

M205 (Topology and Groups)

<i>Year:</i>	2014–2015
<i>Code:</i>	MATHM205
<i>Level:</i>	Masters
<i>Value:</i>	Half unit (= 7.5 ECTS credits)
<i>Term:</i>	1
<i>Structure:</i>	3 hour lectures per week
<i>Assessment:</i>	100% examination
<i>Normal Pre-requisites:</i>	MATH7102, MATH7202
<i>Lecturer:</i>	Dr L Louder

Course Description and Objectives

The course starts with the basics of general topology. The fundamental group is then defined and studied, with an emphasis on examples: graphs and their fundamental groups, free groups, are especially motivating. Fundamental groups of general complexes are discussed, along with group presentations. Some applications will be given, such as winding number and the Brouwer fixed point theorem.

Students should finish the course able to compute: bases for the subgroup of a free group corresponding to an explicit covering space of a graph; presentations for fundamental groups of cell complexes; Cayley graphs of finite groups.

The course could be taken as a complement to the course *Algebraic Topology*.

Recommended Texts

- (i) D. Cohen, *Combinatorial Group Theory: A Topological Approach*, Student Texts 14 (London Mathematical Society, 1989), Chapters 1–7.
- (ii) A. Hatcher, *Algebraic Topology* (CUP, 2001), Chapter. 1.
- (iii) M. Hall, Jr, *The Theory of Groups* (Macmillan, 1959), Chapters. 1–7, 12, 17.
- (iv) D. L. Johnson, *Presentations of Groups*, Student Texts 15 (Second Edition, London Mathematical Society, Cambridge University Press, 1997). Chapters. 1–5, 10, 13.
- (v) W. Magnus, A. Karrass, and D. Solitar, *Combinatorial Group Theory* (Dover Publications, 1976). Chapters. 1–4.
- (vi) William S. Massey, *A Basic Course in Algebraic Topology* (Springer-Verlag, 1991), Chapters 2–6.
- (vii) John Stillwell, *Classical Topology and Combinatorial Group Theory* (Springer-Verlag, 1993).

Detailed Syllabus

- Topological spaces, simple examples and properties: compactness, connectedness, continuous maps and homeomorphisms (in comparison with metric spaces from Analysis 4).

- The quotient topology; cell complexes; more examples, including simplicial complexes (recalled from Algebraic Topology).
- Homotopy and the definition of the fundamental group. The statement of the fundamental group of the circle, and applications (eg winding number, review of the Brouwer Fixed Point Theorem).
- Covering spaces: path lifting and homotopy lifting. Applications to computing fundamental groups, eg of the circle.
- More examples: graphs and free groups. Existence and uniqueness of coverings, the universal cover, regular coverings and normal subgroups. Cayley graphs.
- Applications to the group theory of the free group: reduced words, the Nielsen–Schreier Theorem, computing bases for subgroups of free groups.
- The Seifert–van Kampen Theorem, free products, free products with amalgamation.
- Group presentations: examples and computations.