1. \( n = 77, \; \phi(n) = 6 \cdot 10 = 60. \) \( a = 7. \) \( gcd(60, 7) = 1 \) and \( 2.60 - 7.17 = 1 \), using extended Euclid or extended Binary gcd.

\[
\begin{array}{cccc}
60 & 7 & 1 & 0 \\
7 & 4 & 0 & 1 \\
4 & 3 & 1 & -8 \\
3 & 1 & -1 & 9 \\
1 & 0 & 2 & -17 \\
\end{array}
\]

so \( b = -17 \equiv 43 \pmod{60} \). In order to read the message Bob computes \( 60^{43} \pmod{77} \). Now \( 60 = -17 \pmod{77} \), so \( 60^2 = 17^2 = 58 \), \( 60^4 = 58^2 = -24 \), \( 60^8 = 24^2 = 37 \), \( 60^{16} = 37^2 = 60 = -17 \), and \( 60^{32} = 17^2 = 58 \), all \( \pmod{77} \). As \( 58 = 32 + 8 + 2 + 1 \) one has that the message is \(-19.37. - 19. - 17 \pmod{77} \), \( = 4 \pmod{77} \).

2. The square root of 62821427 is 7926 to the nearest integer. And \( 7926^2 - 7^2 = 62821427 \). So \( p = 7933, \; q = 7919 \) and \( (p - 1)(q - 1) = 62805576 \). Now we must find the multiplicative inverse of \( 5 \pmod{62805576} \), which we do by using the extended Euclid algorithm.

\[
\begin{array}{cccccc}
62805576 & 5 & 1 & 0 & 0 & 1 \\
5 & 1 & 0 & 1 & 1 & -12561115 \\
1 & 0 & 1 & -12561115 & -5 & 62805576 \\
\end{array}
\]

So \( b \), the inverse of \( 5 \pmod{62805576} \) is \(-12561115, = 50244461 \).

3. We check the values of \( b^7 \) for \( b \in \{1, 2, 4, 7, 8, 11, 13, 14\} \).

Note that \( 14 \equiv -1 \pmod{15} \), \( 13 \equiv -2 \pmod{13} \), \( 11 \equiv -4 \pmod{15} \) and \( 8 \equiv -7 \pmod{15} \).

Clearly \( 1^7 \equiv 1 \). \( 2^7 = 128 \equiv 8 \pmod{15} \). \( 4^7 = (2^2)^7 = (2^7)^2 \equiv 8^2 \pmod{15} \equiv 4 \pmod{15} \). and \( 8^7 \equiv (2^7)^3 \equiv 2 \pmod{15} \). So (exactly) 3/4 of the values are neither 1 or \(-1 \).

4. There is some \( k \) such that \( (2/3)^k.n \leq 1 \). Specifically this holds for any \( k \) which is at least as big as \( \log(n)/\log(3/2) \) (just take logs and rearrange).

Let \( t(1) = d \). We have that \( t(n) \leq t(2n/3) + c \) for some constant \( c \). Thus (assuming that \( t \) is increasing) we have that \( t(n) \leq d + (\log(n)/\log(3/2) + 1).c \).