1. Let COMPOSITE be the decision problem:

Input: an integer $n \geq 2$

Question: is $n$ composite?

Show that COMPOSITE is in P if and only if PRIME is in P.

2. Prove that if $A$, $B$ and $C$ are languages, and $A \leq_p B$ and $B \leq_p C$ then $A \leq_p C$.

3. Let $G = (X, E)$ be a graph. $G$ is connected if $\forall x, y \in X \exists x_0, \ldots, x_m \in X$

$(x_0 = x, \ x_m = y \ & \ \forall i < m \{x_i, x_{i+1}\} \in E)$.

Vertices $y_0, \ldots, y_p \in X$ form a cycle if $\forall i < p \{y_i, y_{i+1}\} \in E$ and $\{y_0, y_p\} \in E$. An odd cycle is one with an odd number of distinct vertices (greater than one).

$G$ is 2-colourable if $\exists c : X \rightarrow \{\text{red, blue}\} \ \forall x, y \in X (c(x) = c(y) \implies \{x, y\} \notin E)$

Show $G$ is 2-colourable if and only if $G$ contains no odd cycle.

4. (IF WE HAVE GOT THIS FAR.)

Reduce the 2-colourability problem for the following graph to an instance of SAT. Find a suitable certificate for the problem.