Handout 11 Matrix-vector systems

We consider a system of n equations in n unknowns, which can be written in matrix-vector form as

$$\underline{\underline{A}}\,\underline{x} = \underline{b}$$

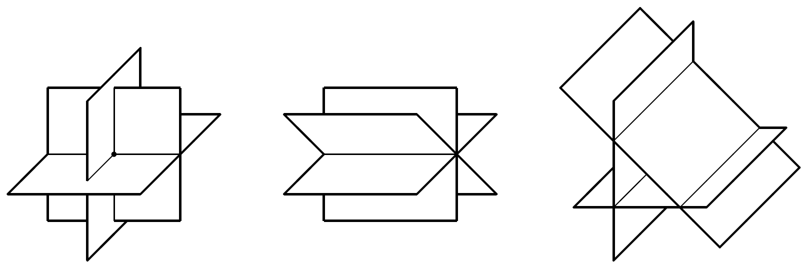
where $\underline{\underline{A}}$ is an $n \times n$ matrix.

How many solutions?

There are three possibilities:

- Unique solution
- Infinite number of solutions
- No solutions

Geometrically we can see these in 3D, where an equation represents a plane, and a **solution** means a point lying on three planes:



Unique solution   Infinite solutions   No solutions

Conditions for a unique solution

Whether or not there is a unique solution depends on the determinant of $\underline{\underline{A}}$:

- $\det(\underline{\underline{A}}) \neq 0$ means there is a unique solution to $\underline{\underline{A}}\,\underline{x} = \underline{b}$.
- If $\det(\underline{\underline{A}}) = 0$ there may be no solutions to the equation, or there may be an infinite number of them.

Homogeneous and inhomogeneous systems

An **inhomogeneous system** is one where $\underline{b} \neq \underline{0}$. Typically we will be looking for a unique solution so we will need $\det(\underline{\underline{A}}) \neq 0$.

A **homogeneous system** is one where $\underline{b} = \underline{0}$. We know one solution: $\underline{x} = \underline{0}$, the *trivial solution*. Typically we will be looking for a non-trivial solution: so we need more than one solution. That means we need an infinite number of solutions and we will need $\det(\underline{\underline{A}}) = 0$ to get them.