MATH1302, Question Sheet 6

Questions 1, 2 and 4 to be handed in Tuesday 4 March before the lecture.

Qu 1 A fisherman, returning from a fishing trip, turns off his engine and lets his dinghy drift slowly at speed $U$ the last few metres to the river bank. When still a few metres from the bank he throws his rucksack mass $m$ towards the bank (in the direction that the boat is drifting) at speed $V$ relative to the boat and at an angle $\alpha$ to the horizontal. Given that the combined mass of the fisherman, the dinghy and its contents, including the rucksack, is $M+m$, find the condition that he will reach the bank without having to restart the engine. (Assume that the river is stationary.)

Qu 2 A runaway railfreight carriage filled with sand is travelling freely down a long, straight and constantly inclined railway track. The empty carriage has mass $M$, it initially holds $S_0$ mass of sand, and the inclination of the track to the horizontal is $\alpha$. There is a crack in the carriage which allows sand to leak out at a constant rate. There is also a partially failed brake that provides a constant resistive force $B$ to slow the carriage. Find the speed of the carriage from rest at time $t < T$ where $T$ is the time taken for all the sand to leak out.

Qu 3 A particle is moving along the $x$–axis under a constant force $F$ gains mass by collecting material that is moving along the $x$–axis with velocity $u$. If $m$ and $v = dx/dt$ are the mass and the velocity of the particle at time $t$ show that

$$\frac{d}{dt} (mv) = F + u \frac{dm}{dt}.$$ 

If $u = 0$ and $m = M + kx$ where $M, k > 0$ are constants, show that $kx = \sqrt{(M^2 + kFt^2)} - M$.

Qu 4 The total mass of a rocket is $M_0 + M_1$, including fuel of mass $M_1$. The fuel is burnt at a constant rate $\lambda$ as is emitted as a stream of gas at a speed $u$ relative to the rocket. The rocket is fired vertically upwards under uniform gravity $g$. If the mass of the fuel and the speed of the rocket at time $t$ are $m(t)$ and $v(t)$ respectively, derive the equation

$$(M_0 + m) \frac{dv}{dt} + u \frac{dm}{dt} = -(M_0 + m)g,$$

$$\frac{dm}{dt} = -\lambda.$$ 

Hence show that for $t < M_1/\lambda$

$$v = u \log \left( \frac{M_0 + M_1}{M_0 + M_1 - \lambda t} \right) - gt,$$

Qu 5 A raindrop falls vertically through a cloud of water particles which are at rest, and accumulates particles at a rate $kv$ units of mass per unit time when its speed is $v$. If the raindrop starts from rest and with mass $M$, show that its speed $v$ after falling through a distance $x$ satisfies

$$v \frac{dv}{dx} + \frac{kv^2}{M + kx} = g.$$ 

Hence find $v$ as a function of $x$. 