High Energy Physics Phenomenology

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Work on making predictions in particle physics using Quantum Field Theory, and comparing to experimental data from colliders.

Main concentration on **Quantum ChromoDynamics** QCD the theory of the strong interaction.

Quarks (fermions) interact via the exchange of gluons (vector bosons) with the physics described by the $SU(3)$ gauge theory with Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -1/4 F^{\mu\nu}{}_a F_{\mu\nu a} + \sum_{f=1}^{n_f} \bar{q}_f \big(i\gamma^\mu D_\mu - m_f\big) q_f,$$

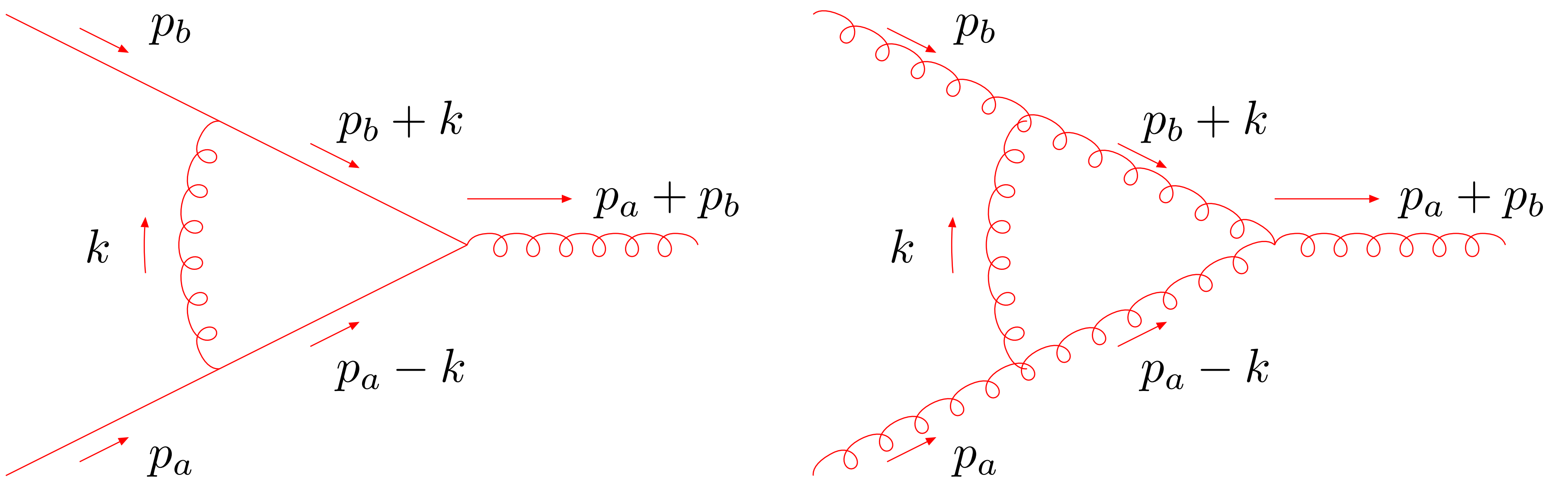
where the covariant derivative is defined by

$$D_\mu q_f = \partial_\mu q_f + i g_s A_{\mu a} 1/2 \lambda_a q_f$$

The sum over f is for the different quark flavours, up. down, strange, charm, bottom and top, each with different masses.

Can formulate Feynman rules to calculate particle interactions as a perturbation series in $\alpha_S = g_s^2/(4\pi)$

At first non-classical order obtain corrections to quark-gluon or gluon-gluon coupling of form



This results in integrals of the form

$$\mathcal{V} \sim \int \frac{d^4k}{(2\pi)^4} \frac{k\ k}{k^2(p_b+k)^2(p_a-k)^2} \rightarrow \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^4} \sim \int \frac{dk}{(2\pi)}\frac{1}{k}$$

when we consider the limit $k \rightarrow \infty$ in the loop. Leads to *ultraviolet* divergence.

In order to obtain a well-defined result must implement some *ultraviolet cutoff* Λ_0 above which QCD is no longer a reliable theory (e.g. Λ_0 is the scale of new physics).

Also introduce a physical *renormalization scale* μ_R – choose to be similar to scale of physics.

Subtract divergences like $\ln(\Lambda_0^2/\mu_R^2)$ and absorb into definition of **bare** parameters, leaving behind finite predictions in terms of physical **renormalised** parameters.

$$g_s^0 = g_s + g_s^3 C \ln(\Lambda_0^2/\mu_R^2) \qquad \sigma(\{p\}, g_s^0, \Lambda_0) \equiv \sigma(\{p\}, g_s)$$

Process known as renormalization. Long been proved that it can be applied successfully to all orders in QCD and rest of the Standard Model.

However, we have introduced artificial renormalization scale μ_R on which renormalised couplings, masses, *etc* depend, though dependence disappears (at all orders in physical quantities), e.g.

$$\frac{d}{d\ln\mu_R^2}\left(\alpha_S(\mu_R^2)\sigma_1(\{p\},\mu_R) + \alpha_S^2(\mu_R^2)\sigma_2(\{p\},\mu_R)\right) = \mathcal{O}(\alpha_S^3(\mu_R^2)).$$

By calculating previous diagrams representing coupling find that coupling satisfies evolution equation

$$\frac{d\alpha_S}{d\ln\mu_R^2} = -\beta_0\alpha_S^2 - \beta_1\alpha_S^3 + \cdots, \qquad \beta_0 = \frac{(11 - 2/3N_f)}{4\pi}$$

Negative β-function means strong at low scales but weaker at higher scales.

Ignoring the $\mathrm{O}(\alpha_s^3)$ corrections this may be solved

$$-\int_{\mu_0^2}^{\mu_R^2} d\,\ln\tilde{\mu}_R^2 = \frac{1}{\beta_0}\int_{\alpha_s(\mu_0^2)}^{\alpha_s(\mu_R^2)} \frac{d\,\tilde{\alpha}_s}{\tilde{\alpha}_s^2},$$

where μ_0 is some fixed scale. Hence,

$$-\ln(\mu_R^2/\mu_0^2) = \frac{1}{\beta_0}\left[\frac{1}{\alpha_s(\mu_0^2)} - \frac{1}{\alpha_s(\mu_R^2)}\right].$$

This leads to

$$\alpha_s(\mu_R^2) = \frac{1}{\beta_0} * \frac{1}{\ln(\mu_R^2/\mu_0^2) + \frac{1}{\beta_0\alpha_s(\mu_0^2)}}.$$

From this expression we can indeed see that $\alpha_s(\mu_R^2)$ decreases as μ_R^2 increases, and that $\alpha_s(\mu_R^2) \to 0$ as $\mu_R^2 \to \infty$. However, the definition relies on an arbitrary boundary condition for the coupling at some fixed scale μ_0^2.

It is simpler, and more illustrate to rewrite the solution for $\alpha_s(\mu_R^2)$ slightly. It may be expressed as

$$\alpha_s(\mu_R^2) = \frac{1}{\beta_0} * \frac{1}{\ln(\mu_R^2) - (\ln(\mu_0^2) - \frac{1}{\beta_0 \alpha_s(\mu_0^2)})}.$$

Defining a scale Λ_{QCD} by

$$\ln(\mu_0^2) - \frac{1}{\beta_0 \alpha_s(\mu_0^2)} = \ln(\Lambda_{QCD}^2),$$

Λ_{QCD} is the value of μ_0^2 for $\alpha_s(\mu_0^2) \to \infty$. Results in the solution.

$$\alpha_s(\mu_R^2) \approx \frac{4\pi}{(11 - 2/3N_f)\ln(\mu^2/\Lambda_{QCD}^2)}$$

Binds partons into hadrons at low scales, i.e. why QCD is theory of strong force. But can do perturbative calculations at higher scales, i.e. scales $\mu_R^2 \gg \Lambda_{QCD}^2$.

In practice $\Lambda_{QCD} \sim 0.3\mathrm{GeV}$, mass scale of hadrons.

Perturbation theory for a few GeV and above.

But hadrons are bound together by the strong force, described by nonperturbative physics.

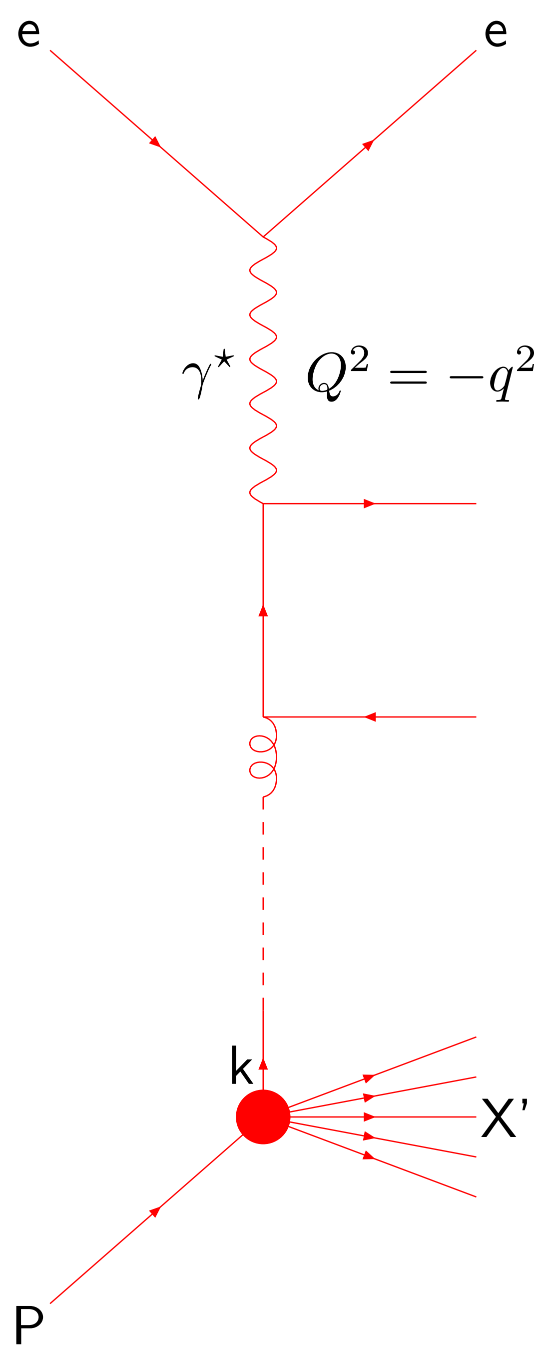
Most important particle colliders use hadrons – HERA was an ep collider, the Tevatron is a $p\bar{p}$ collider, the LHC (large hadron collider) at CERN is a pp collider.

Consider scattering of electrons of protons, which then fragment

$Q^2 = -q^2$ – Scale of scattering

$x = \frac{Q^2}{2P\cdot q}$

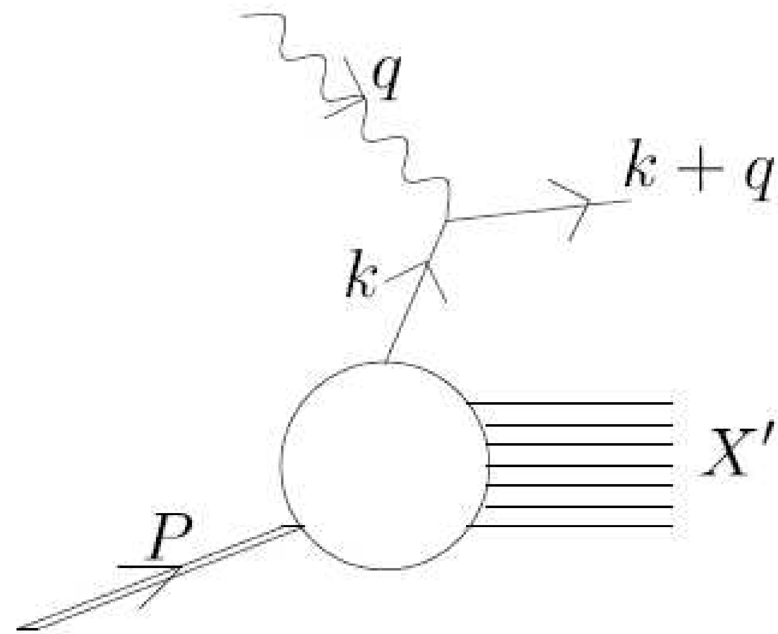
Factorization Theorem – separates processes into nonperturbative parton distributions and coefficient functions for the particular process.



coefficient function
$C_i^P(Q^2, x, \alpha_S)$

parton distribution
$f_i(x, Q^2, P, k, p_{X'}, \alpha_S)$

Consider LO DIS as in figure.



On-shell parton $\delta((k+q)^2)$.

In light-cone kinematics and for Q^2 large $\rightarrow \delta(k/P - x)$, i.e. x is fractional momentum of proton carried by parton.

Leads to parton density.

$$q_f(x) = \frac{1}{2P} \int d^4k \, \delta\left(\frac{k}{P} - x\right) \mathrm{tr}\left(\gamma \Gamma_f(P,k)\right)$$

$$\Gamma_f(P,k)_{\beta\alpha} = \sum_{X'} \delta^4(P - k - p_{X'}) \, \langle P|\bar{q}_{f\alpha}|X'\rangle \, \langle X'|q_{f\beta}|P\rangle,$$

where $q_f(x)$ represents the probability to find a quark of falvour f carrying a fraction x of the momentum of the hadron.

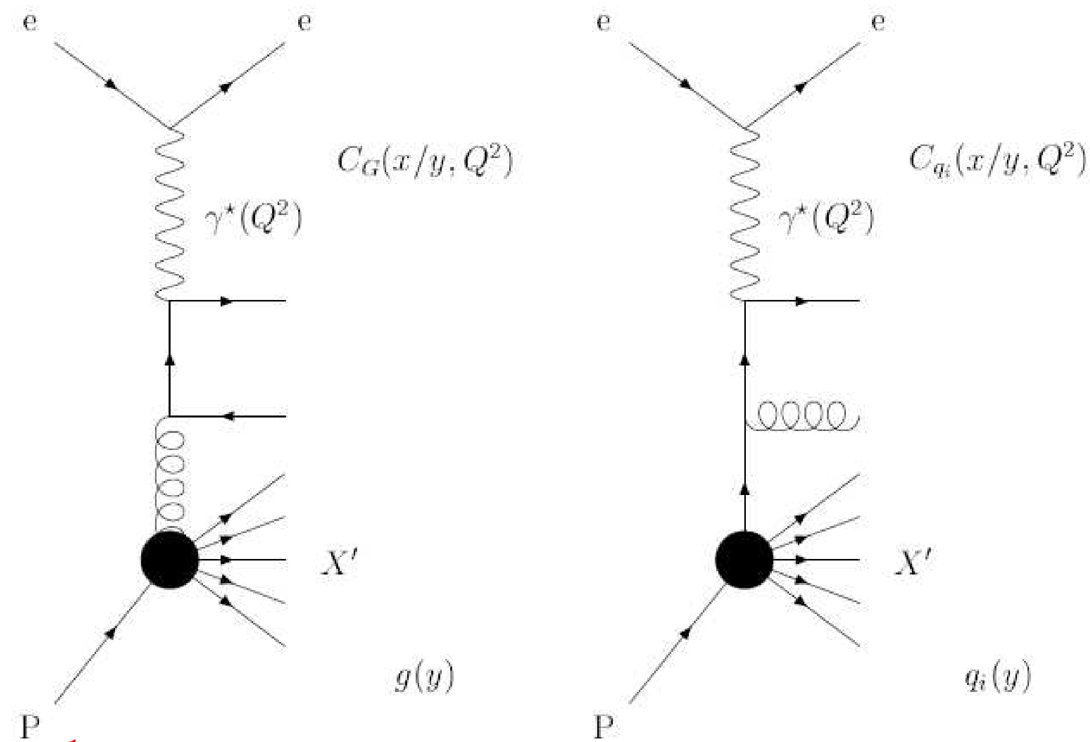
Corrections to above of size $\Lambda^2_{\mathrm{QCD}}/Q^2$.

Beyond LO contributions from extra partons in final state.

However, emission of *soft* or *collinear* final state massless partons (gluon, $m_u, m_d, m_s \ll \Lambda_{QCD}$) lead to *infrared divergences*.

Regularised in perturbative coefficient functions by *factorisation* scale μ_F.

Terms of form $P_{ij}(y)\ln(\mu_F^2/m_q^2)$ absorbed into parton distributions,



$$\sigma(ep \to eX) = \sum_{i=q_f,\bar{q}_f,g} \int_x^1 \frac{dy}{y} C_i\left(\frac{x}{y}, \frac{Q^2}{\mu_F^2}; \alpha_S\right) f_i(y, \mu_F^2)$$

The partons are intrinsically nonperturbative. However, once μ_F^2 is large enough they do evolve with in a perturbative manner.

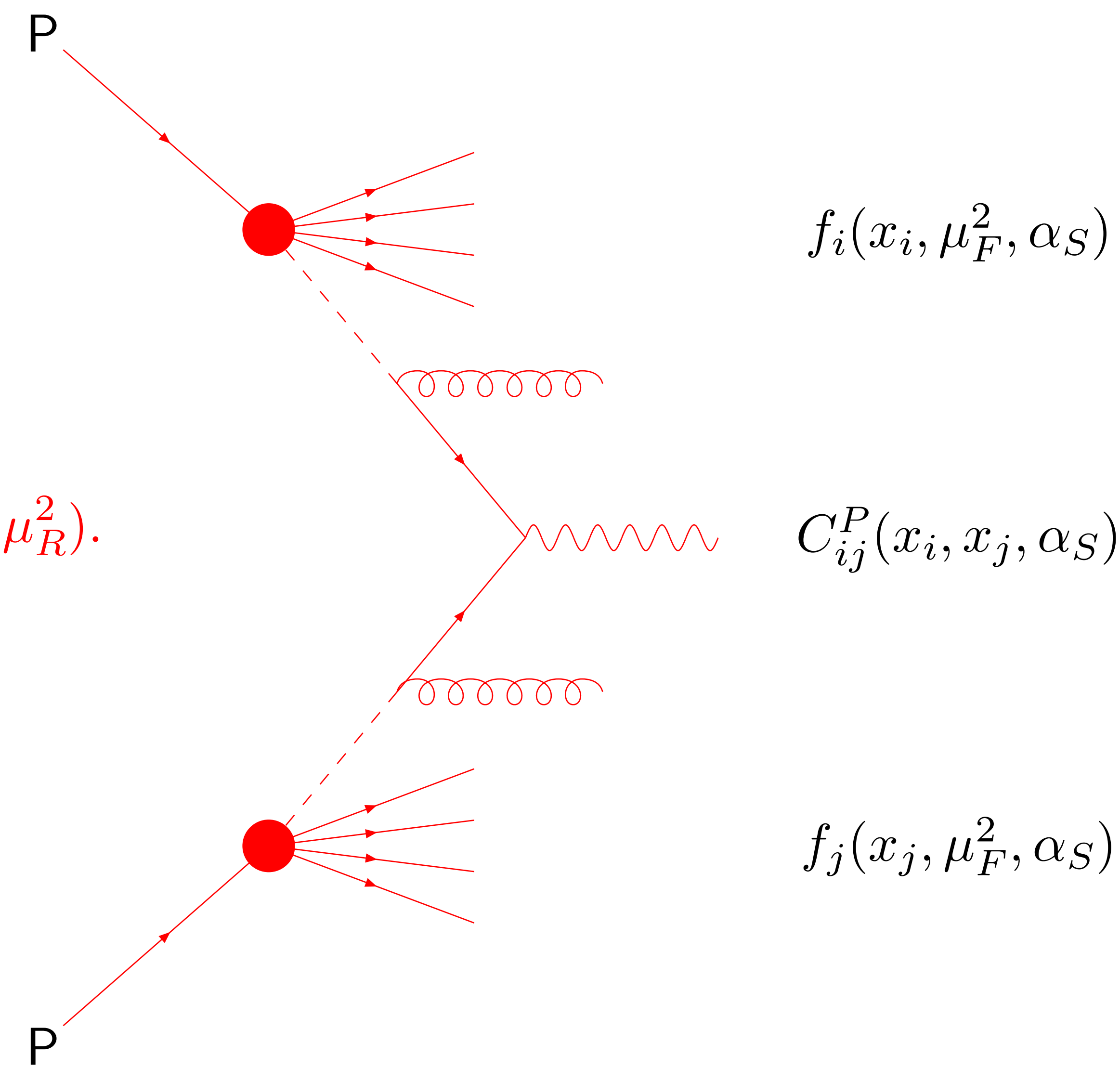
$$\frac{df_i(x, \mu_F^2, \alpha_S)}{d\ln\mu_F^2} = \sum_j P_{ij}(x, \alpha_S) \otimes f_j(x, \mu_F^2, \alpha_S)$$

where the splitting functions $P_{ij}(x, \alpha_S)$ describing how a parton splits into more partons are calculable order by order in perturbation theory.

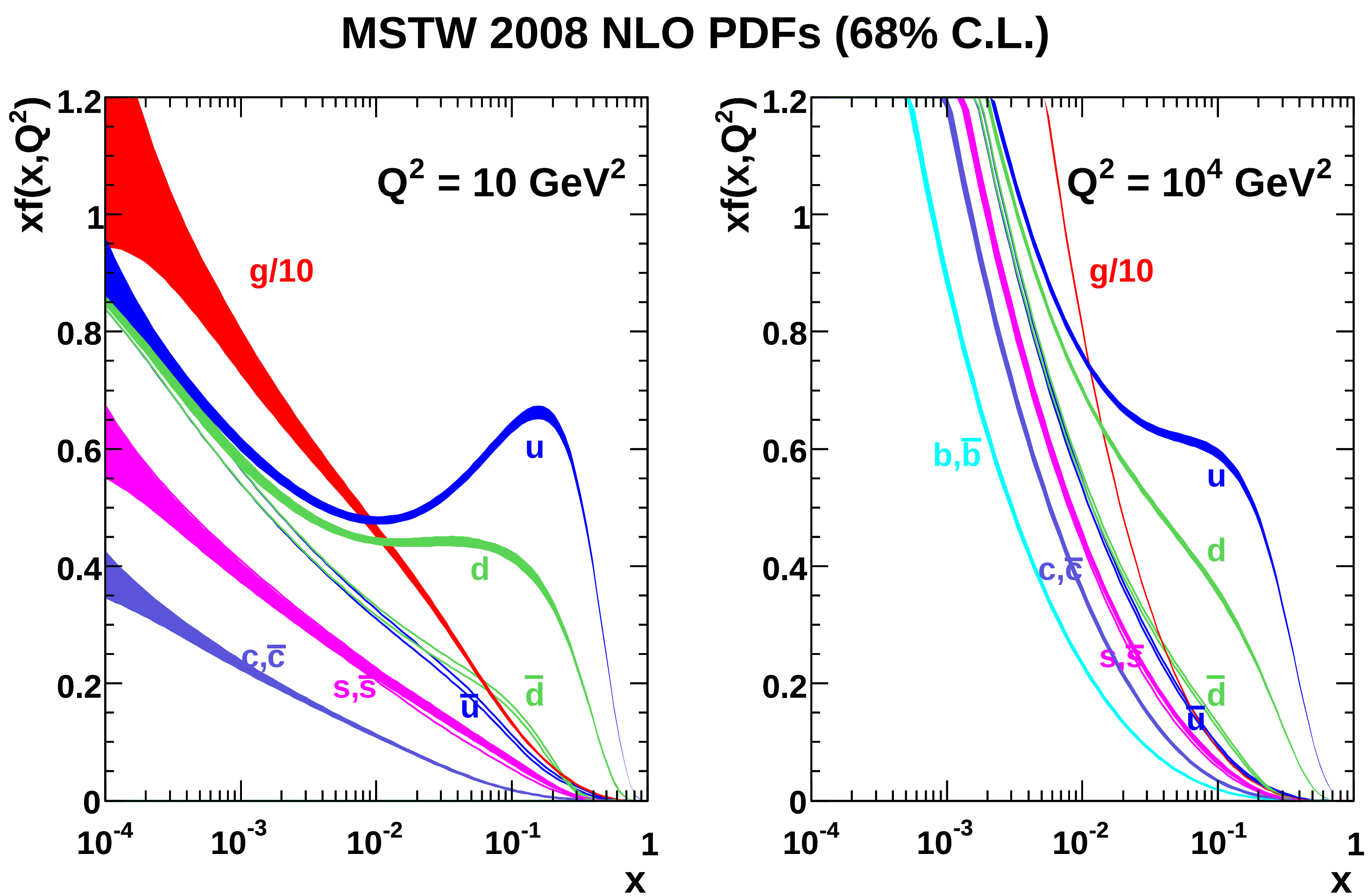
The coefficient functions $C_i^P(x, \alpha_S, Q^2/\mu_F^2)$ are process dependent (new physics) but are calculable as a power-series in $\alpha_S(\mu_R^2)$.

$$C_i^P(x, \alpha_S(\mu_R^2)) = \sum_k C_i^{P,k}(x)\alpha_S^k(\mu_R^2).$$

Since the parton distributions $f_i(x, \mu_F^2, \alpha_S)$ are process-independent, i.e. **universal**, once they have been measured at one experiment, one can evolve and predict many other scattering processes.



Leading member of MSTW (previously MRST) group. Use all available data – more than 30 different types of data set, and most up-to-date QCD calculations to determine parton distributions and their consequences. A large-scale, ongoing project. Vital input for hadron collider physics – used by experiments and theorists worldwide.



Heavy Quarks

charm $\sim 1.5\mathrm{GeV}$, bottom $\sim 4.3\mathrm{GeV}$, top $\sim 175\mathrm{GeV}$. Two distinct regimes:

Near threshold $Q^2 \sim M_H^2$ massive quarks not partons. Created in final state. However, $\ln(Q^2/m_H^2)$ divergences.

High scales $Q^2 \gg M_H^2$ massless partons. Behave like up, down, strange. Sum $\ln(Q^2/M_H^2)$ terms via evolution.

$$
\begin{aligned}
F(x,Q^2) &= C_k^{FF}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2) \\
&= C_j^{VF}(Q^2/m_H^2) \otimes f_k^{n_f+1}(Q^2), \\
&= C_j^{VF}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^{n_f}(Q^2),
\end{aligned}
$$

Perturbative matrix elements $A_{jk}(Q^2/m_H^2)$ containing $\ln(Q^2/m_H^2)$ terms relate $f_k^{n_f}(Q^2)$ and $f_k^{n_f+1}(Q^2) \rightarrow$ correct evolution for both.

$C_j^{VF}(Q^2/m_H^2)$ only uniquely defined in massless limit $Q^2/m_H^2 \rightarrow \infty$.

Have developed a theoretically correct method, Thorne-Roberts variable flavour number scheme (**TR-VFNS**) by imposition of physically motivated constraints $\rightarrow$ precise definition of parton distributions and scattering at all scales.

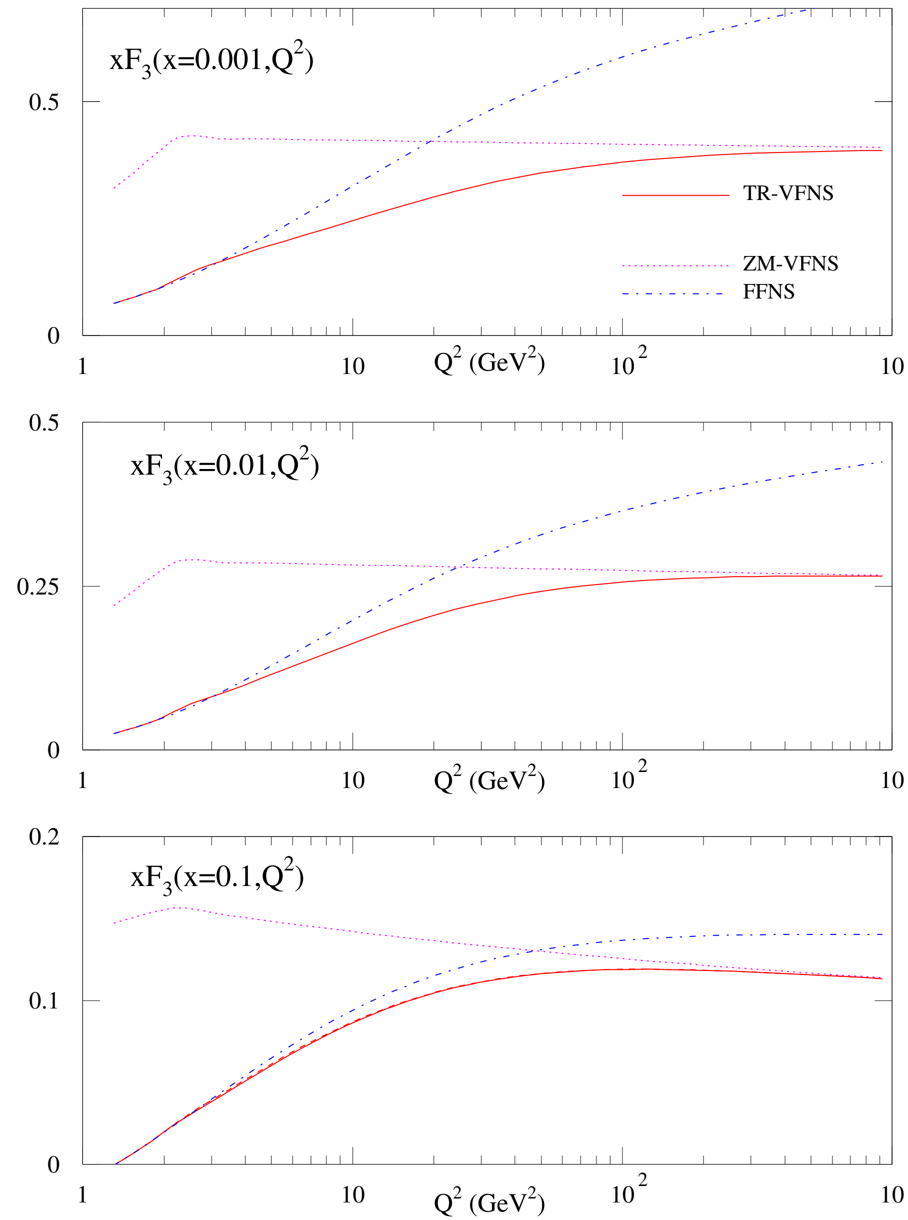
Extrapolation between the two simple kinematic regimes for xF_3 measured using neutrino scattering at NuTeV.

Widely used for structure functions.

In principle defined for $p\,p \rightarrow X + F_Q$ and other processes. In practice details require further work.

Can also be extended for more exclusive final states (not described too well for charm at HERA).

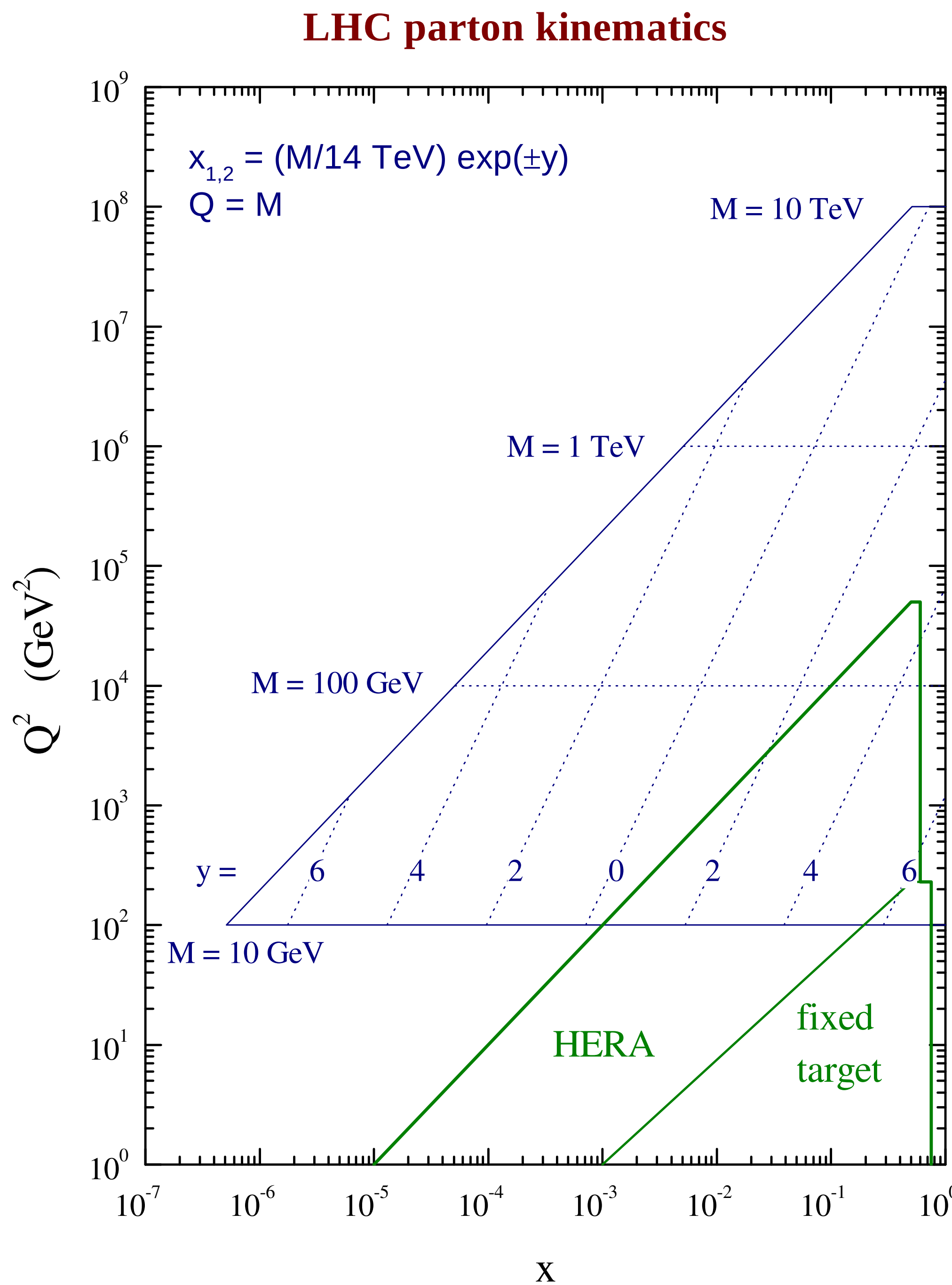
Important to test c-quarks and b-quarks at Tevatron, HERA and LHC.



Small x

Small x parton distributions, corresponding to very high-energy scattering, are interesting within QCD.

Also vital for understanding the standard production processes at the LHC, and perhaps some of the more exotic ones.



Small x Theory

It is known that at each order in α_S each splitting function and coefficient function obtains an extra power of $\ln(1/x)$ (some accidental zeros in P_{gg}), i.e.

$$P_{ij}(x, \alpha_s(Q^2)), \quad C_i^P(x, \alpha_s(Q^2)) \sim \alpha_s^m(Q^2) \ln^{m-1}(1/x).$$

$\rightarrow$ no guarantee of convergence at small x!

$$x < 0.01, \qquad \ln(1/x) > 5, \qquad \rightarrow \alpha_S \ln(1/x) > 1.$$

The global fits usually assume that this turns out to be unimportant in practice, and proceed regardless.

Fits work fairly well at small x, but could be better.

Some predictions unstable from LO $\rightarrow$ NLO $\rightarrow$ NNLO.

Try alternative perturbative organisation – resummation of leading $\ln(1/x)$ terms and running coupling effects (resummation of β_0 terms). Obtained by solving running-coupling BFKL equation for unintegrated gluon $f(k^2)$.

$$f(k^2, x) = f_I(Q_0^2) + \int_x^1 \frac{dx'}{x'} \bar{\alpha}_S(k^2) \int_0^\infty \frac{dq^2}{q^2} K_0(q^2, k^2) f(q^2, x)$$

Solve using double Mellin transformation w.r.t. Q^2 and $\ln(1/x)$. Obtain factorised solution for gluon (N conjugate to $\ln(1/x)$).

$$\mathcal{G}(Q^2, Q_0^2, N) = \mathcal{G}_E(Q^2, N)\mathcal{G}_I(Q_0^2, N) + \mathcal{O}(Q_0^2/Q^2)$$

$\mathcal{G}_I(Q_0^2, N)$ input at low scale $Q_0 \sim \Lambda_{QCD}$. Contaminated by infrared physics.

$\mathcal{G}_E(Q^2, N)$ completely controls evolution with Q^2. Calculable so it is possible to obtain splitting functions unambiguously.

Results in splitting functions of the form

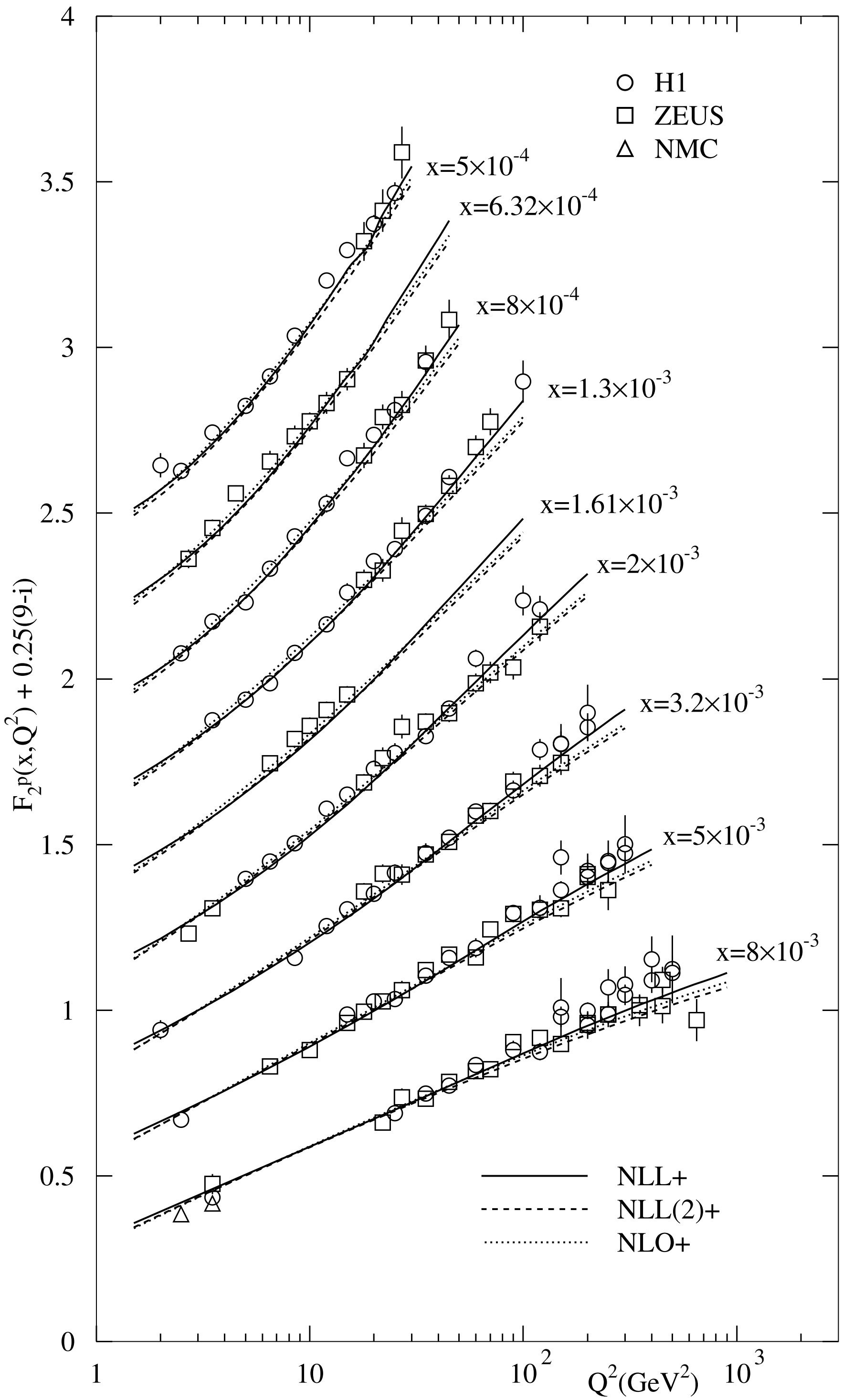
$$xP_{gg}(x, Q^2) = \sum_{n=1}^{\infty} \sum_{m=0}^{n-1} a_{nm} \alpha_S^n(Q^2) \ln^{n-1-m}(1/x) \beta_0^m.$$

The series is asymptotic but well defined ($\sim (-1)^n n!$).

Leads to better fit than NLO-in-α_S, particularly in terms of $dF_2(x, Q^2)/d\ln Q^2$.

Improvement of fit to small x HERA data (within global fit), due to $\ln(1/x)$ and β_0 double resummation compared to standard NLO in α_S.

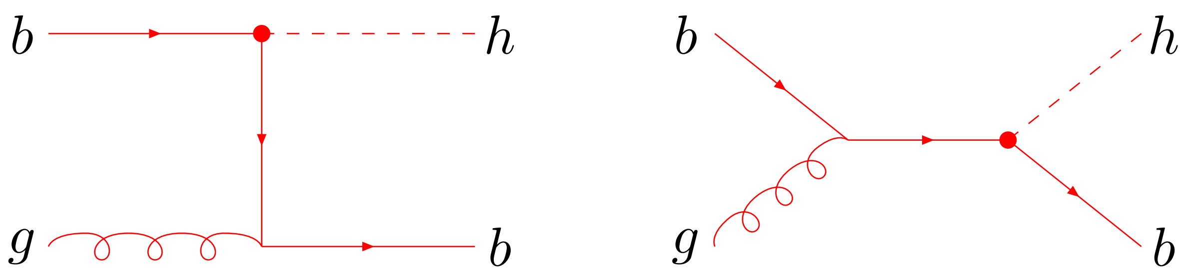
Difference when evolved to LHC region.



main LHC region

Example of need to understand both heavy flavours and small x physics for LHC.

Consider bottom production along with a Higgs boson.



In Standard Model tiny since Higgs-bottom coupling $g_{b\bar{b}h} = m_b/v$, (v Higgs vacuum expectation value.) $m_b = 4.5\mathrm{GeV}$, $v = 246\mathrm{GeV}$.

In Minimal Supersymmetric Standard Model two Higgs doublets coupling separately to d-type and u-type quarks. Expectation values v_d and v_u.

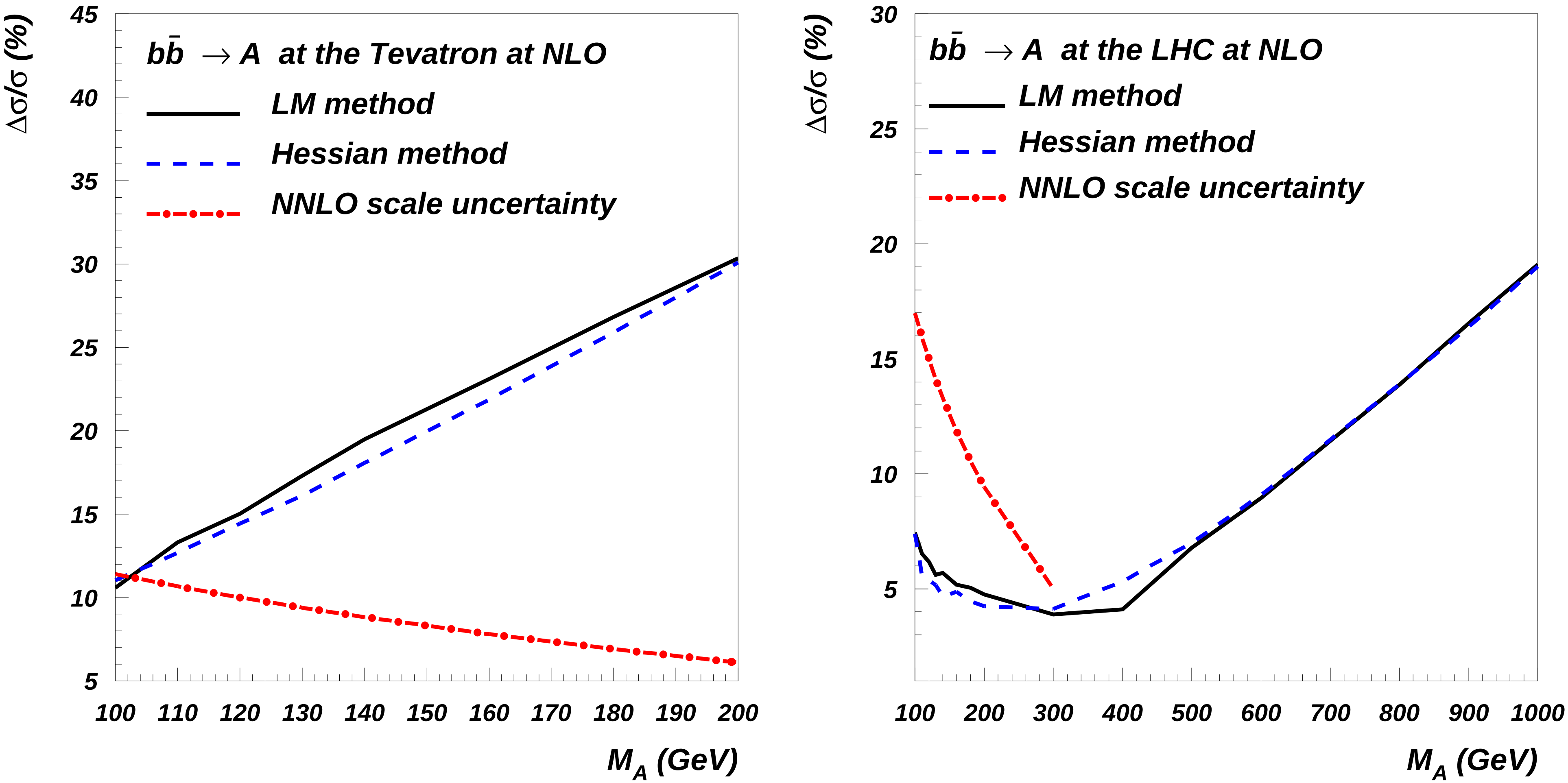
Ratio $\tan\beta = v_u/v_d$.

Enhancement of Higgs-bottom coupling

$$g_{b\bar{b}h} \propto \frac{g^{SM}_{b\bar{b}h}}{\cos\beta}.$$

Bounds from LEP, $\tan\beta$ large $\rightarrow$ $\cos\beta$ small. Enhancement of Higgs-bottom coupling.

Production of supersymmetric Higgs depends on parton uncertainties, heavy flavour procedure and high-energy (small-x) physics.



Summary.

Phenomenology relevant for the Extremely recent (HERA), existing (Tevatron) and existing/upcoming (LHC) hadron colliders (and others).

Main emphasis on QCD-related issues at present, with full understanding of initial state structure with implications for final state physics.

Concentration on theory of heavy flavours and small x physics. Intend to look more at Electroweak final states and effects.

Necessary for understanding of any of the physics – Standard Model or exotic – at these colliders.