

M302 (Asymptotic Methods and Boundary Layer Theory)

<i>Year:</i>	2014–2015
<i>Code:</i>	MATHM302
<i>Level:</i>	Masters
<i>Value:</i>	Half unit (= 7.5 ECTS credits)
<i>Term:</i>	2
<i>Structure:</i>	3 hour lectures per week
<i>Assessment:</i>	90% examination, 10% coursework
<i>Normal Pre-requisites:</i>	MATH3301
<i>Lecturer:</i>	Dr JG Esler

Course Description and Objectives

This course is a natural follow-on to 3301, where the equations of incompressible viscous flow were derived, and many exact solutions obtained and discussed. Here we concentrate on the high Reynolds number limit, and interpret boundary-layer theory as the leading term of a rational approximation to the Navier-Stokes equations. The mathematical basis of such an interpretation is singular perturbation theory and as part of this course the method of matched asymptotic expansions will be taught in the context of ordinary differential equations. Subsequently various steady and unsteady viscous flows will be studied: these will include examples of internal and external flows and of jets and wakes. This is an advanced course that reflects the research interests of many of the Applied Mathematics staff.

Recommended Texts

Batchelor, G. K., *An Introduction to Fluid Mechanics*, CUP.
Hinch, E. J., *Perturbation Methods*, CUP.
Schlichting, H., *Boundary Layer Theory*, McGraw Hill.
Van Dyke, M., *Perturbation Methods in Fluid Mechanics*, Parabolic Press.
Rosenhead, L. (ed)., *Laminar Boundary Layers*, OUP.
Nayfeh, A., *Introduction to Perturbation Techniques*, Wiley.
Sobey, I. J., *Introduction to Interactive Boundary Layer Theory*, OUP.

Detailed Syllabus

- Ideas of asymptotics. Exact solutions of the Navier-Stokes equations that exhibit a boundary layer at small values of the viscosity.
- The boundary-layer equations in the spirit of Prandtl. Scaling, non-dimensionalisation and Reynolds number. Prandtl's boundary-layer equations in two dimensions deduced by order-of-magnitude arguments. Blasius solution: flow past a flat plate.
- Introduction to perturbation theory. Regular and singular perturbations. Matched asymptotic expansions. Examples from algebraic equations and ordinary differential equations. The classical boundary-layer equations of Prandtl as the leading term in a matched asymptotic expansion.
- Exact solutions of the classical boundary-layer equations. Examples selected from: Flow past a wedge; Falkner Skan; Far wake of a flat plate; Two-dimensional jet; Prandtl transformation; Axisymmetric flows: Mangler's transformation. Ekman and Stewartson layers.

- Separation in adverse pressure gradients. Concept of and occurrence in steady and time-dependent flows. Form of skin friction near separation point: Goldstein singularity.
- Introduction to interactive boundary layers. Goldstein near wake. Triple deck theory.