Math 3502: Combinatorial Optimisation. Example sheet 3.

1. For each of the following decision problems describe a certificate that shows the problem bnelongs to NP.

(i) SUBSET SUM

Input: a finite set $A \subseteq \mathbb{N} \setminus \{0\}$ and an integer k.

Question: is there a subset of A whose sum is exactly k?

(ii) HAMPATH

Input: graph G, vertices a and b

Question: Is there a Hamiltonian path from a to b in G?

(**Definition**: Let $G = (X, E)$ be a graph. If a, $b \in X$ are vertices a *path* from a to b is a finite sequence a_0, a_1, $\ldots a_n$ with $a = a_0$, $b = a_n$ and $\{a_i, a_{i+1}\} \in E$ for each $i < n$. A *Hamiltonian path* from a to b is a path from a to b that visits every vertex in the graph exactly once.)

2. Show that NP is closed under union and intersection.

3. Show that the following problem is NP-complete:

BOUNDED HALTING

Instance: A non-deterministic Turing machine M, a string x and a string of t 1s

Question: Does M accept x in at most t steps?

4. (Satisfiability of DNF formulas). A formula F is in *disjunctive normal form (DNF)* if it is of the form $\phi = \bigwedge_i \bigvee_j l_{ij}$ where the l_{ij} are Boolean literals.

Let $SAT_{DNF} = \{\phi \mid \phi \text{ is in DNF and is satisfiable}\}$.

Prove that the language L_{DNF} is in P, that is that there is a polynomial-time algorithm that, on input a formula ϕ in DNF, decides whether ϕ is satisfiable.

(**Note the contrast with CNF!**)