

# Learning Nominal Automata

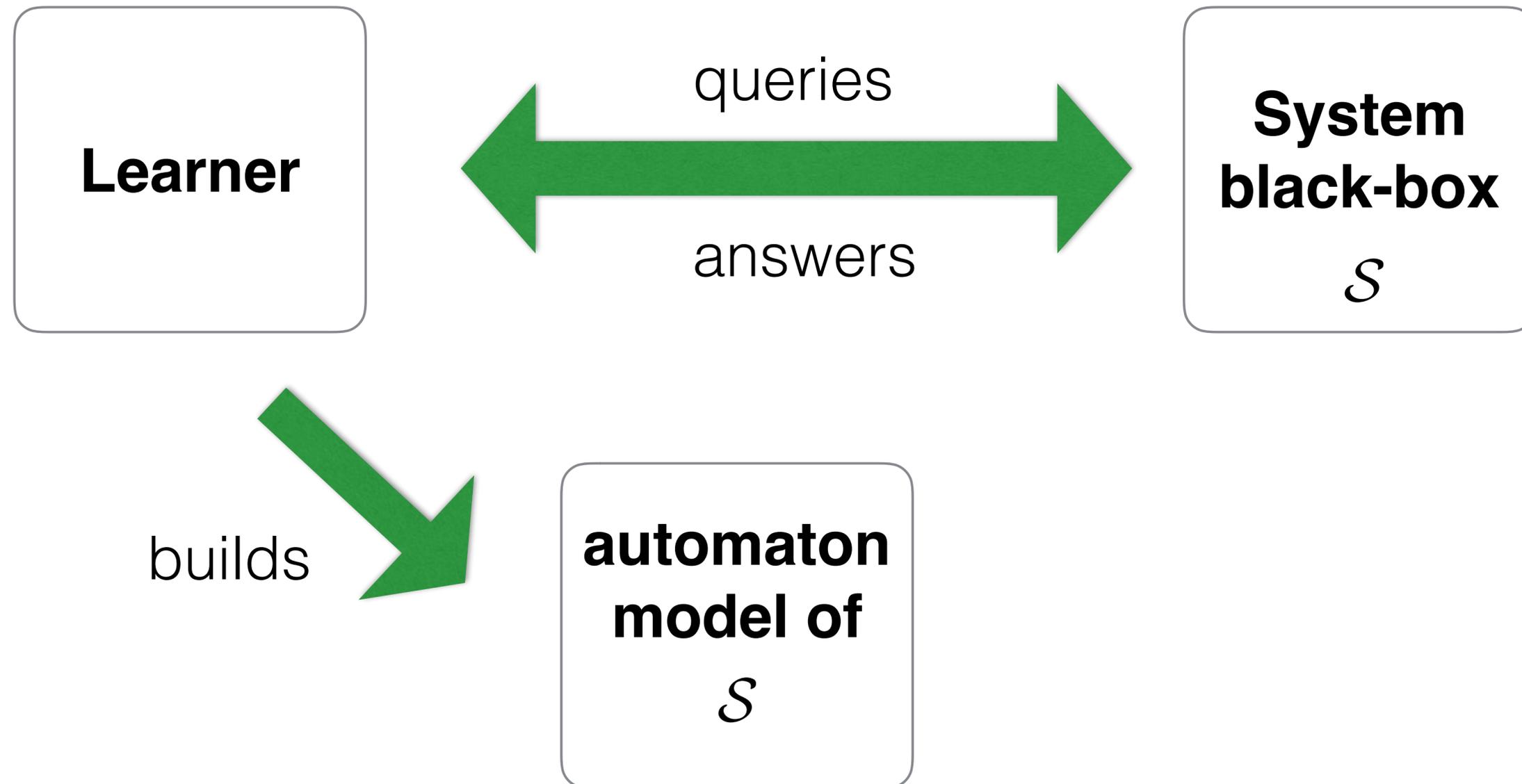
Joshua Moerman  
(Radboud University)

**Matteo Sammartino**, Alexandra Silva  
(University College London)

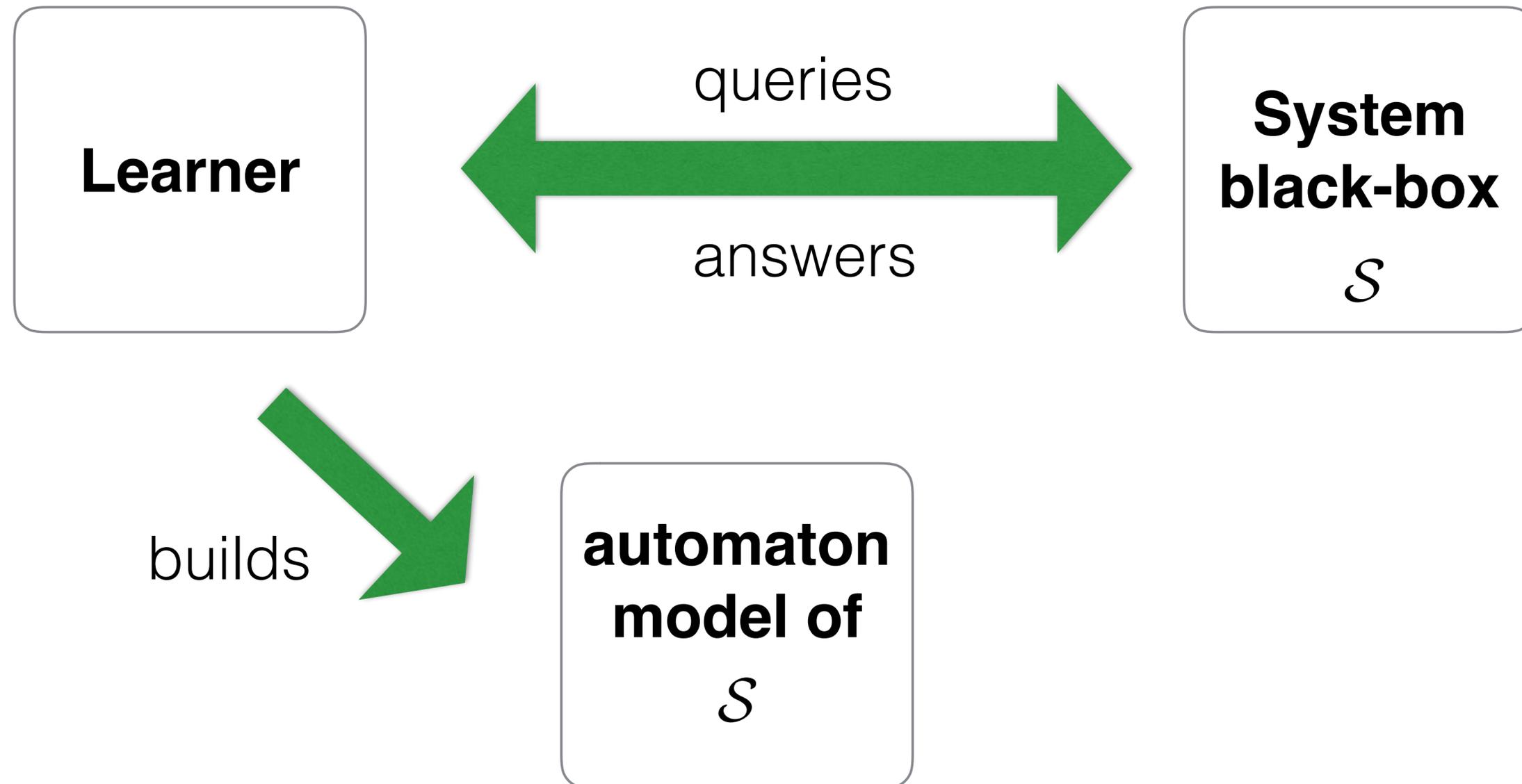
Bartek Klin, Michał Szynwelski  
(Warsaw University)

POPL 2017  
Paris

# Active learning



# Active learning



No formal specification available? **Learn it!**

# $L^*$ algorithm (D.Angluin '87)

**Finite alphabet** of system's actions  $A$

set of system behaviors is a **regular language**  $\mathcal{L} \subseteq A^*$

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**Learner**

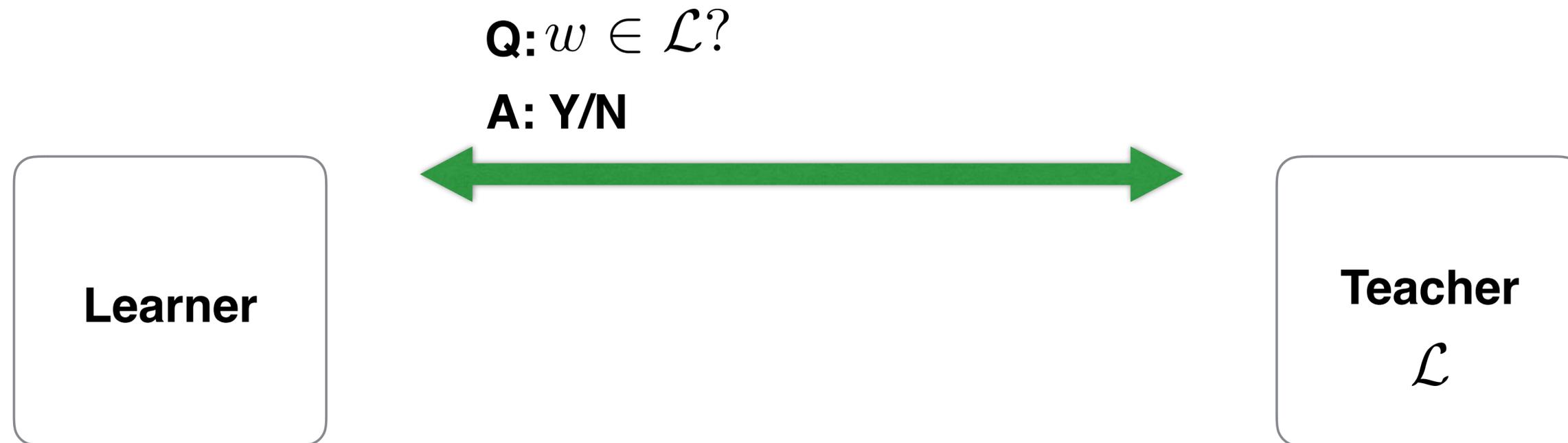
**Teacher**

$\mathcal{L}$

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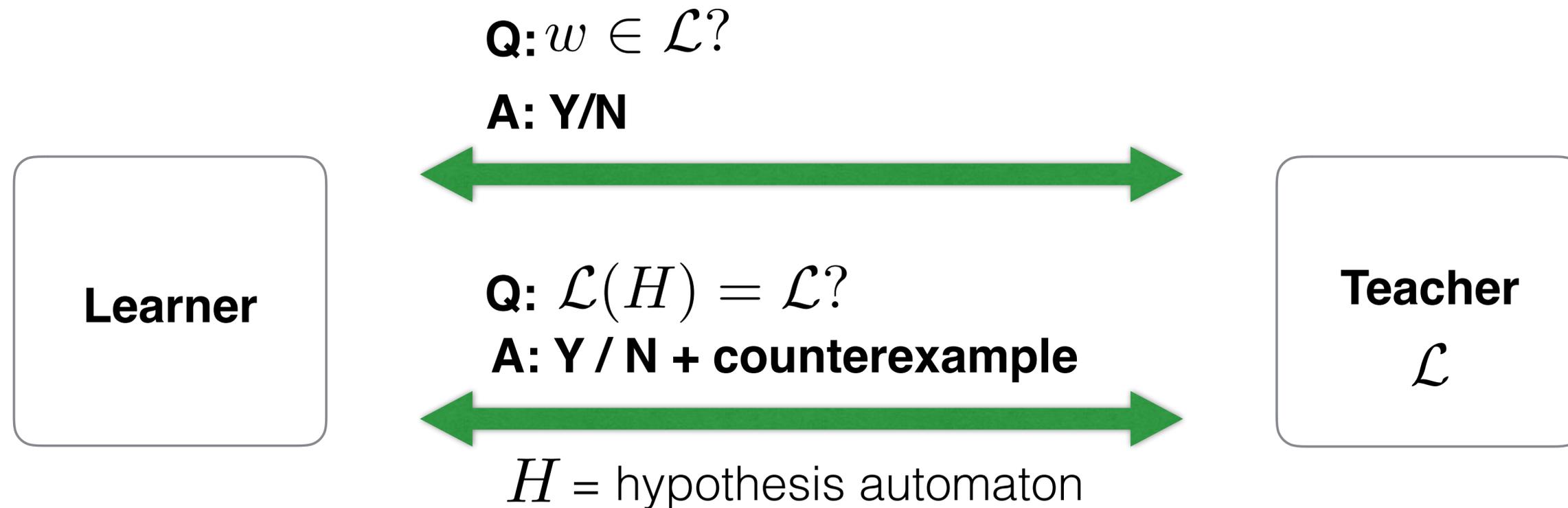
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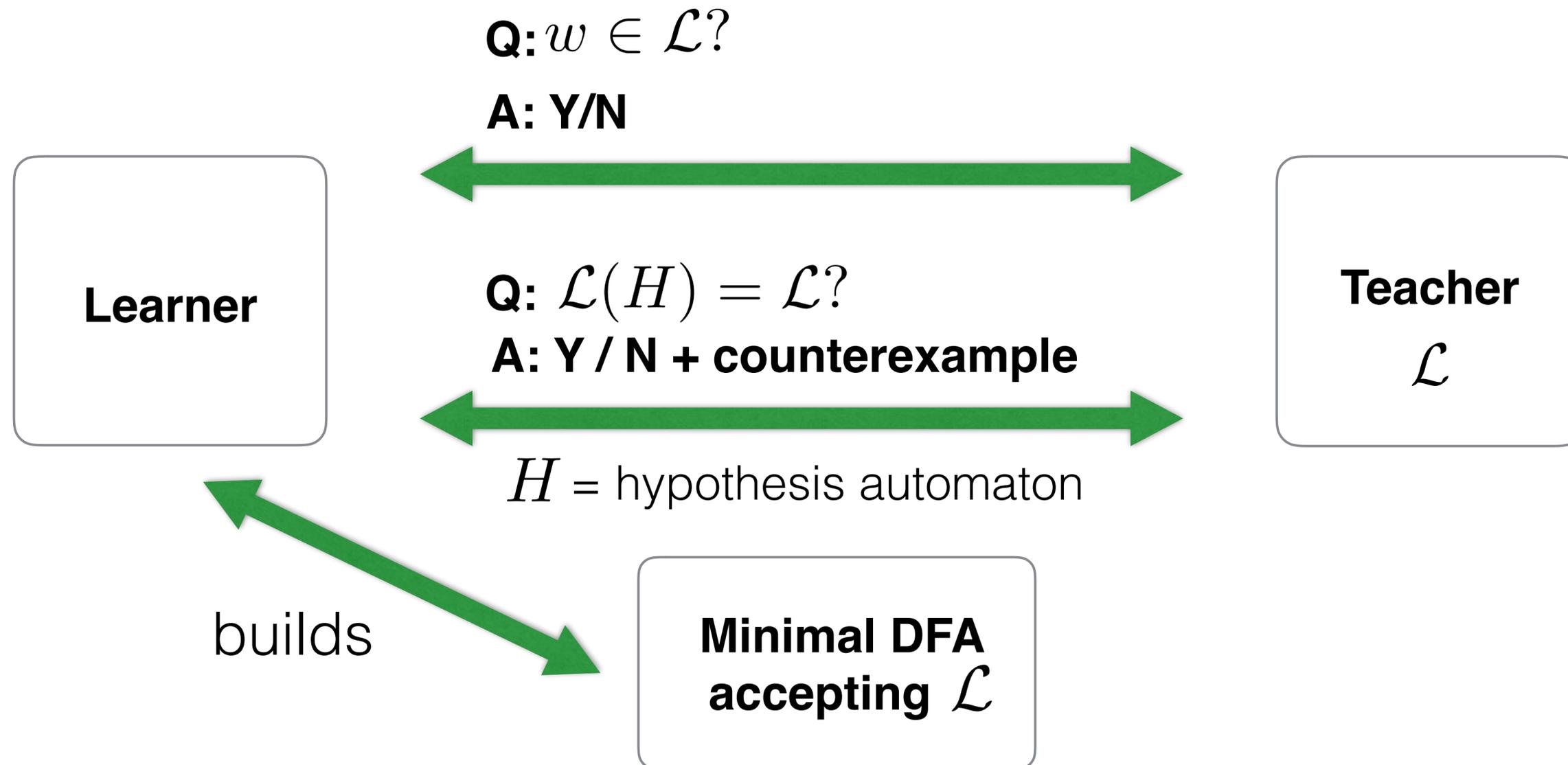
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# Observation table

		$E$			
		$\epsilon$	$a$	$aa$	
$S$	$\epsilon$	0	0	1	
	$a$	0	1	0	
$S \cdot A$	$b$	0	0	0	

$S, E \subseteq A^* \quad A = \{a, b\}$

$$row: S \cup S \cdot A \rightarrow 2^E$$

$$row(s)(e) = 1 \iff se \in \mathcal{L}$$

# Observation table

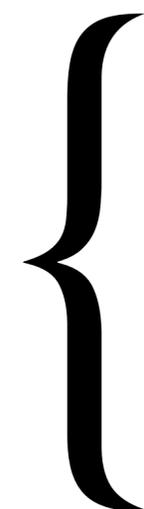
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**Hypothesis automaton**



$$\text{states} = \{row(s) \mid s \in S\}$$

$$\text{final states} = \{row(s) \mid s \in S, row(s)(\epsilon) = 1\}$$

$$\text{initial state} = row(\epsilon)$$

$$\text{transition function} \quad row(s) \xrightarrow{a} row(sa)$$

# Observation table

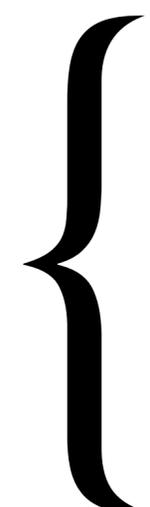
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final states =  $\{row(s) \mid row(s)(\epsilon) = 1\}$

initial state

**Why is this correct?**

transition function  $row(s) \xrightarrow{a} row(sa)$

# Table properties

## Closed

$$\forall t \in S \cdot A \quad \exists s \in S \quad row(t) = row(s).$$

**next state exists**

## Consistent

$$\forall s_1, s_2 \in S \quad row(s_1) = row(s_2) \implies \forall a \in A \quad row(s_1 a) = row(s_2 a)$$

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**Fixed by extending the table**

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# Pros of $L^*$ ...

simple is  
beautiful

&

**POWERFUL**

**Applications** : Hardware verification, security/network protocols...

**Generalizations** : Mealy machines, I/O automata, ...

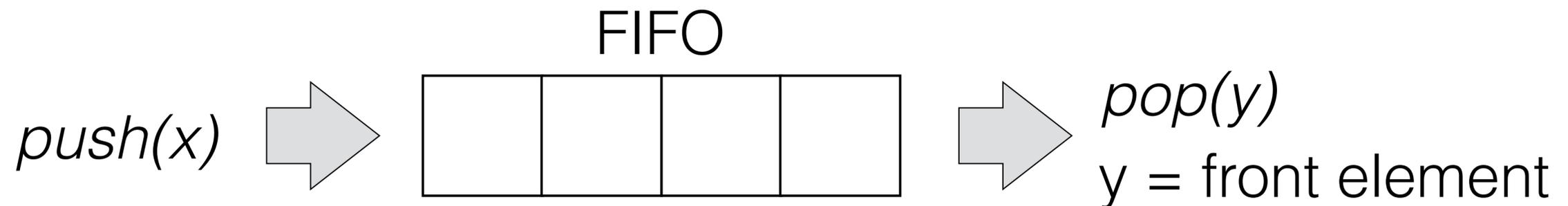
# ... and shortcomings

L\* learns **control-flow**

What if program model needs to express **data-flow**?

operations on **data values**

**comparisons** between data values



# Automata over infinite alphabets (nominal automata)

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$A = \{a, b, c, d, \dots\}$       **infinite alphabet**

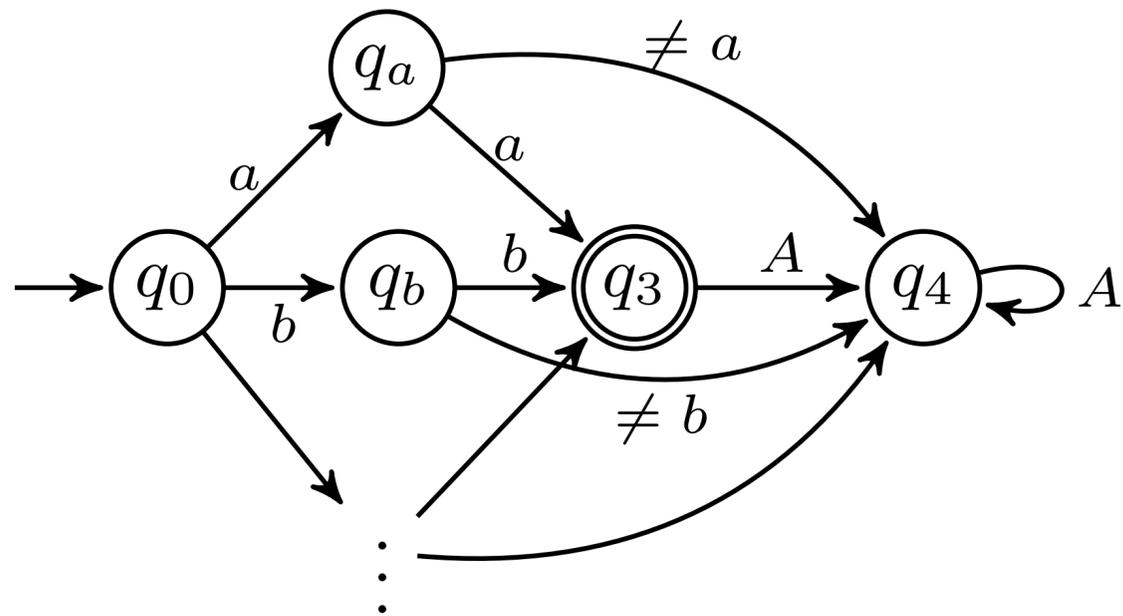
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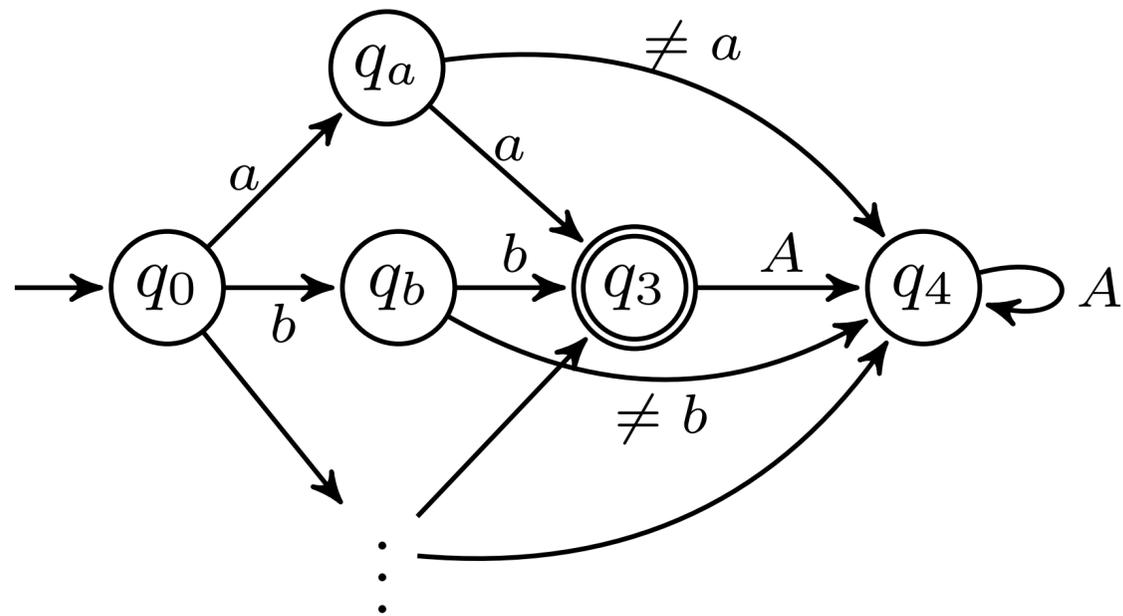
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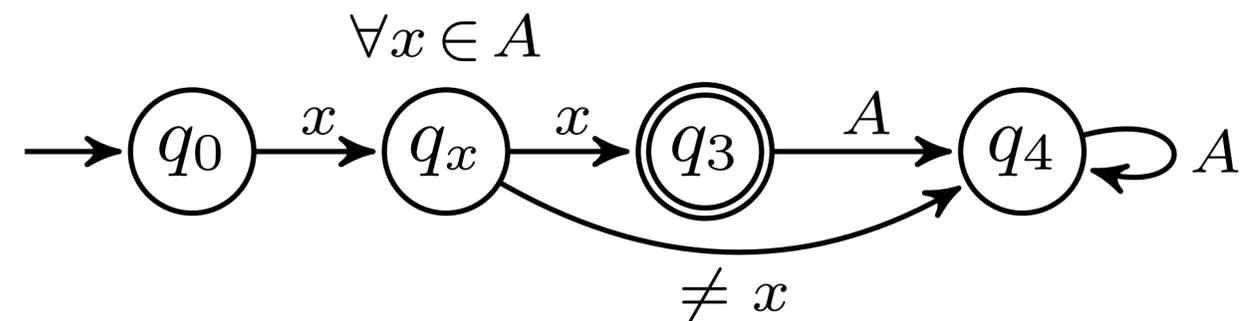
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infinite automaton



but with a finite representation

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- table needs to be **infinite**
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$$\forall t \in S \cdot A \quad \exists s \in S \quad \text{row}(t) = \text{row}(s).$$

**Everything is “finitely representable”**

$$\forall s_1, s_2 \in S \quad \text{row}(s_1) = \text{row}(s_2) \implies \forall a \in A \quad \text{row}(s_1 a) = \text{row}(s_2 a)$$

# A paradigm shift

(finite) sets        (orbit-finite) nominal sets  
functions        equivariant functions

(change category from **Set** to **Nom**)

**Nominal** automata theory

Mikolaj Bojanczyk, Bartek Klin, Slawomir Lasota:  
**Automata with Group Actions**. LICS 2011

**Nominal** Programming languages

Bartek Klin, Michal Szyrwelski:  
**SMT Solving for Functional Programming over Infinite Structures**. MSFP 2016

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**N** First non-trivial application of a new  
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Eryk Kopczynski, Szymon Torunczyk:  
**LOIS: syntax and semantics**. POPL 2017

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Algorithm correctness and termination **from scratch?**

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NLambda guarantees that **each line of code terminates**

Algorithm correctness and termination **from scratch?**

**Not really**

Set-based proofs as **guidelines**

$L^*$  enjoys a nice **category-theoretic** generalization

Bart Jacobs, Alexandra Silva

**Automata Learning: A Categorical Perspective**, Horizons of the Minds 2014

# What we've done

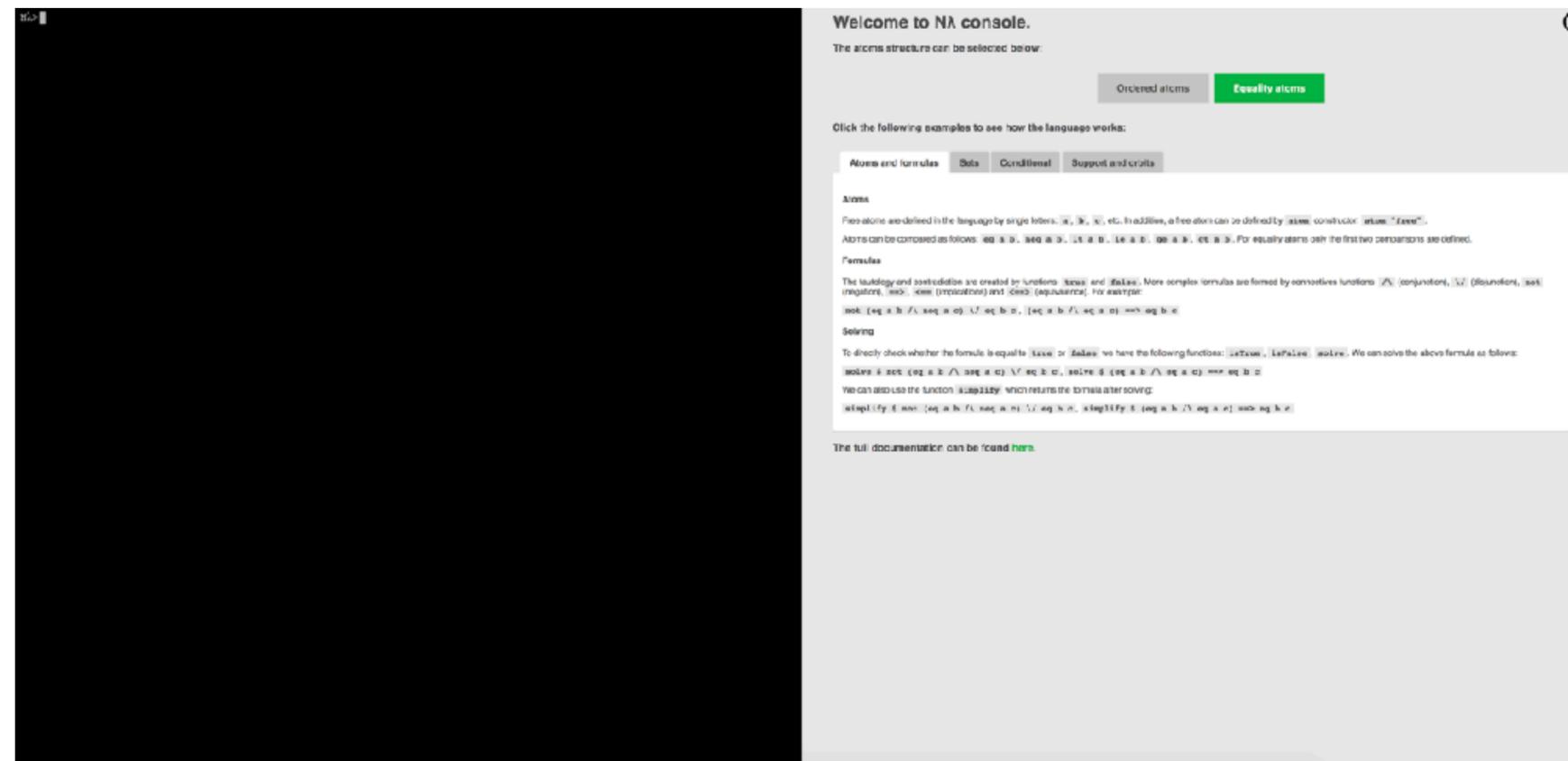
- Nominal  $L^*$
- **More in the paper:** variations, Nominal  $NL^*$
- NLambda (Haskell) Implementation
- Experimental results

# What's next...

- Improve NLambda
- Other active learning algorithms
- Other optimizations
- Applications: large-scale software, crypto protocols...

# Try it yourself

<https://www.mimuw.edu.pl/~szynwe1ski/n1lambda/>



<https://github.com/Jaxan/nominal-1star>

