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# Modelling burglaries in streets

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**Summary.** Basic modelling of the evolution of burglary probabilities along a street or system of streets is described. The central cases investigated are for a single street, a system of streets in series, a system of streets in parallel, and the effects of security.

## 1 Introduction.

Interest in understanding and modelling certain aspects of crime has been growing significantly in recent times. Studies include those in [1, 2, 3, 4, 5, 6]. The present work arose from a workshop in 2007 and is motivated by links established with the Jill Dando Institute at UCL which in turn is linked closely with the London Metropolitan Police in research terms.

The modelling here is described in section 2 for a single street, then extended in section 3 to two or more streets in series and to junctions. This is followed by section 4 which is on special interactions concerned with streets in an in-parallel configuration. Section 5 addresses the modelling of security or watchfulness effects. The investigation is concentrated on the discrete versions of models rather than continuum ones. Also the methodology was tested for analytical cases and compared with limit continuum calculations. Section 6 provides further comments.

## 2 Basic model for one street

This is a simple first model for a rectilinear street consisting of  $K$  houses or other dwellings. The reasoning here is that the probability  $p_k$  of a burglary at the  $k^{\text{th}}$  house of the street evolves according to the differences between the probability at that house and those at its nearest neighbours. The integer  $k$  runs from 1 to  $K$ , and in addition superscripts  $i, i+1$  are to be used, standing for the values at the current time level and at the next time level in turn.

The typical time step in mind is one day say. Thus the normalised evolution equation has the form

$$p_k^{(i+1)} = p_k^{(i)} + \mu(p_{k+1}^{(i)} - p_k^{(i)}) + \mu'(p_{k-1}^{(i)} - p_k^{(i)}). \quad (2.1)$$

The coefficients  $\mu, \mu'$  are taken as constants and are expected to be positive, such that if the probability at  $k + 1$  (or at  $k - 1$ ) is higher than that at  $k$  then the latter is likely to increase in proportion during the next time step. For definiteness we then suppose the coefficients  $\mu, \mu'$  to be equal, effectively yielding a diffusivity of crime through the relation

$$p_k^{(i+1)} = p_k^{(i)} + \mu(p_{k+1}^{(i)} - 2p_k^{(i)} + p_{k-1}^{(i)}). \quad (2.2)$$

The case of unequal coefficients is also of interest however and is mentioned in the final section of the paper. Next we note that (2.2) is of course the diffusion equation in discretised form, controlling heat conduction for example; that is, the continuum limit of (2.2) for many houses is

$$\partial p / \partial t = \kappa \partial^2 p / \partial x^2, \quad (2.3)$$

where the coefficient  $\kappa = \mu(\Delta x)^2 / (\Delta t)$ , subject to the appropriate limiting process involving the effective step lengths  $\Delta x, \Delta t$  in space  $x$  and time  $t$  respectively. Development in space here corresponds to movement from one house to the next. Most of our concern in this article is with the form (2.2).

Initial conditions (set at  $i = 0$  for all  $k$ ) and boundary conditions (usually at  $k = 1, K$  for all positive  $i$ ) are imposed appropriately on (2.2). In addition a refined model of the single street case is currently under investigation in which the burglar is modelled as an agent. In brief, the probabilities evolve as above (and below) in the absence of a burglary. Concerning the probability of a burglary on day  $i$  (as a first step only one burglary is envisaged), the agent proceeds down the street from house 1 to 2, . . .  $k$  . . . and commits a crime with a certain probability which is a function of security etc, as considered in section 6. The crime itself acts to produce new initial conditions for the evolution in (2.2). An object-oriented C++ code for this is under development.

Figure 1 shows a sample solution of (2.2), in which the diffusive effect is clear as the discrete time  $i$  increases. The probabilities of crime are initially zero in all the houses, save House 6, where we model a crime having just occurred by raising the probability to the value of 0.5. This value may be artificially high, but it serves to illustrate the diffusion of probability of crime in a manner directly analogous to thermal diffusion. It can be seen how the probability of crime in neighbouring houses rises rapidly in close proximity to the burgled house and more slowly and steadily further away. It is worth remarking that, in the present discrete setting, if the initialised  $p$  values are set as nonzero only towards the middle of the street with  $p$  imposed as zero at the two ends then  $p$  remains zero near those ends for a finite time.

### 3 Two or more streets in series or with junctions

The first extension now to two streets in series, say street  $P$  (with  $K_p$  houses) leading in to street  $Q$  (with  $K_q$  houses), has the corresponding probabilities being  $p, q$ . Here  $p$  evolves in the form (2.2) with a coefficient now written  $\mu_p$  and  $q$  evolves in a similar form, namely

$$q_k^{(i+1)} = q_k^{(i)} + \mu_q(q_{k+1}^{(i)} - 2q_k^{(i)} + q_{k-1}^{(i)}), \quad (3.1)$$

where  $k$  now runs from 1 to  $K_p$  in (2.2) but from 1 to  $K_q$  in (3.1). The junction of the streets is at  $k = K_p$  in  $P$  street,  $k = 1$  in  $Q$  street. The un-joined ends are taken to have zero probability conditions,

$$p_1^{(i)} = 0, q_{K_q}^{(i)} = 0, \text{ for all } (i), \quad (3.2)$$

whereas a modelled interaction condition applies at the junction. This is

$$(q_2^{(i)} - q_1^{(i+1)}) = \alpha_3(p_{K_p}^{(i+1)} - p_{K_p-1}^{(i)}). \quad (3.3)$$

Here  $\alpha_3$  is a prescribed constant, reflecting proportionality between the probability rates on either side of the junction, with  $p_{K_p}, q_1$  being taken to be identical purely for convenience in these first studies. For certain situations, e.g. involving a change in housing type, it can be argued that  $\alpha_3$  should be the ratio of the  $\mu$  values but other situations such as at crossroads suggest a wider range of  $\alpha_3$  values can hold; again, taking continuity of  $p, q$  is open to some debate. The junction condition allows a type of leakage to occur from one street to the other.

Examples have been calculated for particular values of the coefficients involved and will be in [6]. For the scenario of all the  $q$ 's initially being zero and the  $p$ 's likewise except at a finite number of houses in the middle, we observe that  $Q$  street remains at zero probability until the diffusion in  $P$  street makes the  $p$  values near the junction become nonzero. Further, even after such a delay, an extreme value for the leakage coefficient in (3.3) can make the response in  $Q$  street grow dramatically.

A similar description holds (second) for more streets, say  $P, Q, R, S, \dots$ , in series:  $P$  leading in to  $Q$ , then  $Q$  in to  $R$  and so on. The evolution equation in each street is as in (2.2), (3.1), while the interaction conditions at each junction are (3.3) between  $p, q$ , then  $q, r$ , etc. Examples are in [6].

A similar account can also be made of junctions in which several streets join together, for instance with street  $P$  leading in to both  $Q$  and  $R$  at the same junction, or with a crossroads. Here again interaction of the kind in (3.3) seems reasonable as a first step.

### 4 Streets in parallel

When two streets  $P, Q$  are in effect in parallel it is considered that there is not necessarily any interaction of the above type directly between them; however

interaction may occur at a finite number of special positions, corresponding to an alleyway or parkland for instance offering possibly enhanced access. At such a position  $k$  the probability  $p_k$  at the house in  $P$  street is taken to be influenced positively by the probability  $q_k$  at the corresponding house in  $Q$  street and likewise for the effect of  $P$  street on  $Q$  street. In consequence the evolution system has the form

$$p_k^{(i+1)} = p_k^{(i)} + \mu_p(p_{k+1}^{(i)} - 2p_k^{(i)} + p_{k-1}^{(i)}) + \beta_p(q_k^{(i)} - \epsilon p_k^{(i)}), \quad (4.1)$$

$$q_k^{(i+1)} = q_k^{(i)} + \mu_q(q_{k+1}^{(i)} - 2q_k^{(i)} + q_{k-1}^{(i)}) + \beta_q(p_k^{(i)} - \epsilon q_k^{(i)}), \quad (4.2)$$

at such  $k$  values, where the access factors  $\beta_p$ ,  $\beta_q$  are constants which are generally positive.

The constant factor  $\epsilon$  plays an interesting role. If  $\epsilon$  is unity then the evolution remains as before dependent on various differences between probabilities, whereas if  $\epsilon$  is zero then there is a stronger linkage between the two streets. In fact exponential growth in time arises whenever  $\epsilon$  is between zero and unity. This can be seen readily if there is no significant diffusion in the system, so that the  $\mu$  values are negligibly small, since then (4.1), (4.2) give the property that at large times

$$p_k^{(i)}, q_k^{(i)} \text{ grow as } b^i, \text{ with } b - 1 = [-\epsilon(\beta_p + \beta_q) + \{\epsilon(\beta_p - \beta_q)^2 + 4\beta_p\beta_q\}^{\frac{1}{2}}]/2, \quad (4.3)$$

at the access positions  $k$ . Hence exponential growth is encountered ( $b > 1$ ) except when  $\epsilon$  is unity; the unit case is associated with all previous interactions addressed, by the way. Even if there is significant diffusion essentially the same conclusion is reached, by virtue of a single or double summation over all  $k$  values, depending on the end conditions, although the exponent is then altered from that in (4.3).

Numerical studies support the result (4.3) and its extension to three or more streets in parallel when they interact in the special access way. An example is presented in figure 2. This and other cases examined allow for special access occurring at a number of locations; thesis studies are being conducted by XY. It might be argued that the accesses for the current in-parallel situations should merely be treated as crossroads as in the previous section, but against that these accesses are believed to have a special status.

## 5 Effects of security

Extending the model to allow for security or watchfulness  $s_k$  in the same simple-minded way we first have the system

$$p_k^{(i+1)} = p_k^{(i)} + \mu_p(p_{k+1}^{(i)} - 2p_k^{(i)} + p_{k-1}^{(i)}) - a_3 s_k^{(i+1)}, \quad (5.1)$$

$$s_k^{(i+1)} = s_k^{(i)} + b_1 p_k^{(i)} - b_2, \quad (5.2)$$

where  $a_3, b_1, b_2$  are non-negative constants. The main idea is that security put in at the typical  $k^{\text{th}}$  house reduces the crime probability there, as reflected in (5.1), while any increase in crime probability tends to induce further security as represented in (5.2) but subject to security being reduced if there is negligible probability.

An example is presented in figure 3. It shows crime probability decreasing when security is imposed, along with neighbouring probability levels which may decrease or increase relatively depending on the parameters present. We note that under some circumstances the system can produce apparently unrealistic negative values for the  $p_k$  and /or the  $s_k$  at certain positions although this feature can be prevented by sensible adjustments of the model, one of which is discussed in the next section.

## 6 Final comments

- According to the modelling the crime probabilities mostly diffuse with time for the single street and multiple street cases, with the diffusion rates depending sensitively on the parameters involved. A possible exception is for the in-parallel scenarios described in section 3 which admit temporal growth of the probabilities.
- In the multiple street cases the junctions between streets can play a key role, as is well known in other network settings. We see this aspect in all the models of sections 3,4.
- For the most part linear effects have been addressed so far. An exception should probably be made in future studies for the setting of section 5 concerning security effects. The relations in (5.1), (5.2) could be better treated as giving growth or decay rates, which would make the interactions nonlinear in addition to ensuring that the probabilities and security levels remain sensibly non-negative.
- In the conference itself there were several interesting points made by audience members at the talk associated with this article. Most had already been accommodated in the research, including (first) the continuum version (2.3) and its solution properties and (second) the possibility of unequal coefficients in (2.1) which provokes a directional preference corresponding to a convective effect proportional to  $(\mu-\mu')$  in (2.2). Other comments are covered by this text or will be covered in later research including [6]. There are indeed many further issues to be studied.
- Acknowledgements. We thank especially Shane Johnson, Steve Bishop, John Ockendon and the audience for helpful comments and interest. JPC acknowledges gratefully a Royal Society Industry Fellowship.

## 7 Figures

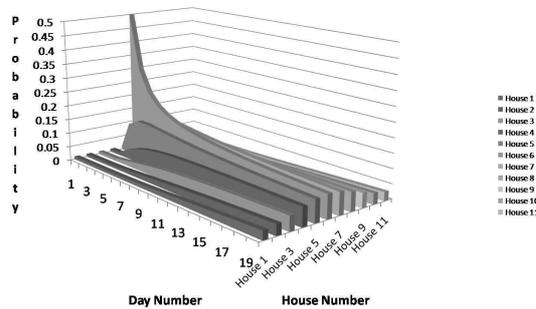


Fig. 1. The evolution of probability of crime in a street of 11 houses is shown.

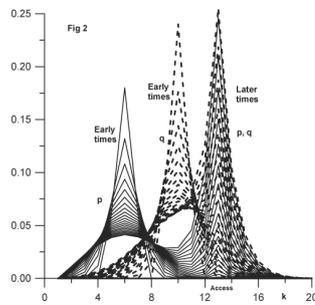


Fig. 2. For two in-parallel streets.

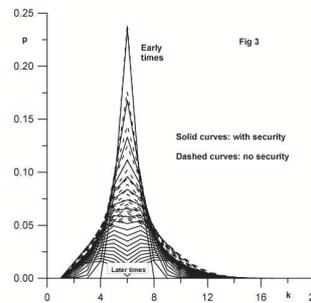


Fig. 3. Showing effects of security.

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