1. Show how MSIS1 acts on the following graph to find the maximum size of an independent set:

![Graph 1](image1)

2. Show how MSIS2 acts on the following graph to find a maximum size of an independent set:

![Graph 2](image2)

3. (a) Of what must a connected graph in which every vertex has degree \( \leq 2 \) consist? Prove your answer.

(b) Let \( G \) be any graph in which every vertex has degree \( \leq 2 \). Describe \( G \). How would you calculate the size of a maximal independent set of vertices of \( G \)?

4. Write an algorithm MSIS3 for calculating the size of the largest independent set of vertices: its trivial case is when no vertex has degree \( \geq 3 \), otherwise it proceeds as in MSIS1 or MSIS2.

5. Show how ChromPoly acts on the following graph to find its chromatic polynomial.

![Graph 3](image3)
6. Show how ChromPoly acts on the following graph to find its chromatic polynomial.

![Graph Image]

7. A chain on \( n \) vertices, \( S_n \), is a connected graph with exactly two vertices of degree 1 and all others of degree 2. E.g. here is a chain, \( S_4 \) with 4 vertices.

![Chain Graph Image]

Prove by induction on \( n \) that if a chain \( S_n \) has \( n \) vertices then \( P(k; G) = k(k - 1)^{n-1} \).

8. Let \( G = (X, E) \) be a graph with \( n \) vertices. The chromatic number, \( \chi(G) \), is the smallest number \( k \) such that there is a function \( f : X \rightarrow \{0, 1, \ldots, k-1\} \) such that for all \( x, y \in X \) (where \( x \neq y \)) one has that \( \{x, y\} \in E \) implies that \( f(x) \neq f(y) \). Recall that \( \alpha(G) \) the largest size of an independent set in \( G \), that is a set \( Y \subseteq X \) such that for all \( x, y \in Y \) (where \( x \neq y \)) one has \( \{x, y\} \notin E \).

Show that \( \alpha(G) \geq n/\chi(G) \).