1. For each of the following decision problems describe a certificate that shows the problem belongs to \( NP \).

(i) **SUBSET SUM**

Input: a finite set \( A \subseteq \mathbb{N} \setminus \{0\} \) and an integer \( k \).

Question: is there a subset of \( A \) whose sum is exactly \( k \)?

(ii) **HAMPATH**

Input: graph \( G \), vertices \( a \) and \( b \)

Question: Is there a Hamiltonian path from \( a \) to \( b \) in \( G \)?

*(Definition:)* Let \( G = (X,E) \) be a graph. If \( a, b \in X \) are vertices a path from \( a \) to \( b \) is a finite sequence \( a_0, a_1, \ldots, a_n \) with \( a = a_0 \), \( b = a_n \) and \( \{a_i, a_{i+1}\} \in E \) for each \( i < n \). A Hamiltonian path from \( a \) to \( b \) is a path from \( a \) to \( b \) that visits every vertex in the graph exactly once.*

2. Show that \( NP \) is closed under union and intersection.

3. Show that the following problem is \( NP \)-complete:

**BOUNDED HALTING**

Instance: A non-deterministic Turing machine \( M \), a string \( x \) and a string of \( t \) 1s

Question: Does \( M \) accept \( x \) in at most \( t \) steps?

4. (Satisfiability of DNF formulas). A formula \( F \) is in disjunctive normal form (DNF) if it is of the form \( \phi = \bigwedge_i \bigvee_j l_{ij} \) where the \( l_{ij} \) are Boolean literals.

Let \( SAT_{DNF} = \{ \phi \mid \phi \text{ is in DNF and is satisfiable} \} \).

Prove that the language \( L_{DNF} \) is in \( P \), that is that there is a polynomial-time algorithm that, on input a formula \( \phi \) in DNF, decides whether \( \phi \) is satisfiable.

*(Note the contrast with CNF!)*