Handout 14 Non-standard elimination

In non-standard elimination, we can do any of the legal row operations (i.e. things that make sense with the original equations):

- multiply a row by a constant;
- swap two rows;
- add a multiple of one row to another row; or
- subtract a multiple of one row from another row.

At the end of the process, we can still safely check whether the determinant is zero, but we will not find the value of it from the echelon-form matrix.

There are several variants.

Keeping integers in the matrix

If we have an example with integers to start with, we can avoid fractions by using row multiples:

$$\left(\begin{array}{cc|c} 3 & 1 & 4 \\ 5 & -1 & 4 \end{array}\right) \quad r_2 \to 3r_2 - 5r_1 \quad \left(\begin{array}{cc|c} 3 & 1 & 4 \\ 0 & -8 & -8 \end{array}\right).$$

Row echelon form

To get true (as opposed to generalised) row echelon form we can carry out standard Gaussian elimination, but at the end we need to multiply each row by a constant so that the first nonzero element in each row is a 1.

Pivoting

Swap rows so that we use the largest magnitude element in each column to eliminate entries in the rows below. Helps minimise numerical errors. This example (fixed precision at 3 significant figures, errors in the entries at the same level) shows the effect this can have.

Without pivoting:

$$\left(\begin{array}{cc|c} 0.01 & 1.02 & 4.32 \\ 5.00 & 2.03 & 1.21 \end{array}\right) \qquad \left(\begin{array}{cc|c} 0.02 & 1.01 & 4.31 \\ 5.00 & 2.02 & 1.22 \end{array}\right)$$

$$r_2 \to r_2 - 500r_1 \qquad r_2 \to r_2 - 250r_1$$

$$\left(\begin{array}{cc|c} 0.01 & 1.02 & 4.32 \\ 0.00 & -508 & -2160 \end{array}\right) \qquad \left(\begin{array}{cc|c} 0.02 & 1.01 & 4.31 \\ 0.00 & -250 & -1080 \end{array}\right)$$

$$-508y = -2160 \implies y = 4.25 \qquad -250y = -1080 \implies y = 4.32$$

$$0.01x + 1.02y = 4.32 \implies x = -1.5 \qquad 0.02x + 1.01y = 4.31 \implies x = -2.66$$

and with: because a_{11} is small, we swap rows 1 and 2.

$$\left(\begin{array}{cc|c} 5.00 & 2.03 & 1.21 \\ 0.01 & 1.02 & 4.32 \end{array}\right) \qquad \left(\begin{array}{cc|c} 5.00 & 2.02 & 1.22 \\ 0.02 & 1.01 & 4.31 \end{array}\right)$$

$$r_2 \to r_2 - 0.002r_1 \qquad r_2 \to r_2 - 0.004r_1$$

$$\left(\begin{array}{cc|c} 5.00 & 2.03 & 1.21 \\ 0.00 & 1.02 & 4.32 \end{array}\right) \qquad \left(\begin{array}{cc|c} 5.00 & 2.02 & 1.22 \\ 0.00 & 1.00 & 4.31 \end{array}\right)$$

$$1.02y = 4.32 \implies y = 4.24 \qquad 1.00y = 4.31 \implies y = 4.31$$

$$5.00x + 2.03y = 1.21 \implies x = -1.48 \qquad 5.00x + 2.02y = 1.22 \implies x = -1.50$$

Now the errors in both variables are roughly the same size as the rounding errors.