline{A}}\,\underline{\underline{B}}^2$.

3. Find m, n and p such that

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -3 & 6 \\ 3 & 6 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & p & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & n \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & p \\ 0 & 0 & 1 \end{pmatrix}.$$

4. Find a matrix $\underline{\underline{A}}$ such that $\underline{\underline{A}}^2 = \underline{\underline{B}}$, where

$$\underline{\underline{B}} = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}.$$

5. (i) Find $\underline{\underline{A}}\,\underline{\underline{B}}$ and $\underline{\underline{B}}\,\underline{\underline{A}}$ for these two matrices:

$$\underline{\underline{A}} = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix} \qquad \underline{\underline{B}} = \begin{pmatrix} 6 & 4 & -3 \\ 2 & 1 & 2 \\ -4 & 5 & 7 \end{pmatrix}.$$

   (ii) Express each of the following matrices as a product of two square matrices:

$$\underline{\underline{C}} = \begin{pmatrix} 2a & 5b & 8c \\ 3a & -b & 4c \\ a & 6b & c \end{pmatrix} \qquad \underline{\underline{D}} = \begin{pmatrix} p & 9p & -7p \\ 4q & 2q & 5q \\ r & -r & 3r \end{pmatrix}.$$

6. Let $\underline{\underline{A}}$ be a 2x2 matrix such that $\underline{\underline{A}}^2$ is the zero matrix. Find the possible forms of its elements.