Math 3502: Combinatorial Optimisation. Example sheet 2.

1. Let COMPOSITE be the decision problem:

Input: an integer $n \geq 2$

Question: is n composite?

Show that COMPOSITE is in P if and only if PRIME is in P.

2. Prove that if A, B and C are languages, and $A \leq_p B$ and $B \leq_p C$ then $A \leq_p C$.

3. Let $G = (X, E)$ be a graph. G is *connected* if $\forall x,\ y \in X\ \exists x_0, \ldots, x_m \in X$
$(x_0 = x,\ x_m = y\ \&\ \forall i < m \{x_i, x_{i+1}\} \in E)$.

Vertices $y_0, \ldots, y_p \in X$ form a *cycle* if $\forall i < p\ \{y_i, y_{i+1}\} \in E$ and $\{y_0, y_p\} \in E$. An *odd* cycle is one with an odd number of of distinct vertices (greater than one).

G is 2-colourable if $\exists c : X \longrightarrow \{\text{red}, \text{blue}\}\ \forall x,\ y \in X\ (c(x) = c(y) \Longrightarrow \{x, y\} \notin E)$

Show G is 2-colourable if and only if G contains no odd cycle.

4. (IF WE HAVE GOT THIS FAR.)

Reduce the 2-colourability problem for the following graph to an instance of SAT. Find a suitable certificate for the problem.