Electrical and optical properties of materials

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Part 5: Optical properties of materials

5.1 Reflection and transmission of light

5.1.1 Normal to the interface

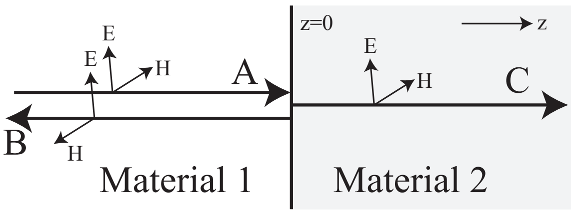

Figure 5.1: Reflection and transmission normal to the interface

We shall now examine the properties of reflected and transmitted electromagnetic waves. Let's begin by considering the case of reflection and transmission where the incident wave is normal to the interface, as shown in Figure 5.1. We have been using $E = E_0 \exp[i(kz - \omega t)]$ to describe waves, where the speed of the wave (a function of the material) is ω/k. In order to express the wave in terms which are explicitly a function of the material in which it is propagating, we can write the wavenumber k as:

(5.1)

where n is the refractive index *of the material* and k_0 is the wavenumber of the wave, were it to be travelling in free space. We will therefore write a wave as the following (with z replaced with whatever direction the wave is travelling in):

(5.2)

In this problem there are three waves we must consider: the incident wave A, the reflected wave B and the transmitted wave C. We'll write down the equations for each of these waves, using the impedance $Z = E_x/H_y$, and noting the flip in the direction of H upon reflection (see Figure 5.1), and the different refractive indices and impedances for the different materials.

$$\begin{array}{lll} E(A) = E_A e^{i(n_1 k_0 z - \omega t)}, & E(B) = E_B e^{i(n_1 k_0 z - \omega t)}, & E(C) = E_C e^{i(n_2 k_0 z - \omega t)} \\ H(A) = \frac{E_A}{Z_1} e^{i(n_1 k_0 z - \omega t)}, & H(B) = -\frac{E_B}{Z_1} e^{i(n_1 k_0 z - \omega t)}, & H(C) = \frac{E_C}{Z_2} e^{i(n_2 k_0 z - \omega t)} \end{array} \tag{5.3}$$

At the boundary ($z = 0$), and for any time t, the total E field on each side of the interface must be equal (and similarly for H). Thus:

(5.4)

Rearranging, we obtain the ratio of the reflected and transmitted amplitudes:

$$\text{reflected} \qquad \frac{E_B}{E_A} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{5.5}$$

$$\text{transmitted} \qquad \frac{E_C}{E_A} = \frac{2Z_2}{Z_2 + Z_1} \tag{5.6}$$

For the case where the relative permeability $\mu_r = 1$, a valid assumption for most materials at *optical frequencies*, we can express[1] this in terms of the refractive index n.

$$\text{reflected} \qquad \frac{E_B}{E_A} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{n_1 - n_2}{n_1 + n_2} \tag{5.7}$$

$$\text{transmitted} \qquad \frac{E_C}{E_A} = \frac{2Z_2}{Z_2 + Z_1} = \frac{2n_1}{n_1 + n_2} \tag{5.8}$$

Note that these represent the field amplitudes, and not power (which goes as field squared). Similar expressions can be derived for the ratio of the magnetizing fields H, using the relation that $H_{A,B} = E_{A,B}/Z_1$ and $H_C = E_C/Z_2$. These expressions should look reminiscent of those derived in your quantum mechanics course for a particle incident on a potential barrier.

[1]Reminder: $n = \sqrt{\epsilon_r \mu_r}$ and $Z = \sqrt{\mu/\epsilon}$

5.1.2 At an angle θ to the interface

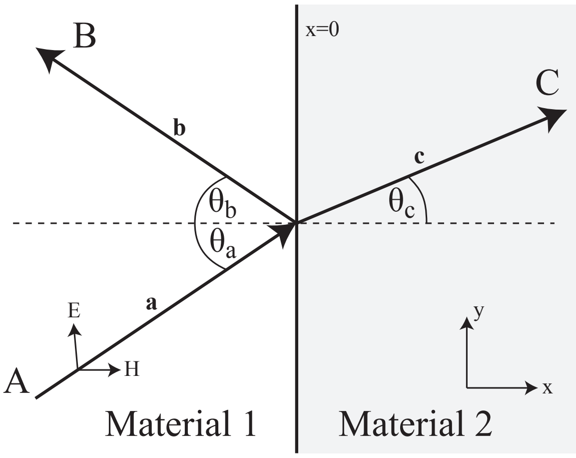

Figure 5.2: Reflection and transmission at an interface

Figure 5.2 shows a diagram of an electromagnetic wave incident on some interface between two materials 1 and 2. Note that for this derivation we must break with our convention of the waves travelling along z. So, for this part, the wave will be travelling in the $x-y$ plane. z is perpedicular and coming out of the page. Here, the three waves A, B and C travel along directions $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively:

$$E(A) = E_A \exp[i(n_1 k_0 \vec{a} - \omega t)] \tag{5.9}$$

$$E(B) = E_B \exp[i(n_1 k_0 \vec{b} - \omega t)] \tag{5.10}$$

$$E(C) = E_C \exp[i(n_2 k_0 \vec{c} - \omega t)] \tag{5.11}$$

$$\vec{a} = \cos\theta_a \vec{x} + \sin\theta_a \vec{y} \tag{5.12}$$

$$\vec{b} = -\cos\theta_b \vec{x} + \sin\theta_b \vec{y} \tag{5.13}$$

$$\vec{c} = \cos\theta_c \vec{x} + \sin\theta_c \vec{y} \tag{5.14}$$

There is the boundary condition that along the interface ($x = 0$), all three must remain in phase. Hence, their arguments must be equal

$$\begin{aligned} i\left(n_1 k_0 \sin\theta_a \vec{y} - \omega t\right) &= i\left(n_1 k_0 \sin\theta_b \vec{y} - \omega t\right) \\ &= i\left(n_2 k_0 \sin\theta_c \vec{y} - \omega t\right) \end{aligned} \tag{5.15}$$

Equating the arguments of these terms, we see that the frequency ω must remain constant, and that for the reflected wave:

$$\tag{5.16}$$

which is know as the Law of Reflection. While for the transmitted wave:

(5.17)

For $\frac{n_1}{n_2}\sin\theta_a \leq 1$, this reduces to $\theta_c = \frac{n_1}{n_2}\theta_a$ which is known as Snell's Law (for refraction) However, for $\frac{n_1}{n_2}\sin\theta_a > 1$, there is no allowed value of θ_c, and there is no transmission (this shall be useful in total internal reflection in optical fibres later on).

For the amplitudes, the algebra gets a little bit more cumbersome, but this is derived in detail in Bleaney & Bleaney (3rd edition, Chapter 8, pp 239-244). Here we will simply quote the results, for the case where $\mu_r = 1$.

$$-\frac{E_B(x)}{E_A(x)} = \frac{E_B(y)}{E_A(y)} = \frac{\sin 2\theta_c - \sin 2\theta}{\sin 2\theta_c + \sin 2\theta} \qquad \text{reflected} \tag{5.18}$$

$$\frac{E_C(x)}{E_A(x)} = \frac{E_C(y)}{E_A(y)} = \frac{4\sin\theta_c\cos\theta}{\sin 2\theta_c + \sin 2\theta} \qquad \text{transmitted} \tag{5.19}$$

Note that on reflection, the x- and y-components of the electric field E go to zero for the case where $\tan\theta = n_2/n_1$, which is known as the *Brewster angle* Θ_B. The component of E along z does not go to 0, and so an electromagnetic wave becomes *plane polarised* when incident at the Brewster angle — more on this later.

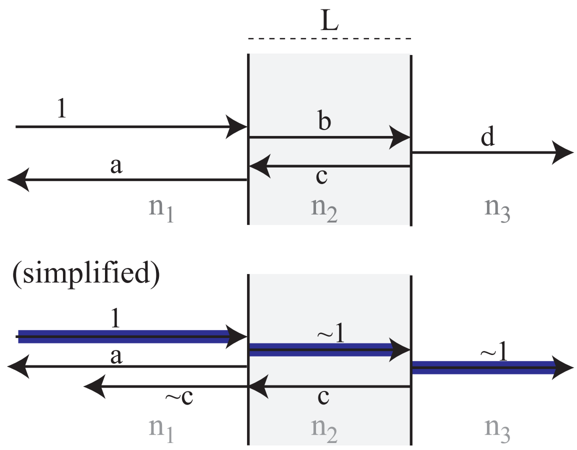

Figure 5.3: Coating an interface to reduce reflection. The lower diagram shows a simplified way of approaching the problem where reflection is assumed to be small

5.2 Lens blooming

The fact that there must be some reflection at any interface means that we must use interference between two or more beams to reduce it to zero. The general case is indicated schematically in Figure 5.3 for a coating or *bloom* of refractive index n_2 on the lens of index n_3. Using the results above, we note the amplitude reflection and transmission coefficients are:

$$\tag{5.20}$$

We can simplify the problem by assuming reflection is already reasonably small (as shown in the lower part of Figure 5.3). For zero reflection we need to ensure that the two components marked a and c are equal in amplitude and π out of phase so as to destructively interfere. The amplitude of $a = r_{12}$ and that of $c = r_{23}$, so these reflections coefficients will have to be equal for any chance of complete destructive interference. Thus:

$$\tag{5.21}$$

This gives the solution $n_2 = \sqrt{n_1 n_3}$, so we know what the refractive index n_2 of the coating must be. But what about the thickness? Wave c has travelled an additional length $2L$, and so will have picked up an extra phase (think about the $\exp(i n_i k_0 z)$ term for a wave):

$$\tag{5.22}$$

For complete cancellation, we need this phase difference to be π:

(5.23)

(5.24)

where $\lambda_0 = 2\pi/k_0$ is the wavelength of the wave, were it to be travelling in free space and λ_2 is the wavelength of the wave when in material 2. Thus, blooming with a layer of the appropriate thickness will remove all reflection *at a particular wavelength*. This is also called a 'quarter-wave' coating.

5.3 Dielectric mirrors

What if we want to do the opposite, and have *only* reflection — i.e. create a mirror? This can be achieved using multiple layers of very low loss dielectric, as shown in Figure 5.4. The total reflected wave is the sum of reflections from each interface in the layered structure. When the refractive index of the incident material is less than the current one ($n_j > n_{j+1}$), the reflection coefficient $r_{j,j+1}$ is positive and there no phase shift on reflection. In the opposite case ($n_j < n_{j+1}$), the reflection coefficient $r_{j,j+1}$ is negative and reflection is accompanied by an instantaneous π phase shift in the wave. Let's count up all the contributions to the reflected wave (assuming $n_1 > n_2 > n_0$):

	phase		reflection		propagation
reflection at entrance	ϕ_1	$=$	π	$+$	0
from interface 2	ϕ_2	$=$	0	$+$	$2L_1n_1k_0$
from interface 3	ϕ_3	$=$	π	$+$	$\phi_2 + 2L_2n_2k_0$
from interface 4	ϕ_4	$=$	0	$+$	$\phi_2 + 2(L_1n_1 + L_2n_2)k_0$
.					
.					
from interface 2n+1	ϕ_{2n+1}	$=$	π	$+$	$\phi_{2n} + 2L_2n_2k_0$
from interface 2n+2	ϕ_{2n+2}	$=$	0	$+$	$\phi_{2n} + 2(L_1n_1 + L_2n_2)k_0$

If we set both layers to 'quarter-wave' thickness (i.e. $L_1 = \lambda_1/4$ and $L_2 = \lambda_2/4$), then:

$$\phi_1 = \phi_2, \quad \phi_{2n+1} = \phi_{2n} + 2\pi \quad \text{and} \quad \phi_{2n+2} = \phi_{2n} + 2\pi \tag{5.25}$$

In other words, all waves are in phase and the assembly acts as a mirror. Efficiencies can be very high (e.g. 99.8% compared with 98.5% for the best silvered mirror).

If instead we used 'half-wave' layers ($L_i = \lambda_i/2$), then we'd have:

$$\phi_2 = \phi_1 + \pi, \quad \phi_{2n+1} = \phi_{2n} + 3\pi \quad \text{and} \quad \phi_{2n+2} = \phi_{2n} + 4\pi \tag{5.26}$$

The waves are alternately in anti-phase (phase difference π) and the system is non-reflecting.

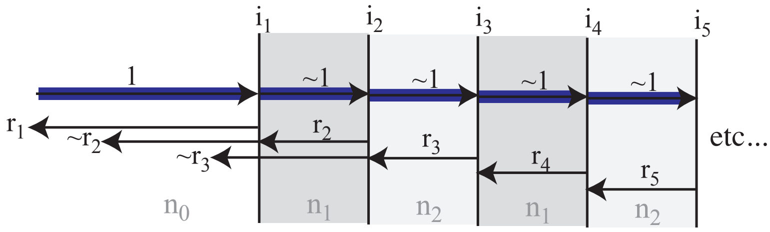

Figure 5.4: Multiple layers of alternating dielectric to create a dielectric mirror

5.4 Optic fibres

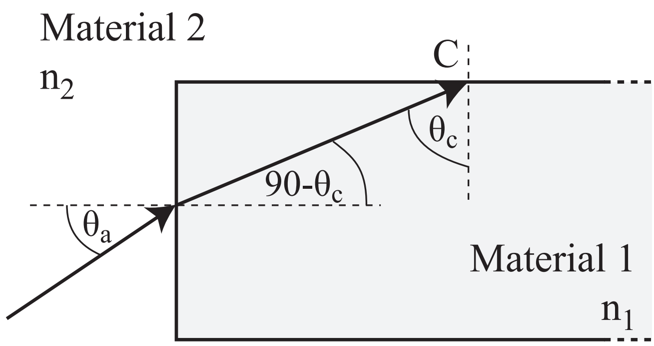

Figure 5.5: Basic theory of light pipes

Optic fibres or 'light pipes' operate on the principle of *total internal reflection*. Above we showed that there is no transmission at an interface (from region 1 into region 2) when the angle of incidence θ is such that:

$$\tag{5.27}$$

Take some light entering a light pipe with an angle of incidence θ_a, as shown in Figure 5.5. At its front surface, light is accepted into the material provided:

$$\sin\theta_a \leq \frac{n_1}{n_2}, \quad \text{with} \quad \sin(90-\theta_c) = \cos\theta_c = \frac{n_2}{n_1}\sin\theta_a \tag{5.28}$$

The light remains trapped within the material if:

$$\sin \theta_c > \frac{n_2}{n_1} \tag{5.29}$$

Or, expressed in terms of the incidence light:

(5.30)

Note that this inequality is satisfied *for all* θ if $\frac{n_1}{n_2} > \sqrt{2}$, such that all incident light would be accepted into the guided cone.

Provided that angles of incidence are well within these limits, the light may be guided around curved paths, such as illustrated in Figure 5.6.

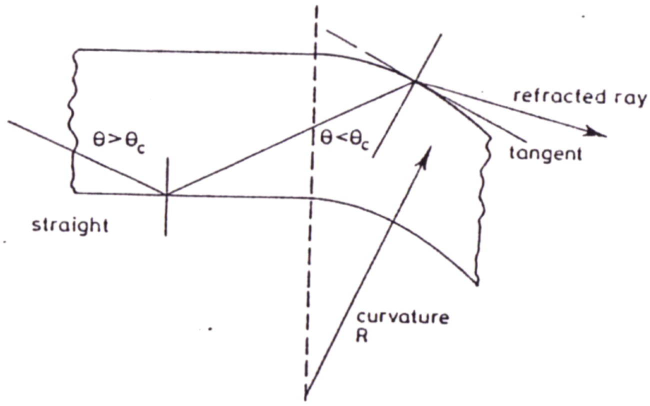

Figure 5.6: A curved light pipe

For most real optic fibers, the surrounding material (with n_2) is not air, but rather some cladding to provide mechanical support to the light pipe. The basic requirements for an optical fibre are:

* high core refractive index (n_1)
* low cladding refractive index (n_2)
* low core dispersion
* smooth interface between core and cladding
* sufficient cladding thickness
* good flexibility

* good transmission efficiency

The requirement for low *dispersion* (the constancy of refractive index) becomes important when the light pipe is used for telecommunications etc, when modulated signals of slightly different frequencies must not spread out as they travel along the pipe. The smooth interface is to prevent leakage and/or scattering from imperfections. The requirement of sufficient cladding thickness is to prevent leakage from the *evanescent* wave present in the cladding[2]. A thick cladding layer is also handy for protection. Good flexibility is an obvious requirement for something to be mechanically stable, while good transmission efficiency is essentially saying the imaginary part of the refractive index must be small so that there is negligible decay as the light passes through the material. For $\mu_r \sim 1$,

(5.31)

So this is equivalent to saying the relative permittivity ϵ_r must be real, positive (see Figure 2.12 from earlier for a reminder of how this varies with frequency — e.g. resonances and Debye relaxation). Low absorption tends to go with low dispersion, so a good dielectric benefits both criteria.

In practice, standard glass and silica fibres are adequate for optical frequencies and the short distances involved in medical applications, (Christmas trees!) etc., but significant improvements have been necessary for the multi-km distances needed by telecommunications. These improvements have been required in the whole system, from the emitter to connection to fibre to connection to detector.

The attenuation (or absorption) characteristics of a typical fibre as a function of wavelength are shown in Figure 5.7. Also shown in the figure are which source and detector materials are available for different wavelengths. Note the windows of good transmission in the near infrared at 1.2 and 1.55 μm, accessible by use of an InGaAs ternary and InGaAsP quaternary semiconductors.

A typical emitter/fibre system is shown in Figure 5.8, with dimensions indicated.

[2]We have said there is total internal reflection, when in fact there is always decaying standing wave which penetrates the interface. As long as it is of negligible amplitude by the time the refractive index changes again, one is alright. However, for a thin layer of material it is possible for the light to leak out, just as in quantum tunneling through a barrier

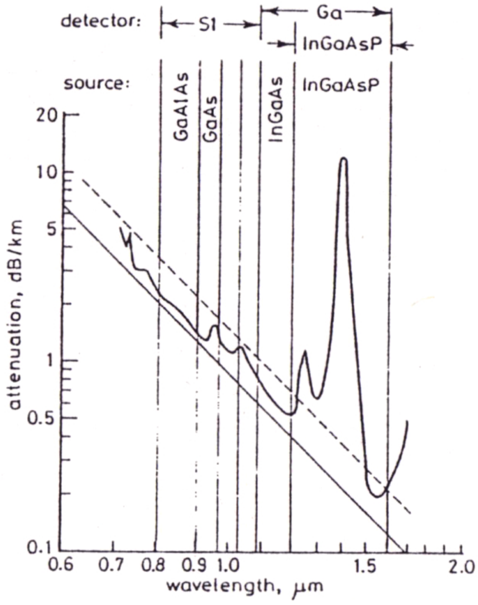

Figure 5.7: Attenuation versus wavelength for a typical fibre. Also shown are some available sources and detectors

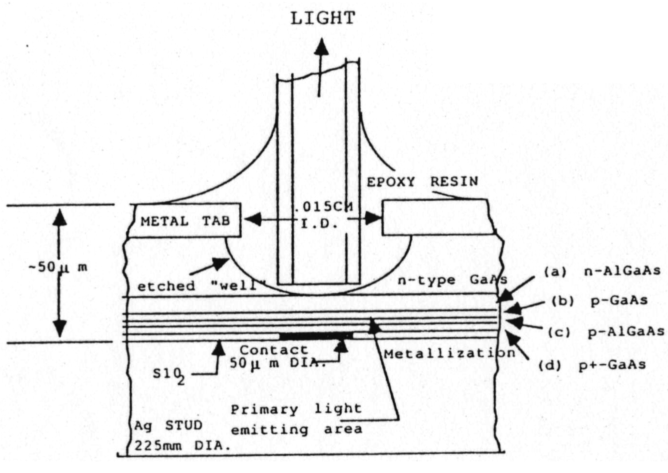

Figure 5.8: A typical LED emitter fibre system

5.5 Polarised light

In our work on electromagnetic waves we have assumed the electric and magnetic field are fixed along x and y, respectively, perpendicular to the direction of propagation z. In this sense we have already been assuming one particular polarisation (though everything we've done could be generalised). The kind of polarisation we have used is *plane polarised*, and we could rotate E_x and H_y around the $x - y$ plane to generate other kinds of plane polarisation.

Ordinary (unpolarised) light has equal intensity of electric field vectors along both x and y and so there is no net polarisation, and can be thought of as a combination of two light beams of orthogonal polarisations (see Figure 5.9).

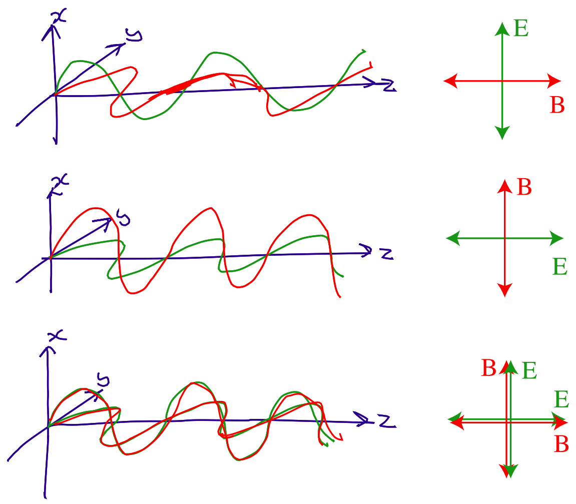

Figure 5.9: Pictoral representation of plane polarised and ordinary light beams

5.5.1 Selective reflection/transmission

We found when treating the problem of light incident at some angle θ to an interface, that there was a particular angle, the *Brewtser angle*: $\tan \Phi_B = n_2/n_1$, where only the component of the electric field perpendicular to the directions of propagation (i.e. out of the page for Figure 5.10) remains. Therefore, incident light at the Brewster angle becomes plane-polarised in reflection. For a typical glass with refractive index 1.54, the Brewster angle is around 57°, as shown in Figure 5.10.

The reflected wave, although polarised, is of rather small amplitude. The transmitted wave is partially polarised and, to a first approximation, retains

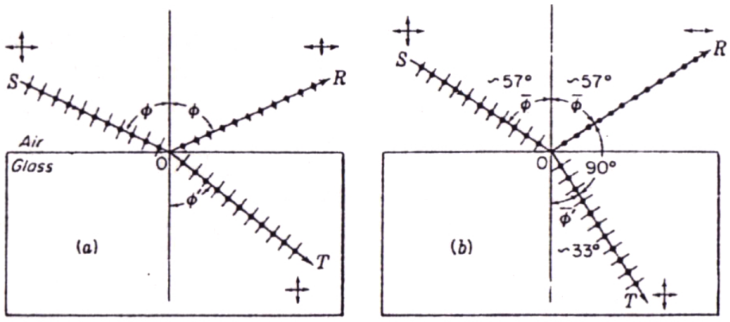

Figure 5.10: (a) Partial polarisation of light by reflection and transmission. (b) Complete polarisation in reflection at the Brewster angle

the full amplitude of the incident light (at that polarisation). Thus, as an alternative, polarisation may be obtained as an increasing fraction of the transmitted wave, as illustrated in Figure 5.11.

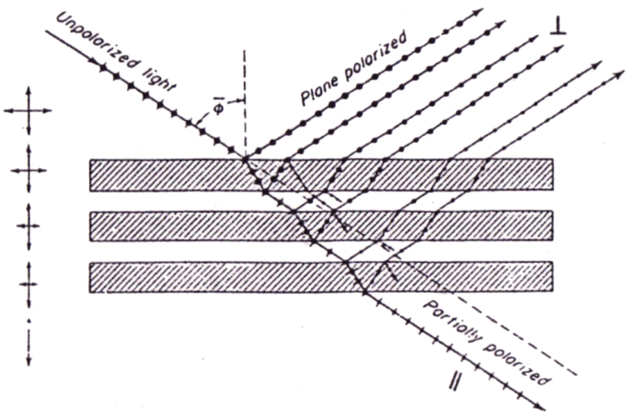

Figure 5.11: Polarisation by a pile of glass plates

5.5.2 Double diffraction

Birefringence is the property of a material whereby the refractive index depends on the polarisation of the light. This must rely on some non-cubic symmetry in the crystal structure (e.g. tetragonal, hexagonal or trigonal, with the two directions of interest being parallel or perpendicular to the c-axis). When unpolarised light enters such a crystal, it splits into two plane-polarised portions which then propagate according to the two different refractive indices. This opens up several possibilities for their separation, illustrated by the Nicol prism in Figure 5.12. The incoming light S splits into the ordinary

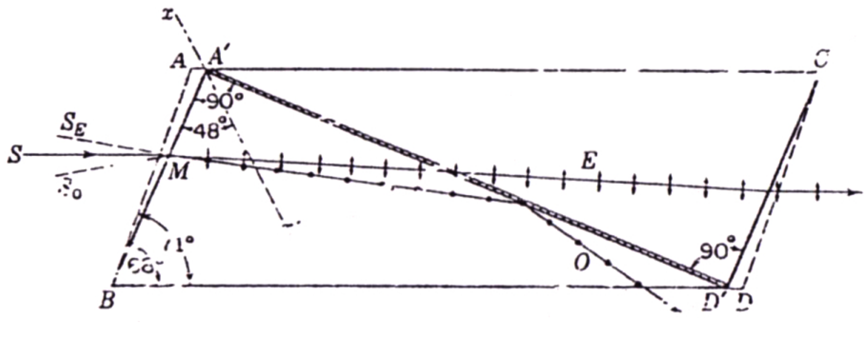

Figure 5.12: The Nicol prism for the production of polarised light

O and extraordinary E rays inside the calcite crystal, with refractive indices $n(O) = 1.66$ and $n(E) = 1.49$. Two pieces of the crystal are glued together using Canada balsam, which is not birefringent and has $n = 1.55$). Thus, at an appropriate range of angles, one of the rays (O say) is totally internally reflected while the other (E say) is transmitted. Note that the reflected ray contains O and a component of E which was reflected also at the interface, so it contains only partial polarisation.

5.5.3 Selective absorption

The phenomenon of *dichroism* refers to a material absorbing light at different strengths depending on its polarisation. A commercial implementation of this came from Polaroid$^{\mathrm{TM}}$ who developed the system of dichroic crystals in a plastic sheet which were aligned during production. A pair of parallel polaroid sheets transmit about 40% of the incident light (of the possible 50%) and less than 0.01% when crossed (i.e. at 90°).

5.6 Transmission through a polariser and analyzer

We may take two polarising devices (e.g. two sheets of polaroid) and place them in series in the path of an initially unpolarised beam of light. If they are perpendicular then we expect 'no' light to get through. However, if we put some test system between them which has the potential to rotate the polarisation of light, we will see this in the transmitted light. This kind of system is referred to as a *polariser* and *analyzer*, with the incident light falling on the polariser, passing through the test system, and then the analyzer. In the absence of any experimental system, transmission obeys a fairly obvious law, known as Malus' Law whereby the transmitted field strength goes as:

$$ \tag{5.32} $$

The intensity (proportional to field squared) goes as:

(5.33)

where θ is the angle between the polarisers and I_0 is the intensity of incident light (assuming no losses at all from the transmitted polarisation, which is never really the case).

5.7 Rotation of plane of polarisation

Two primary methods for rotating the polarisation of light use i) optically active chemicals or ii) magnetic fields (known as the Faraday or Kerr effect depending on whether transmission or reflection is used). We shall not be studying these, but rather looking at transmission through a series of n analyzers, each rotated by a small angle $\delta\theta$ with respect to its predecessor (as shown in Figure 5.13). Following Eq. 5.32,

(5.34)

(5.35)

Using a Taylor expansion for cos (assuming $\delta\theta$ is small)[3]:

(5.36)

and another[4] for $(1+x)^m$:

(5.37)

If the total angle between the first and the last analyzer is $\phi = n\delta\theta$, then:

(5.38)

The polarisation of the light coming out of the final analyzer is determined by the angle of the final analyzer, and hence a total polarisation rotation of ϕ has been accomplished, but at some cost in intensity. Plugging some numbers in, for $\phi = \pi/2$ and $n = 10$, 75% of the intensity is transmitted, while for $n = 40$, the transmission is 94%. So, provided the 'twist' angle is small enough, the plane of polarisation can be 'guided' round without too much loss.

[3]Taylor expansion: $\cos\theta \approx 1 - \theta^2/2$

[4]Taylor expansion: $(1+x)^m \approx 1 + xm$

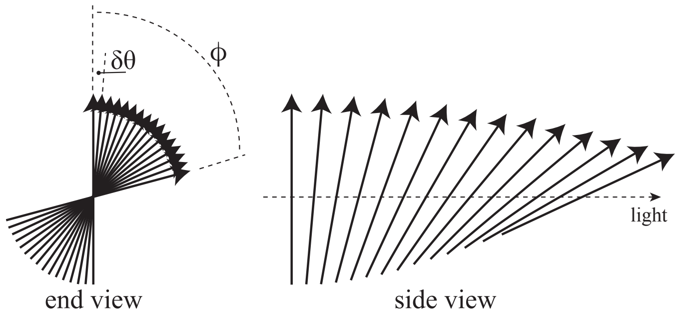

Figure 5.13: Slowly rotating analyzers can rotate the polarisation of light

5.8 Liquid crystals

Liquid crystals are a particular group of materials which show a 'double' melting point: at the lower melting point the solid 'melts' but remains ordered in at least one dimension until some higher melting temperature is reached and the material becomes truly liquid. Between these two temperatures the material is called a *liquid crystal*. There are three types of liquid crystal: nematic, cholesteric and smectic, though only the first is widely used for its electrical properties. In the *nematic* class, the rod-shaped (or long ellipsoidal) molecules tend to line up parallel to each other locally — the extent of alignment may be enhanced by the application of relatively small electric fields. In terms of the various polarisation mechanisms described in Part 2 of this course, liquid crystals behave a bit like large permanent dipole fluid particles at low temperature (although the molecules are not permanently polarised). Nematic liquid crystals have the further property of aligning with respect to a true solid surface with which they are in contact (e.g. Figure 5.14) with either perpendicular (*homeotropic*) or parallel (*homogeneous*) ordering.

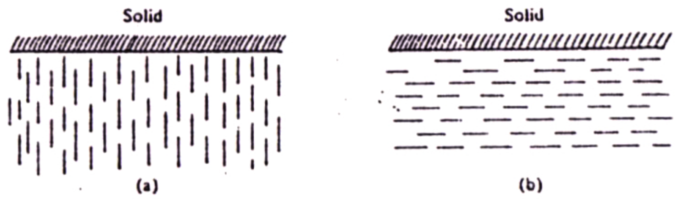

Figure 5.14: (a) Homeotropic and (b) homogeneous ordering of a nematic liquid crystal with respect to a solid surface

If we take a thin cell ($\sim$10 μm) of homogeneously ordered nematic liquid crystal, as illustrated in Figure 5.15, then we can control the alignment through the application of an electric field E between the plates of the cell.

Although the required field 150 kV/m may seem high, this can be achieved with a relatively small voltage (1.5 V) across the 10 μm gap. The response time of the realignment is $<$ 100 ms, which is adequate for most display purposes. This time dependence arises from Debye relaxation.

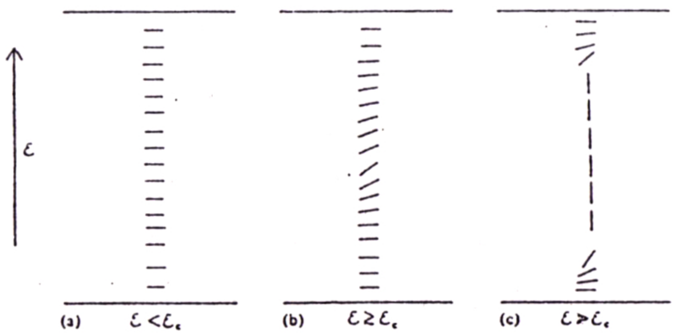

Figure 5.15: Realignment of a homogeneously ordered liquid crystal by an applied electric field, as might be used in liquid crystal displays

5.9 Liquid crystal displays

Liquid crystal displays, or LCDs, have undergone a phenomenally rapid rate of development during the last 40 years. The first publications in this area appeared in 1968 where the authors were already hoping for applications in televisions, for example. However, early devices were unreliable and slow throughout the 1970s. In the 1980s they began their domination of the static/slow monochrome and colour display markets. In the late 1990s they became competitive for TV and computer displays. One principle drawback is the intensity — they are always present between the viewer and the light source and in reality always absorb some of the transmitted, even when 'on'. Furthermore some complain that the blacks are not competitive with other technologies, such as plasma TVs, because they are not perfectly absorbing when in the 'off' state. With the advent of LED backlighting in displays, it may be that the next 20 years sees a decline in the use of LCDs in favour of fast-switching sources (rather than fast-switching filters).

That said, let's look at basic principles of operation of the LCDs currently in use. A good example is the 90° twisted nematic display. The extra property used here is the ability of certain molecules to rotate the plane of polarisation of light, keeping the E-vector of the light aligned along the major axis of the molecule. This works as long as there is only a very small change in angle from one molecule to the next (c.f. Section 5.7 and Eq. 5.38). So, if the liquid crystal is held between two plates and then twisted by 90°, in the absence of any electric field, polarised light will follow the molecular axes and undergo a 90° rotation. If an electric field is now applied so as to alter the nematic alignment, the polarisation no longer rotates (see Figure 5.16). Suitably arranged polariser/analyser combinations can thus distinguish between the two cases, and light can controllably be allowed to transmit through the device, or vice versa. A typical arrangement of the actual components is shown in Figure 5.17, while Figure 5.18 shows how this might be constructed in practice. The reflection mode is typically used in watches or pocket calculators, allowing them to work well around sources of bright ambient light (e.g. outside).

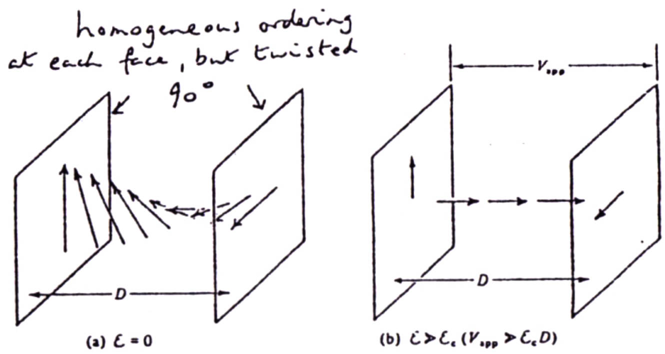

Figure 5.16: The principle of a twisted nematic display

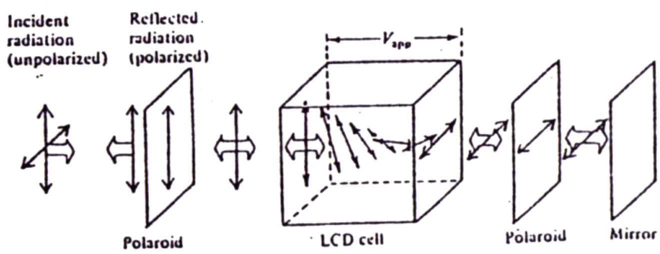

Figure 5.17: Light path in a reflection nematic display

A.1 Acknowledgements

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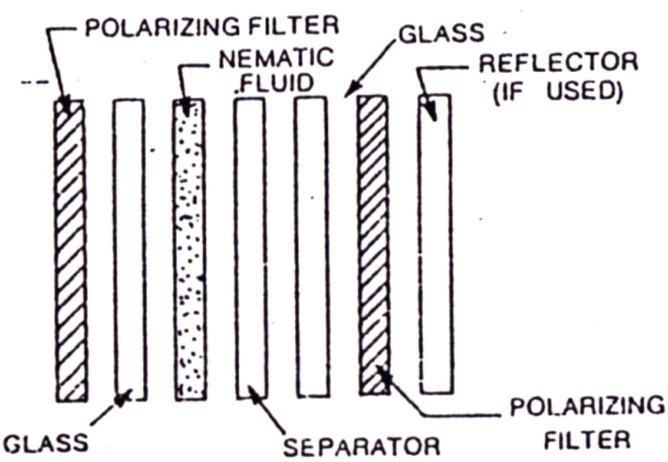

Figure 5.18: Typical construction of a liquid crystal display