

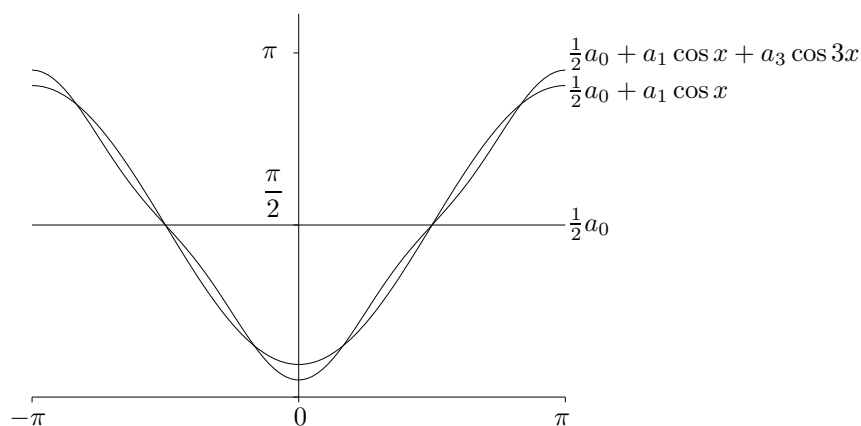
Handout 4 Fourier series examples

Sawtooth function

$$f(x) = \begin{cases} -x & -\pi \leq x \leq 0 \\ x & 0 \leq x \leq \pi \end{cases} \quad \text{an even function.}$$

We calculated: $a_0 = \pi$ $a_n = \frac{2}{\pi n^2}(\cos n\pi - 1) = \frac{2}{n^2\pi} \begin{cases} 0 & n \text{ even} \\ -2 & n \text{ odd} \end{cases}$ $b_n = 0$.

The Fourier series begins: $f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left(\cos x + \frac{1}{9} \cos 3x + \frac{1}{25} \cos 5x + \dots \right)$.



The coefficients in this case decrease fast, like $1/n^2$: this sort of decay occurs when f is *continuous*.

Square wave

$$f(x) = \begin{cases} -1 & -L < x \leq 0 \\ 1 & 0 < x \leq L \end{cases} \quad \text{an odd function.}$$

This function is periodic, but it is not continuous: there are jumps at $0, L, 2L, 3L, \dots$

We calculated: $a_n = 0$ $b_n = \frac{2}{n\pi}(1 - \cos n\pi) = \frac{2}{n\pi} \begin{cases} 2 & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$

and the Fourier series starts: $f(x) = \frac{4}{\pi} \left(\sin \left(\frac{\pi x}{L} \right) + \frac{1}{3} \sin \left(\frac{3\pi x}{L} \right) + \frac{1}{5} \sin \left(\frac{5\pi x}{L} \right) + \dots \right)$.

Here the coefficients decrease slower, like $1/n$: this happens when f is discontinuous (has a jump).

