We have argued that time plays a unique role in quantum mechanics. It is unlike other observables and one cannot naively assume it to be measurable. We have examined a number of different types of measurements of the time of an event, including measurements which involve continual monitoring of the system, coupling to physical clocks, measuring of current operators, and time-of-arrival operators. These various types of measurements given different results, and there does not appear to be any canonical method for measuring the time of an event.

In the context of the time-of-arrival t_A, we have found a basic limitation on the accuracy (as opposed to uncertainty) that t_A can be determined reliably: $\delta t_A > 1/\bar{E}_k$. This limitation is quit different in origin from that due to the uncertainty principle; here it applies to the inference of the value of time for a *single* event. Furthermore, unlike the kinematic nature of the uncertainty principle, in our case the limitation is essentially dynamical in its origin; it arises when the time-of-arrival is measured by means of a continuous interaction between the measuring device and the particle.

While we know of no formal proof that this relation holds for time-of-arrival, our arguments are fairly general in nature. For the case of traversal time, we have argued that the limitation does not depend on any particular measurement procedure.

We have also argued that monitoring whether the particle is at the location of arrival x_A at various times, and also measuring the current operator, do not allow one to construct a probability distribution which one could interpret as representing the probability that the particle will arrive at a certain time.

We would also like to stress that continuous measurements differ both conceptually and quantitatively from a measurement of the time-of-arrival operator. While the time-of-arrival operator is a formally constructed operator which can be measured by an impulsive von-Neumann interaction, continuous measurements are much closer to actual experiments. Furthermore, we have seen that the result of these two measurements do

not need to agree. In particular, at high accuracy, continuous measurements give rise to entirely different behavior – the particle never arrives. The time-of-arrival on the other hand, can be measured to any accuracy.

However, the time-of arrival operator is not self-adjoint. Attempts to modify the time-of-arrival operator in such a way as to make it self-adjoint result in the problem that the particle does not arrive on time with probability 1/2. Operators which classically might give the time of an event cannot be given a physical interpretation. While several authors [9][25] have maintained that the problems with defining an operator for the time of an event are technical, and can be circumvented by slightly modifying these operators. We have argued that probabilities in time are fundamentally different from traditional probabilities in quantum mechanics, and that there is a limitation on these measurements. As is the case with "time operators" [20] in closed quantum systems, the time-of-arrival operator has a somewhat limited physical meaning.

We have also seen that one cannot determine the temporal ordering of events to arbitrary accuracy. The limitation on these measurements is once again given by $1/\bar{E}$ where $\bar{E}$ is the typical total energy of the system. Nor can one prepare a two particle system in a state in which the two particles always arrive within a time $1/\bar{E}$ of each other.

However as with most research, this thesis raises more questions than it answers. Does a formalism exist where time is an element of reality? If not, does there exist a proof of the minimum inaccuracy bounds we have proposed? More intriguing, are some of the connections between this research, and the problem of time in quantum gravity and quantum cosmology.

In the Introduction we briefly discussed the canonical approach to quantizing gravity. One immediately encounters the problem that relative to the external parameter time in the Schrödinger equation, the state of the universe does not evolve. This is because the

system must satisfy constraints which are equivalent to reparametrization of the time variable.

The situation is somewhat analogous to being inside a box, and having some external observer weigh the box with high accuracy [40]. In order to keep the box at this fixed weight, the external experimenter cannot measure observables which evolve in time. Quantum mechanics also dictates that the observer will see people inside the box in a superposition of many different ages. This is because observables which would allow one to infer the time are (in a sense) conjugate to energy (they can't be exactly conjugate to the energy as we learned in Chapter 4). This gives us a rather interesting way to perform the Schrödinger cat experiment [41] (see the Figure at the beginning of this Chapter). Take an animal (Schrödinger's poodle, for example), stick her in a box, and weigh the box accurately. If the box is sufficiently isolated from the environment (a very difficult task), the poor poodle will be in a superposition of herself at different stages of her life. If we weigh the box very accurately, and later look at the age of the poodle, we will sometimes find that the poodle is so old that she died (or so young that she was just matter waiting to be born) – she is in many superpositions of being alive and dead. If we measure the weight of the box with infinite accuracy, then essentially any time we look inside the box, we will find nothing but poodle dust.

In general relativity, which describes the entire universe, all of us, observers and the observed, are in some sense living inside a box of fixed energy. In this regard, the recent results of Aharonov and Reznik [42] are interesting. They have shown that if one attempts to measure the energy of a closed system from within that system, then the time required to make this measurement must be at least $1/\delta E$ where δE is the precision with which one desires to make the measurement of energy. This result is very closely related to the measurements discussed in this thesis. Of course, an observer outside the box can measure the box's energy in as short a time and as accurately as desired [43].

These examples, which arise out of trying to understand quantum gravity, have led us to examine the role that time plays in ordinary quantum mechanics. We have argued that measurements of the time of an event are fundamentally different from ordinary observables in quantum mechanics. Certainly quantum mechanics is entirely self-consistent, and yet, questions remain about the role of time in the theory. Not only do we not understand the role of time in quantum gravity, but it appears that we do not quite understand the role of time in ordinary quantum mechanics. It is hoped that by understanding *quantum time*, some new insight can be gained into a quantum theory of gravity.